Amending the Constitution:

Article V and The Effect of Voting Rule Inflation

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Abstract

It is generally believed that amendment to the United States Constitution has proven to be unduly onerous. Consistent with that belief, we show that, even if the original Article V amendment requirement were deemed optimal, the *effective* super-majority required by Article V has substantially increased over time, as the size of the relevant voting bodies has increased. We demonstrate this and other comparative statics in a general model of supermajority voting rules. Calibrating the model based on the optimality of the original requirements, we show that the requirement in Article V that an amendment be supported by 2/3 of each House of Congress and 3/4 of State Legislatures would now be equivalent to a requirement of support by 53% of the House, 59% of the Senate, and 62% of State Legislatures. Without this "voting rule inflation" effect, we find that several proposed Amendments to the Constitution would likely have passed. Voting rule inflation is thus shown to be an important consideration for any constitutional designer.

"**Article V**: *The Congress, whenever two thirds of both Houses shall deem it necessary, shall propose Amendments to this constitution, or, on the Application of the Legislatures of two thirds of the several States, shall call a Convention for proposing Amendments, which, in either Case, shall be valid to all Intents and Purposes, as Part of this Constitution, when ratified by the Legislatures of three fourths of the several States, or by Conventions in three fourths thereof, as the one or other Mode of Ratification may be proposed by Congress; Provided that no Amendment which may be made prior to the Year One thousand eight hundred and eight shall in any Manner affect the first and fourth clauses in the Ninth Section of the first Article; and that no State, without its Consent, shall be deprived of its equal Suffrage in the Senate.*"

"*...for it is, in my conception, one of those rare instances in which a political truth can be brought to the test of a mathematical demonstration.*" Alexander Hamilton, Federalist No. 85

1   Introduction

Legal rules and institutions are fundamental determinants of economic outcomes. Moreover, the fact that *constitutions* can have important effects on economic outcomes is now well established (Persson and Tabellini (2003)). One important part of a constitution is its amendment provisions, which govern the way in which the constitution may adapt to changing and unforeseen circumstances. Amendment provisions need to strike a delicate balance. On the one hand, a permissive amendment rule may undermine the very purposes of entrenched constitutional protections; on the other, a stringent one may hinder adaptation

to changing circumstances.

In adopting Article V of the United States Constitution, the Framers knew the importance of the amendment provision. It was widely understood by the Framers that the requirement of unanimous state consent to amend the Articles of Confederation had been a major obstacle to the capacity of the Confederation to respond to changing circumstances.[1] For instance, several proposals between 1781 and 1783 to give Congress a power of taxation had been defeated by the veto of a single state.[2] Conversely, the Framers were also concerned to guard against what Madison described in Federalist 43 as "the extreme facility, which would render the Constitution too mutable". The initial proposal to require ratification by only two-thirds of the states was therefore abandoned in favor of an amendment proposed by Madison to Article V, requiring approval by three-quarters of the states (Lash (1994)).

The final requirements of Article V of the Constitution, in which either the states (acting through their legislatures or conventions) or 2/3 of both Houses of Congress may propose amendments, and amendments must be ratified by 3/4 of the states, were seen by the Framers to strike the optimal balance between concerns about flexibility and stability.

In contemporary constitutional scholarship, however, there is a widely held perception that the Article V process has proved excessively difficult (Griffin (1995b), Lutz (1995)). Those scholars point to the fact that no proposal for amendment has ever been initiated by the states (Stokes-Paulsen (1993), Levinson (1995)). The effect, in practical terms, is that Constitutional amendment will always require approval of 2/3 of both Houses and 3/4 of the states (Levinson (1995)).

The result has been that many constitutional scholars have focused attention on more informal mechanisms through which Constitutional change may be achieved (Ackerman (1991), Ackerman (1996), Amar (1994), Griffin (1995a), Levinson (1995), Strauss (2001)). While

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[1]See for example., Charles Pickney, Observations on the Plan of Government Submitted to the Federal Convention, reprinted in The Records of the Federal Convention 1787 (Max Farrand ed., 1937); Alexander Hamilton, The Records of the Federal Convention 1787 (Max Farrand ed., 1937); The Federalist No. 85 (Hamilton); James Madison, The Virginia Ratification Debates, reprinted in The Debates of the Several State Conventions on the Adoption of the Federal Constitution (Jonathan Elliott, ed., 1836).

[2]Rhode Island and New York respectively: see Lash (1994).

continuing to employ Article V processes in a highly visible way, political actors have also increasingly turned to alternative mechanisms, such as control of Supreme Court appointments, as important for effecting Constitutional change (Strauss (2001)),.

We develop a model of super-majority voting rules in which the optimal rule depends on the distribution of preferences, number of decision-makers, and the importance of the issues at stake. We identify two possible sources of "voting rule inflation": first, changes in the underlying distribution of voter preferences and second, changes in the size of the voting pool. We focus, however, on the second source of change, considering that changes in the heterogeneity of voter preferences are notoriously difficult to measure. We hold the distribution of voter preferences constant, and analyze the second potential source of change by use of comparative statics.

Using these tools, we develop a general model of super-majority voting rules and show that increasing the number of voters on a Constitutional amendment, both in terms of representatives voting in Congress and the number of states voting, materially increases the difficulty of Article V amendment. This conforms to the conclusion of Lutz (1995), but gives a theoretical explanation for this fact, and a way to quantify the magnitude of voting rule inflation.

Calibrating the model by using parameter values which would make the 1789 Article V (2/3 of the Senate, 2/3 of the House, 3/4 of the States) rule optimal, we show that for the 2008 voting pool, the optimal Article V voting rule would in fact have been 59%, 53% and 62% respectively.

There is a simple intuition for why the voting rule inflation we identify has occurred. In the context of our model, the optimal super-majority rule is determined by a trade-off between the blocking power of a small minority of voters and the possibility of a majority taking an action which adversely impacts a minority.[3] On the one hand, a permissive super-majority rule is desirable because it reduces the probability of a small minority blocking

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[3]We rule out the possibility of monetary transfers/side payments.

a change to the social decision which could benefit a large number of others ("blocking"). On the other, a strict rule is also desirable because it reduces the chance that an individual will be affected by the majority making a change to the social decision which hurts them a great deal, but benefits the majority, even if only by a very small amount ("oppression"). The trade-off between these two considerations determines the optimum. The trade-off may also change over time, the number of voters in a particular body increases. As the size of the pool of voters increases, the probability of being in a potentially oppressed minority decreases, and the probability of being blocked increases. Unless adjusted downward, any given super-majority rule will therefore over-protect against the danger of oppression, as compared to blocking, over time: or be subject to a form of constitutional inflation.

In the Article V context itself, we acknowledge that it is difficult to determine what the precise effect of this inflation has been. Members of Congress may have engaged in strategic voting when supporting proposed amendments to the Constitution in light of increasing obstacles to Article V amendment. However, we identify four proposed amendments to the Constitution since 1973 which have gained sufficiently strong support in Congress as to have been potential candidates for successful amendment, but for the inflationary effect identified. Those amendments are the Equal Rights Amendment, the Balanced Budget Amendment, the Flag Burning Amendment and a same-sex marriage amendment.

At a more general level, we also suggest that the inflationary effect identified implies that any static super-majority rule will not be optimal at all points in time. It can be optimal at the outset, or at some particular point in the future, but not both unless the rule itself, *in percentage terms*, changes with the number of voters.

The remainder of this paper is organized as follows. Section 2 discusses the relevant literature on voting rules. Section 3 contains our model and theoretical results. Section 4 calibrates this model in the context of Article V. Section 5 concludes.

2 Related Literature

This model of super-majority voting we use in this paper builds on a large literature in both economics and political science examining voting rules and on super-majority rules in particular[4]. In economics, interest in super-majority voting rules can be traced to Black (1948). Attention to super-majority requirements has also been an important part of subsequent work in social choice theory. Arrow himself conjectured (Arrow (1951)) that a sufficient degree of social consensus could overcome his impossibility theorem[5]. This conjecture was formalized by Caplin and Nalebuff (1988) and with greater generality by Caplin and Nalebuff (1991).

More recent work on incomplete contract also develops a framework for analyzing optimal super-majority requirements in certain specific contexts. Aghion and Bolton (1992) show that some form of majority voting dominates a unanimity requirement in a world of incomplete social contracts. They highlight the fact that if a contract could be complete then the issue of super-majority requirements is moot if rules are chosen behind the veil of ignorance. Aghion, Alesina and Trebbi (2004) utilize a related framework, in the spirit of public good provision analyzed by Romer and Rosenthal (1983). In a similar model, Erlenmaier and Gersbach (2001) consider "flexible" majority rules whereby the size of required super-majority depends on the proposal made by the agenda setter. Babera and Jackson (2004) consider "self-stable" majority rules, in the sense that the required super-majority does not wish to change the super-majority rule itself *ex post*. A related paper is Maggi and Morelli (2003), which finds that unanimity, in certain settings, is usually optimal if there is imperfect enforcement.

Despite this large literature there is no general exposition of optimal super-majority rules. In our model we consider a general formulation where the policy set is a continuum. This

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[4]Early works by economists using this notion include Vickrey (1945), Harsanyi (1953) and Harsanyi (1955). Mirrlees (1971) and, of course, Rawls (1971) analyze profound questions within this framework.

[5]"The solution of the social welfare problem may lie in some generalization of the unanimity condition..." (quoted in Caplin and Nalebuff (1988))

allows us to study the effect of risk and risk-aversion on the voting rule. As discussed in section 3, we consider a particularly strong form of incompleteness of the social contract. The social contract is not permitted to specify a state-contingent super-majority rule, nor are monetary transfers / side payments allowed. In the context of the model this means that the super-majority requirement cannot differ based on realized draws from the distribution of types.

3 The Model

3.1 Statement of the Problem

Let there be n voters, with n finite. The policy space is assumed to be the unit interval $[0, 1]$. Voters preferences over this policy space are drawn from the distribution function $F(x)$.

Definition 1. *A Social Decision is a scalar, $\theta \in [0, 1]$.*

Assumption A1. *Each voter i has a utility function of the form*

$$U_i = -u\left(|\theta - x_i|\right),$$

where $u\left(\cdot\right)$ is an increasing, convex function, and x_i is voter i's preferred policy.

We are thus assuming that voters are *ex ante* identical, but not (generically) *ex post*.

Definition 2. *A Super-majority Rule is a scalar $\alpha \in [\frac{1}{2}, 1]$ which determines the proportion of voters required to modify the social decision.*

There are two time periods in the model. In period 1 voters know the distribution of preferences, $F(x)$, but they do not know their draw from the distribution. In this period they determine, behind the veil of ignorance, a social choice and a super-majority rule. In

period 2, after preferences are realized, the social decision can be changed if a coalition of at least αn voters prefer a new social decision.

As mentioned before, we restrict the (social) contracting space. State contingent super-majority rules are not permitted. An example of such a rule would be any kind of utilitarian calculus which would vary the super-majority requirement to change the status quo according to the aggregate utility to be gained *ex post*. We also rule out monetary transfers / side payments. Let $\hat{\theta}$ be the *ex ante* optimal social decision.

Definition 3. *The ex post optimal social decision is:*

$$\theta^* = \arg\max_{\theta} \sum_{i=1}^{n} u\left(|\theta - x_i^*|\right).$$

With a finite number of voters the *ex post* optimal decision may well differ from the *ex ante* optimal decision because of the realized draws from $F(x)$. It is this wedge between *ex ante* and *ex post* optimality which creates complexity in the choice of the optimal super-majority rule.

We make the following technical assumption which enables us to avail ourselves of several useful results from the theory of order-statistics.

Assumption A2. *The parent distribution of voter types $F(x)$ is absolutely continuous.*

By using order-statistics we are able to fully characterize the aggregate expected utility of a given voter for an arbitrary distribution of the population, number of voters, degree of risk-aversion and super-majority rule. We are, therefore, able to determine which rule yields the highest expected utility, and is hence optimal.

There is an obvious issue of how the *ex post* social decision is determined if a coalition has a sufficient number of members relative to the required super-majority who would be made better-off by a change to the *ex ante* social decision. In principle, any *ex post* social decision within the interval spanned by their preferences improves each of their payoffs.

For simplicity we make the following assumption about how the bargaining power amongst members of such a coalition.

Assumption A3. *If a coalition has the required super-majority ex post then the social decision is that preferred by the "final" member of the coalition. That is, the member of the coalition whose preference is closest to the ex ante social decision.*

3.2   Analytic Results

Theorem 1. *Assume A1-A3. Then the optimal super-majority rule is decreasing in the number of voters, n.*

Proof. See appendix. ■

As the number of voters increases, the probability of being part of an expropriated minority decreases. The benefit gained from avoiding blocking, however, is unchanged and the probability of this increases. The risk-averse agents therefore require less insurance and hence the optimal super-majority rule decreases.

Theorem 2. *Assume A1-A3. Then the optimal super-majority rule is increasing in the coefficient of importance, $\beta \equiv -u''(\cdot)/u'(\cdot)$.*

Proof. See appendix. ■

As the coefficient of importance/risk-aversion increases voters are progressively more concerned with being expropriated. They essentially purchase insurance against this by requiring that the size of the majority required to expropriate them be large, thereby reducing the probability of that event occurring. In fact, when the coefficient of importance is sufficiently high a unanimity requirement is always optimal. If there is the prospect of a sufficiently bad payoff then voters require a veto in order to insure themselves against this outcome[6].

Before stating our final result, the following definition is useful.

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[6] And as $\beta \to \infty$ expected utilty$\to -\infty$.

Definition 4. *A distribution $\widehat{F}(\cdot)$ is Rothschild-Stiglitz Riskier than another distribution $F(\cdot)$ if either (i) $F(\cdot)$ Second Order Stochastically Dominates $\widehat{F}(\cdot)$, (ii) $\widehat{F}(\cdot)$ is a Mean Preserving Spread of $F(\cdot)$, or (iii) $\widehat{F}(\cdot)$ is an Elementary Increase in Risk from $F(\cdot)$.*

As is well known, Rothschild and Stiglitz (1970) showed that these three statements are equivalent.

Theorem 3. *Assume A1-A3. Then the optimal super-majority rule is larger for a distribution of voter types, $\widehat{F}(x)$ than for the distribution $F(x)$ if $\widehat{F}(x)$ is Rothschild-Stiglitz Riskier than $F(x)$.*

Proof. Trivial, since a Rothschild-Stiglitz increase in risk has the same effect as an increase in the coefficient of importance. ∎

This result obtains for reasons closely related to those of the two previous theorems. As the spread of voter types increases more insurance is desired, which is effected by requiring the super-majority rule to be higher. This is, however, only the case if the voters' utility is more than proportionally decreasing as the social decision moves away from their ideal point (i.e. $\beta > 0$).

We now provide two examples which serve two purposes: (i) they illustrate the analytic results in a less abstract setting, and (ii) provide the basis for calibrating the model as we do in section 4.

3.2.1   Example 1

Voters' types are drawn from the uniform distribution on $[0, 1]$, $u_i = -\exp\{\beta |\theta - x_i|\}$, and $n = 5$.

First note that the *ex ante* optimal social decision is simply $\theta^* = \frac{1}{2}$. First we focus on the outcome under majority rule, which is simply that the *ex post* social decision is the median

of the voters' draws. Consider voter i and let the other voters' draws be:

$$x_1^* \leq x_2^* \leq x_3^* \leq x_4^*$$

where x_k^* is the kth order-statistic. Now note that the density of (x_2^*, x_3^*) on $[0,1] \times [0,1]$ is[7]:

$$f(a_2, a_3) = 24a_2(1 - a_3)$$

Note that in considering the median we need only be concerned with voter i's position relative to x_2^* and x_3^*. If they are between x_2^* and x_3^* then they are the median. If $x_i^* \leq x_2^*$ then the expected loss is $\int_0^{a_2} - \exp\{\beta |t - a_2|\} dt$ and if $x_i^* \geq x_3^*$ it is $\int_{a_3}^1 - \exp\{\beta |t - a_3|\} dt$. If $x_2^* \geq x_i^* \geq x_3^*$ then the expected loss is $- \exp(0) = -1$. The expected utility of voter i is therefore:

$$E\left[u_i^M\right] = \int_0^1 \int_0^{a_3} \left(\begin{array}{c} \int_0^{a_2} - \exp\{\beta(a_2 - t)\} dt \\ + \int_{a_2}^{a_3}(-1)dt \\ + \int_{a_3}^1 - \exp\{\beta(t - a_3)\} dt \end{array} \right) 24a_2(1 - a_3)da_2 da_3$$

$$= -\frac{\beta(\beta^4 - 10\beta^3 + 120\beta + 480) + 240e^\beta(\beta - 3) + 720}{5\beta^5}.$$

Now consider the expected utility of voter i if we require unanimity in order to change the social decision *ex post*. Denote the *ex post* social decision as t. Let B be the event where $0 \leq x_1^* \leq x_4^* < \frac{1}{2}$ and let B' be the event where $\frac{1}{2} \geq x_1^* \geq x_4^* \geq 1$. Let $A = \Omega \backslash (B + B')$. It is clear that $\Pr(A) = \frac{7}{8}$ and that $\Pr(B) = \Pr(B') = \frac{1}{16}$. The expected utility of voter i

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[7]For an absolutely continuous population the joint density of two order statistics $i < j$, from n statistics, is given by:

$$\frac{n!}{(i-1)!(j-i-1)!(n-j)!}F(x_i)^{i-1}\left(F(x_j) - F(x_i)\right)^{j-i-1}\left[(1 - F(x_j))^{n-j} f(x_i)f(x_j)\right]$$

(See Balakrishnan and Rao (1998)). For the uniform distribution this implies:

$$f(x_i, x_j) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}u_i^{i-1}\left(u_j - u_i\right)^{j-i-1}\left(1 - u_j\right)^{n-j}$$

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conditional on event A is:

$$
\begin{aligned}
E\left[u_i^U|A\right] &= 2\int_0^{\frac{1}{2}} -\exp\left\{\beta\left(\frac{1}{2}-t\right)\right\}dt \\
&= \frac{2\left(1-e^{\beta/2}\right)}{\beta}.
\end{aligned}
$$

The density[8] of x_4^* is $f(a_4) = 4\left(a_4\right)^3$. We now need the density of x_4^* on $[0, \frac{1}{2}]$, which is found by applying the Change of Variables Theorem, yielding $g(a_4) = 2\times 4(2a_4)^3 = 64(a_4)^3$. Therefore:

$$
\begin{aligned}
E\left[u_i^U|B\right] &= \int_{\frac{1}{2}}^1 -\exp\left\{\beta\left(t-\frac{1}{2}\right)\right\}dt \\
&\quad + \int_0^{\frac{1}{2}}\left(\int_0^{a_4} -\exp\left\{\beta\left(a_4-t\right)\right\}dt - \int_{a_4}^{\frac{1}{2}} 1dt\right)64\left(a_4\right)^3 da_4 \\
&= \frac{2\left(1-e^{\beta/2}\right)}{\beta} + \frac{1}{\beta} - \frac{8(48+e^{\beta/2}(\beta^3-6\beta^2+24\beta-48)}{\beta^5} - \frac{1}{10}.
\end{aligned}
$$

where $\int_{\frac{1}{2}}^1 -\exp\left\{\beta\left(t-\frac{1}{2}\right)\right\}dt$ is the term associated with $x_i \geq \frac{1}{2}$ and the term associated with $x_i \leq \frac{1}{2}$ is $\int_0^{\frac{1}{2}}\left(\int_0^{a_4} -\exp\left\{\beta\left(a_4-t\right)\right\}dt - \int_{a_4}^{\frac{1}{2}} 1dt\right)64\left(a_4\right)^3 da_4$.

Under event B' the expected utility is given by:

$$
\begin{aligned}
E\left[u_i^U|B'\right] &= \int_0^{\frac{1}{2}} -\exp\left\{\beta\left(\frac{1}{2}-t\right)\right\}dt \\
&\quad + \int_{\frac{1}{2}}^1\left(\int_{a_1}^1 -\exp\left\{\beta\left(t-a_1\right)\right\}dt - \int_{\frac{1}{2}}^{a_1} 1dt\right)64\left(1-a_1\right)^3 da_1 \\
&= \frac{2\left(1-e^{\beta/2}\right)}{\beta} + \frac{1}{\beta} - \frac{8(48+e^{\beta/2}(\beta^3-6\beta^2+24\beta-48)}{\beta^5} - \frac{1}{10}.
\end{aligned}
$$

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[8]For the uniform distribution the density of the ith order statstic is:

$$
f_i(u) = \frac{n!}{(i-1)!(n-i)!}u^{i-1}(1-u)^{n-i}
$$

Therefore the total expected utility under unanimity is:

$$
\begin{aligned}
E\left[u_i^U\right] &= \frac{7}{8}\mathbb{E}\left[u_i^U|A\right] + \frac{1}{16}\mathbb{E}\left[u_i^U|B\right] + \frac{1}{16}\mathbb{E}\left[u_i^U|B'\right] \\
&= \frac{-3840 + \beta^4(\beta - 160) + 10e^{\beta/2}(-384 + \beta(192 + \beta(\beta(15\beta + 8)48)))}{80\beta^5}.
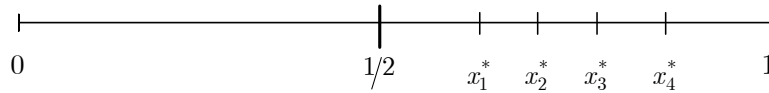\end{aligned}
$$

For majority rule to be preferable to unanimity therefore requires $E\left[u_i^M\right] > E\left[u_i^U\right]$. Solving numerically shows that this is the case if and only if $0 \leq \beta \lessapprox 3.9$. Therefore when the decision is relatively unimportant majority rule dominates, but with a sufficiently high enough degree of importance unanimity is preferred.

Now consider the case where the social decision can be altered *ex post* if four voters agree. In this example with five voters this reflects the only super-majority which is greater than simple majority but less than unanimity.

Now define events B, B', C and C' as follows. B is the event where $0 \leq x_1^* \leq x_2^* \leq x_3^* \leq x_4^* \leq \frac{1}{2}$. B' is the event where $\frac{1}{2} \leq x_1^* \leq x_2^* \leq x_3^* \leq x_4^* \leq 1$. C is the event where $0 \leq x_1^* \leq \frac{1}{2} \leq x_2^* \leq x_3^* \leq x_4^* \leq 1$. C' is the event where $0 \leq x_1^* \leq x_2^* \leq x_3^* \leq \frac{1}{2} \leq x_4^*$. Also, let $A = \Omega \backslash (B + B' + C + C')$.
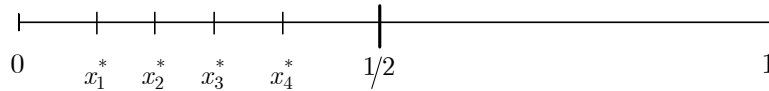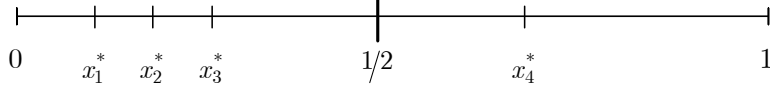

Event B

Event B'

Event C

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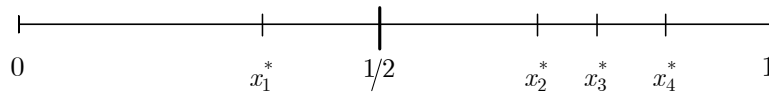
Event C'

Note that $\Pr(B) = \Pr\left(x_4^* \leq \frac{1}{2}\right) = \frac{1}{16} = \Pr(B')$. $\Pr(C') = \Pr\left(x_3^* \leq \frac{1}{2} \wedge x_4^* \geq \frac{1}{2}\right) = \frac{1}{4} = \Pr(C')$. Also note that $\Pr(A) = \frac{3}{8}$.

As before, if event A occurs then there is no change to the *ex ante* social decision and hence the expected utility of voter i is:

$$
\begin{aligned}
E\left[u_i^S | A\right] &= 2\int_0^{\frac{1}{2}} -\exp\left\{\beta\left(\frac{1}{2} - t\right)\right\} dt \\
&= \frac{2(1 - e^{\beta/2})}{\beta}.
\end{aligned}
$$

Note[9] that the density of x_2^* conditional on event C is simply the density of the first order-statistic of three on $[\frac{1}{2}, 1]$. In fact, order statistics from a continuous parent form a Markov Chain. It follows that the density of the first-order statistic of three on $U[0, 1]$ is $3(1 - a_2)^2$. By a change of variables, the density on $[\frac{1}{2}, 1]$ is therefore $24(1 - a_2)^2$. Hence the expected utility conditional on event C is:

$$
\begin{aligned}
E\left[u_i^S | C\right] &= \int_{\frac{1}{2}}^1 \left(-1\int_{1/2}^{a_2} dt + \int_{a_2}^1 -\exp\left\{\beta(t - a_2)\right\} dt\right) 24(1 - a_2)^2 da_2 \\
&\quad + \int_0^{1/2} -\exp\left\{\beta\left(\frac{1}{2} - t\right)\right\} dt \\
&= -\frac{1}{8} + \frac{1}{\beta} + \frac{1 - e^{\beta/2}}{\beta} - \frac{6(-8 + e^{\beta/2}(8 - 4 + \beta^2))}{\beta^4}.
\end{aligned}
$$

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[9]This fact is quite general. The conditional pdf of an order-statistic is given by:

$$
f_{X_r | X_{s=v}}(x) = \frac{(s - 1)!}{(r - 1)!(s - r - 1)!} \frac{f(x)F(x)^{r-1}(F(v) - F(x))^{s-r-1}}{F(v)^{s-1}}
$$

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The density of x_3^* conditional on event C' is the third of three uniformly distributed order-statistics on $[0, \frac{1}{2}]$, which is $g(a_2|C') = 24\,(a_3)^2$. Hence the expected utility conditional on event C' is:

$$
\begin{aligned}
E\left[u_i^S|C'\right] &= \int_0^{\frac{1}{2}} \left(-1 \int_{a_3}^{\frac{1}{2}} dt + \int_0^{a_3} -\exp\left\{\beta\,(a_3 - t)\right\} dt \right) 24\,(a_3)^2\, da_3 \\
&\quad + \int_{1/2}^1 -\exp\left\{\beta\left(t - \frac{1}{2}\right)\right\} dt \\
&= -\frac{1}{8} + \frac{1}{\beta} + \frac{1 - e^{\beta/2}}{\beta} - \frac{6(-8 + e^{\beta/2}(8 - 4 + \beta^2))}{\beta^4}.
\end{aligned}
$$

Now note that the joint density of (x_3^*, x_4^*) on $[0, 1]$ is $f(x_3, x_4) = 12\,(a_3)^2$ and so on $[0, \frac{1}{2}]$ it is $192\,(a_3)^2$. The expected utility conditional on event B is therefore:

$$
\begin{aligned}
E\left[u_i^S|B\right] &= \int_0^{\frac{1}{2}} \int_0^{a_4} \left(\begin{array}{l} -\int_{a_3}^{a_4} 1\, dt \\ +\int_0^{a_3} -\exp\left\{\beta\,(a_3 - t)\right\} dt \\ +\int_{a_4}^1 -\exp\left\{\beta\,(t - a_4)\right\} dt \end{array} \right) 192\,(a_3)^2\, da_3 da_4 \\
&= \frac{-3840e^{\beta/2} - \beta^4(\beta - 20) + 1920(\beta + 6) + 80e^{\beta/2}(\beta^4 + 72\beta - 96)}{10\beta^5}.
\end{aligned}
$$

The joint density of (x_1^*, x_2^*) on $[0, 1]$ is $f(x_1, x_2) = f(x_1, x_2) = 12\,(1 - a_2)^2$ and so on $[\frac{1}{2}, 1]$ it is $192\,(1 - a_2)^2$. The expected utility conditional on event B' is:

$$
\begin{aligned}
E\left[u_i^S|B'\right] &= \int_{\frac{1}{2}}^1 \int_1^{a_1} \left(\begin{array}{l} -\int_{a_1}^{a_2} 1\, dt \\ +\int_0^{a_1} -\exp\left\{\beta\,(a_1 - t)\right\} dt \\ +\int_{a_2}^1 -\exp\left\{\beta\,(t - a_2)\right\} dt \end{array} \right) \left(-192\,(1 - a_2)^2\right) da_2 da_1 \\
&= \frac{-3840e^{\beta/2} - \beta^4(\beta - 20) + 1920(\beta + 6) + 80e^{\beta/2}(\beta^4 + 72\beta - 96)}{10\beta^5}.
\end{aligned}
$$

Therefore the total expected utility under a super-majority of four voters (ie. 80% super-majority) is:

$$E\left[u_i^S\right] = \frac{3}{8}E\left[u_i^S|A\right] + \frac{1}{16}E\left[u_i^S|B\right] + \frac{1}{16}E\left[u_i^S|B'\right]$$
$$+\frac{1}{4}E\left[u_i^S|C\right] + \frac{1}{4}E\left[u_i^S|C'\right].$$

Which, upon simplification, is:

$$E\left[u_i^S\right] = \frac{\left(\begin{array}{c} -1920e^\beta + \beta^4(80 - 3\beta) + 1920(\beta + 3) \\ -10e^{\beta/2}(284 + \beta(-192 + \beta(\beta + 4)(5\beta - 12))) \end{array}\right)}{40\beta^5}$$

For an 80% super-majority to be preferable to majority rule therefore requires $E\left[u_i^S\right] > E\left[u_i^M\right]$. Solving numerically shows that this is the case if and only if $\beta \gtrapprox 2.69$. For unanimity to be superior to an 80% super-majority rule requires $E\left[u_i^U\right] > E\left[u_i^S\right]$. Solving numerically reveals that this the case for $\beta \gtrapprox 9.02$. That is, the 80% super-majority rule dominates unanimity until the degree of importance becomes sufficiently large. For sufficiently large degrees of importance unanimity dominates because the fear of expropriation dominates and a veto provides them with insurance against this possibility. Therefore, in this example, for $0 \gtrapprox \beta \gtrapprox 2.69$ majority rule is optimal, for $2.69 \gtrapprox \beta \gtrapprox 9.02$ an 80% super-majority requirement is optimal, and for $\beta \gtrapprox 9.02$ a unanimity requirement is optimal. This is reflected in the following figure.
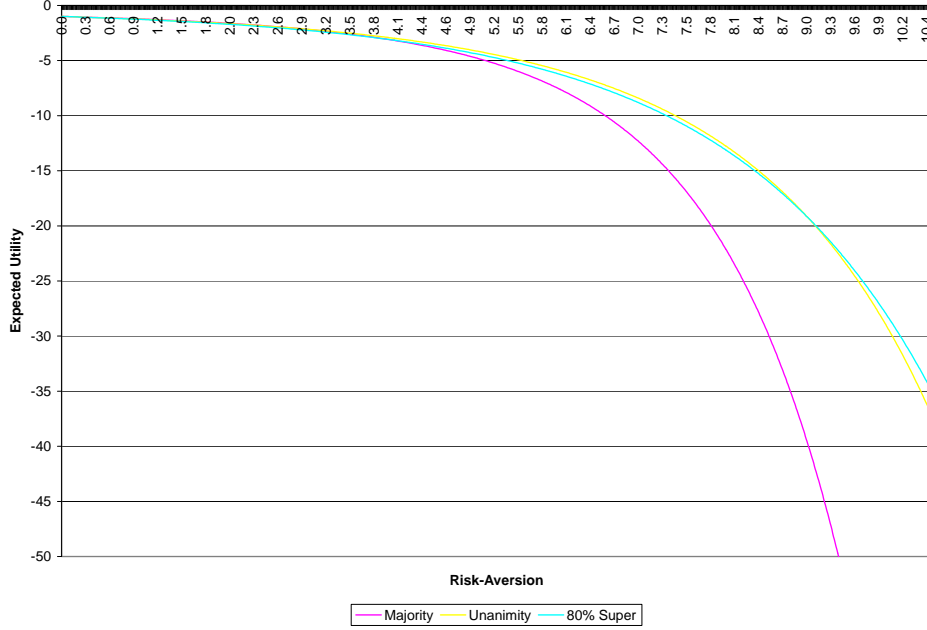
Figure 1: Example of Supermajority Rules and Risk-Aversion

3.2.2 Example 2

Voters' types are drawn from the uniform distribution on $[0, 1]$, $u_i = -\exp\{\beta\,|\theta - x_i|\}$, and $n = 3$.

This example illustrates that as the number of voters increases the optimal super-majority rule decreases. We again use the uniform distribution, but with 3 voters rather than 5.

The expected utility under majority rule (here 2 out of three voters) is[10]:

$$
\begin{aligned}
E\left[U_M\right] &= \int_0^1 \int_0^{a_2} \left[\begin{array}{c} \int_0^{a_1} -\exp\{\beta\,(a_1 - t)\}\,dt \\ +\int_{a_2}^1 -\exp\{\beta\,(t - a_2)\}\,dt \\ -1\int_{a_1}^{a_2} dt \end{array}\right] 2\,da_1\,da_2 \\
&= \frac{12 - 12e^\beta\beta(12 - \beta(\beta - 6))}{3\beta^3}.
\end{aligned}
$$

---

[10]Note that the joint density of (x_1, x_2) where there are just two order statistics is simply 2.

The expected utility under a unanimity requirement is:

$$
\begin{aligned}
E\left[U_U\right] &= \begin{array}{c} \int_0^{1/2} -\exp\left\{\beta(1/2 - t)\right\} dt \\ + \left(\int_0^{1/2} \left(\int_0^{a_2} -\exp\left\{\beta(a_2 - t)\right\} dt - 1 \int_{a_2}^{1/2} dt \right) 8a_2 da_2 \right) \end{array} \\
&= \frac{-48 + \beta^2(\beta - 12) + 6e^{\beta/2}(-8 + \beta(\beta + 4))}{6\beta^3}.
\end{aligned}
$$

Now consider $\beta = 5$. In this case, where $n = 3$, a unanimity requirement is optimal and yields expected utility of approximately -3.44. Where $n = 5$ (ie. example 1) and $\beta = 5$ an 80% super-majority is optimal and the expected utility is approximately -4.12. For majority rule under $n = 3$ expected utility is -4.50. This illustrates the general point made in Theorem 1, that unless adjusted downward, any given super-majority rule will over-protect against the danger of oppression, as compared to blocking, as the voting population increases: or be subject to a form of constitutional inflation, which increases the effective hurdle to achieving constitutional change.

4   Calibration in the Context of Article V

In the context of Article V of the U.S. Constitution, there has been a very clear increase in the size of the relevant voting population from 1789 to the present. From 1789 to 1959, when Alaska entered the Union, the number of states for Article V purposes has increase almost four-fold: from 13 states to 50. The number of Senators increase from 26 to 100, and the number of House members from 65 to 435.[11]

---

[11] To be conservative we use the maximum number of members at any point in the first Congress (including one vacancy in the House). The minimum numbers were 22 Senators and 59 House members. This was due to the delay in ratification of the Constitution in North Carolina (admitted November 21, 1789) and Rhode Island (admitted on May 29, 1790). Using the smaller numbers (22 Senators, 59 House members and 11 States) would obviously increase the magnitude of voting rule inflation.

Table 1: Numbers of Voting Parties in 1789 and 2008

	1789	2008
Senators	26	100
House Members	65	435
States	13	50

This increase has, in turn, meant the that the super-majority rule which would achieve the same balance originally struck by the framers in Article V has in fact decreased, over time.

Table 2 reports the actual voting rule equivalents for the 2008 voting population of the original Article V voting rule.

Table 2: Voting Rule Equivalents

	1789	2008
Senate	67%	59%
House	67%	53%
States	75%	62%

As noted in section 1, these calculations assume that the distribution of preferences has remained unchanged. This allows us to isolate the pure effect of a an increase in numbers on the effective stringency of the voting rule imposed by Article V, rather than conflating that effect with a change in the distribution of preferences. Our theoretical results demonstrate that a less "spread-out" distribution of preferences implies that a lower rule is optimal. To the extent that voter preferences in the Congress and the Senate were more spread-out in 1789 than they are today, our results therefore provide a *lower bound* on the impact of changes on the optimal voting rule. Conversely, of course, if voter preferences in Congress and the Senate are more spread-out now than in 1789 then our results may, to some extent, overstate the degree of voting rule inflation.

The evidence suggests, however, that polarization was quite large at the time the Constitution was adopted. One way to measure this is by examining the difference between the mean DW-nominate scores of the two major parties (McCarty, Poole and Rosenthal (2008)). DW-nominate scores are a unidimensional measure of ideology which are widely used in political science, with a lower value indicating a more left-leaning representative. This difference was 0.52 in the 1st House, 0.57 in the 2nd and 0.75 in the 5th (as party identification arguably became stronger, having not really been envisioned by the Framers (Ackerman (1991), Pildes and Levinson (2006))). If one splits the data by state, instead, the difference in the 1st House was approximately 0.6. The Democrat-Repulican difference was approximately 0.6 in 1984, a time at which the current numbers of voting parties were in place.

We should note that there has been a sharp recent increase in this statistic: it now stands at 0.96. None of the failed amendments we discuss below, however, is affected by this recent increase.

We specify the assumptions underlying the calibration in the appendix.

4.1 Application: Failed Amendments

We now consider what impact, if any, voting rule inflation can be observed to have had on actual amendments to the constitution. A natural question is whether, absent the voting rule inflation, several failed amendments would have passed.

Such a question, of course, is impossible to answer in any definitive way, in part because of the possibility that, contrary to our model (where people receive independent draws behind the veil of ignorance) voters may vote inter-dependently, and more so now than previously[12], or that members of the House and Senate may vote strategically on proposed constitutional amendments. In some cases, strategic considerations may mean that potential amendments are not even proposed. If the probability of an amendment being passed under the cur-

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[12]Formally, this would mean that voter preferences are drawn from a non *iid* parent distribution. Examining DW-nominate scores over time suggests that there is little evidence of an increase interdependence.

rent rule is relatively small then legislators may not risk the "political capital" involved in proposing an amendment. Conversely, in other cases, some Members may vote in favor of a proposed amendment knowing it will not pass, in order to gain an electoral advantage, or because conditional on her vote not being pivotal, she prefers to be on the record as being against the amendment. Behaviors such as this mean that one cannot simply look at the *equilibrium* voting pattern since the voting rule may cause endogenous changes in voting behavior.

It is still possible to gain a sense of the our effect based on observed voting patterns. Equilibrium voting behavior provides at least a useful benchmark against which the impact of strategic considerations can be assessed.

For example, we identify one failed amendment, the Equal Rights Amendment (of 1971), which went to the states for ratification but ultimately failed at that stage, which would almost certainly have passed under the adjusted super-majority requirement.[13] 35 states (i.e. 70 percent of states) ratified the amendment within the required time-frame rather than the 38 required by Article V, as written. Under the adjusted super-majority rule we identify for ratification by the states (i.e. 62 percent), only 31 states would have been required. The sequence in which states did in fact ratify the amendment also suggests that strategic voting considerations are unlikely to have been a factor influencing states' decisions to ratify.

Table 3 documents additional failed amendments voted on by Congress since the 93rd Congress (1973).

[Table 3 Here]

Of the 27 proposed amendments which actually went to a vote of either the House or Senate, but which ultimately failed, 13 might have gained it under the adjusted requirement. When one restricts attention to *distinct* amendments, just 15 were voted on and none

---

[13]H.J.Res 208 passed the House on October 12, 1971, with a vote of 354 yeas and 24 nays and passed the Senate on March 22, 1972 by a margin of 84-8.

passed.[14] We suggest that four might have progressed further under the adjusted requirement (the reintroduced Equal Rights Amendment of 1983 might have passed at least the House, a Balanced Budget Amendment the House and Senate, a Flag Burning Amendment the House, and the same-sex marriage amendment the House) (see Table 3). Of these, the same-sex marriage amendment seems most likely to have been affected by strategic voting, and therefore least likely actually to have passed even under the adjusted rule we identify.

One way to assess the magnitude of strategic voting is to perform a kernel density plot of the DW-nominate scores of the yea and nay voters on a proposed amendment. For example, the following figure does this for H.J.Res.88–the same-sex marriage amendment voted on in the 109th House, which received a 55.8 percent yea vote in the House.
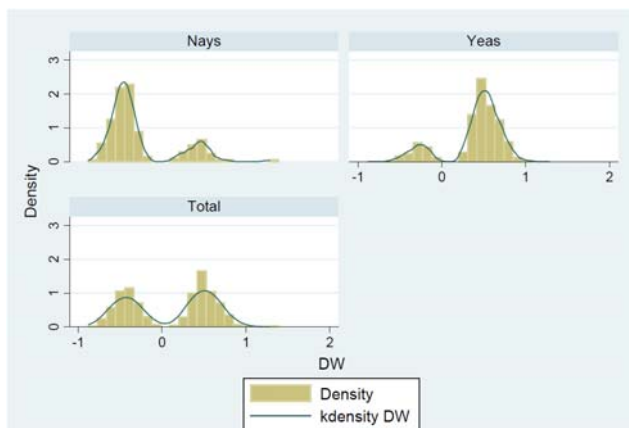
Figure 2: DW Nominate Scores by Vote on HJ Res 88 (109th Congress)

There is a clear discontinuity in DW-nominate scores revealing that some yea votes came from otherwise left-leaning members, but also that some nay votes came from otherwise right-leaning members. These left-wing yea votes in particular indicate the possibility of strategic voting in favor of the bill, and therefore that it might not have gained the necessary

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[14]The 26th amendment occured prior to the sample period. It was ratified by July 1, 1971 and a certificate of validity was granted on July 7.

53% support, had the amendment rule been understood to be lower. By contrast, a similar analysis of the votes in favor of the reintroduced (1983) Equal Rights Amendment[15] and a number of flag burning and balanced budget amendments provides far less evidence of the possibility of such strategic voting having an effect.

5 Discussion and Conclusion

The mechanism by which a Constitution may be amended has crucial importance for both the success and the legitimacy of the system of government it establishes.

First and foremost, as the experience prior to 1789 under the Articles of Confederation demonstrated, if a constitution cannot be amended to respond to changing understandings or circumstances, the polity it establishes may fail.

Of course, where formal mechanisms prove too onerous, informal modes of constitutional change may emerge to prevent a constitutional system from collapsing, as arguably occurred during the New Deal (Ackerman (1996)). However, the move to more informal mechanisms of amendment should not be thought of as costless.

The shift to more informal modes of amendment carries with it a real potential cost in terms of constitutional legitimacy. There is the danger that "amendment" by the United States Supreme Court will be strongly counter-majoritarian, and thus raise substantial questions of democratic legitimacy (c.f. Bickel (1962)). Further, even if, as is perhaps more likely (Dahl (1989)), the Supreme Court acts in a way which is pro-majoritarian over time, this mode of "amendment" represents a much less democratic form of politics than that envisaged by the Framers. Informal modes of constitutional change therefore do not displace the central role of formal modes of amendment (Levinson (1995)). Constitutional scholars continue to question whether the current voting requirements in Article V should not be lowered in some way (Griffin (1995a), Levinson (1995), Lutz (1995)). While this paper

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[15] H.J.Res 208 passed the House on October 12, 1971, with a vote of 354 yeas and 24 nays and passed the Senate on March 22, 1972 by a margin of 84-8. However only 35 states (not the requisite 38) ratified the amendment within the required timeframe.

does not directly address this question, it provides theoretical support for the intuition that the current mechanism is much more onerous than that initially seen as optimal. It formalizes the widely held perception that Article V amendment has become increasingly–and unduly–onerous over time.

Whatever the implications of the finding of "voting rule inflation" in the Article V context, the effect identified has important implications for the practice of constitutional design.

The problem of voting rule inflation will have the potential to arise in a whole range of "constitutional" settings–both commercial as well as governmental.

Where a company grows in size, voting rule inflation can mean that a minority of shareholders can block a sale or (voluntary) takeover, or other change in organizational structure, in a much broader range of circumstances than deemed optimal by the founding shareholders. Likewise in a transnational setting, the increase in the number of members of an organization such that the European Union or United Nations can mean that when it comes to major structural reforms, those changes are (much) more difficult to achieve than was contemplated by the initial members states, in adopting a particular super-majority rule.

In all these settings, the potential for voting rule inflation will mean that constitutional drafters will need to adopt a much more carefully designed voting-rule *standard*, than is currently the norm, if they are to achieve a consistent balance between concerns about flexibility and stability, or blocking and oppression, over time.

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6   Appendix

6.1   Proofs

Proof of Theorem 1. Let event 1 be the event that $x_1^* \leq ... \leq x_n^* \leq \hat{\theta}$, where x_i^* is the ith order statistic. Let event 2 be the event that $x_1^* \leq ... \leq x_{n-1}^* \leq \hat{\theta} \leq x_n^*$, and so on up to event $n+1$. Note then that the probability of event j occurring is given by

$$
\begin{aligned}
\pi_j &= \Pr\left(\text{Event } j\right) \\
&= F^{(j-1)}\left(\hat{\theta}\right)\left[1 - F\left(\hat{\theta}\right)\right]^{(n-j+1)}.
\end{aligned}
$$

When the super-majority rule is $\lambda = \psi\left(\alpha n\right)$, where ψ is the ceiling function which rounds its argument up to the nearest integer, utility conditional on draws $x_1^*, ..., x_n^*$ is

$$
\bar{V} = \sum_{i=1}^{n}\sum_{j=1}^{n+1}\pi_j u\left(\left|\theta^* - x_i^*\right|\right).
$$

By A3, the ex post social choice under super-majority rule λ is x_λ^* for $j \leq (n+1)/2$ and $x_{n+1-\lambda}^*$ for $j > (n+1)/2$. We can thus write $\bar{V}$ as

$$
\bar{V} = -\sum_{i=1}^{n}\left(\sum_{j=1}^{(n+1)/2}\pi_j u\left(\left|x_{\psi(\alpha n)}^* - x_i^*\right|\right) + \sum_{j=((n+1)/2)+1}^{n+1}\pi_j u\left(\left|x_{n+1-\psi(\alpha n)}^* - x_i^*\right|\right)\right).
$$

Expected utility involves integrating over all possible realizations of the order statistics–that is, over their joint pdf. Thus, expected utility is

$$
E\bar{V} = -\int \cdots \int \sum_{i=1}^{n}\left(\begin{array}{c}\sum_{j=1}^{(n+1)/2}\pi_j u\left(\left|x_{\psi(\alpha n)}^* - x_i^*\right|\right) + \\ \sum_{j=((n+1)/2)+1}^{n+1}\pi_j u\left(\left|x_{n+1-\psi(\alpha n)}^* - x_i^*\right|\right)\end{array}\right) f\left(a_1, ..., a_n\right) da_1...da_n.
$$

This can be simplified by noting that the joint pdf of all n order statistics is $n!$, since the unordered sample has density equal to 1 and there are $n!$ different permutations of the sample

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corresponding to the same sequence of order statistics. Thus we have

$$
E\bar{V} = - \int \cdots \int \sum_{i=1}^{n} \left(\begin{array}{c} \sum_{j=1}^{(n+1)/2} \pi_j u \left(\left| x^*_{\psi(\alpha n)} - x^*_i \right| \right) + \\ \sum_{j=((n+1)/2)+1}^{n+1} \pi_j u \left(\left| x^*_{n+1-\psi(\alpha n)} - x^*_i \right| \right) \end{array} \right) n! da_1 ... da_n.
$$

Denote the optimal super-majority rule as $\alpha^* = \arg\max_\alpha \left\{ E\bar{V} \right\}.$ By the Monotonicity Theorem of Milgrom and Shannon (1994), a necessary and sufficient condition for α^* to be nonincreasing in n is that $E\bar{V}$ have decreasing differences in $(\alpha, n).$ This requires that for all $n' \geq n$, $E\bar{V}(n', \alpha) - E\bar{V}(n, \alpha)$ is nonincreasing in α. Assuming for simplicity that n and n' are odd (the generalization to even integers is simply a matter of notation) this entails

$$
\int \cdots \int \sum_{i=1}^{n} \left(\begin{array}{c} \sum_{j=1}^{(n+1)/2} \pi_j(n) u \left(\left| x^*_{\psi(\alpha n)} - x^*_i \right| \right) + \\ \sum_{j=((n+1)/2)+1}^{n+1} \pi_j(n) u \left(\left| x^*_{n+1-\psi(\alpha n)} - x^*_i \right| \right) \end{array} \right) n! da_1 ... da_n, \qquad (1)
$$

$$
- \int \cdots \int \sum_{i=1}^{n'} \left(\begin{array}{c} \sum_{j=1}^{(n'+1)/2} \pi_j(n') u \left(\left| x^*_{\psi(\alpha n')} - x^*_i \right| \right) + \\ \sum_{j=((n'+1)/2)+1}^{n'+1} \pi_j(n') u \left(\left| x^*_{n'+1-\psi(\alpha n')} - x^*_i \right| \right) \end{array} \right) (n')! da_1 ... da_{n'}
$$

to be nonincreasing in α for all $n' \geq n$ and all $\alpha.$ An increase in α makes the term $\sum_{j=1}^{(n+1)/2} \pi_j(n) u \left(\left| x^*_{\psi(\alpha n)} - x^*_i \right| \right)$ larger, since $\left| x^*_{\psi(\alpha n)} - x^*_i \right|$ increases and the probabilities $\pi_j = F^{(j-1)} \left(\hat{\theta} \right) \left[1 - F \left(\hat{\theta} \right) \right]^{(n-j+1)}$ are unchanged. Similarly

$$
\sum_{j=((n+1)/2)+1}^{n+1} \pi_j(n) u \left(\left| x^*_{n+1-\psi(\alpha n)} - x^*_i \right| \right)
$$

is larger and so the first line of (1) is overall larger. Note, however, that the second line of (1) increases by more than the first line for a given change in $\alpha.$ The first term inside the parentheses in the second line increases by more than its corresponding term in the first line since each term $u(|\cdot|)$ is weakly larger in the second line by construction of the ordering of the order statistics, probabilities sum to 1, and $n'! > n!$ This argument is true for all α and $n' > n$, and hence the proof is complete. ∎

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Proof of Theorem 2. By a similar argument to the proof of the above theorem we require $E\bar{V}$ to have increasing differences in (β, α). This requires that for all $\beta' \geq \beta$, $E\bar{V}(\beta', \alpha) - E\bar{V}(\beta, \alpha)$ is nondecreasing in α. That is

$$
\int \cdots \int \sum_{i=1}^{n} \left(\begin{array}{c} \sum_{j=1}^{(n+1)/2} \pi_j(n) u_\beta \left(\left| x^*_{\psi(\alpha n)} - x^*_i \right| \right) + \\ \sum_{j=((n+1)/2)+1}^{n+1} \pi_j(n) u_\beta \left(\left| x^*_{n+1-\psi(\alpha n)} - x^*_i \right| \right) \end{array} \right) n! da_1...da_n, \qquad (2)
$$

$$
- \int \cdots \int \sum_{i=1}^{n} \left(\begin{array}{c} \sum_{j=1}^{(n+1)/2} \pi_j(n) u_{\beta'} \left(\left| x^*_{\psi(\alpha n)} - x^*_i \right| \right) + \\ \sum_{j=((n+1)/2)+1}^{n+1} \pi_j(n) u_{\beta'} \left(\left| x^*_{n+1-\psi(\alpha n)} - x^*_i \right| \right) \end{array} \right) n! da_1...da_n,
$$

is nondecreasing in α for all $\beta' > \beta$. The first line of the above is identical to the second except for the differences in the function u. As in the proof of Theorem 1 an increasing in α makes the term $\sum_{j=1}^{(n+1)/2} \pi_j(n) u \left(\left| x^*_{\psi(\alpha n)} - x^*_i \right| \right)$ larger, since $\left| x^*_{\psi(\alpha n)} - x^*_i \right|$ increases and the probabilities $\pi_j = F^{(j-1)} \left(\hat{\theta} \right) \left[1 - F \left(\hat{\theta} \right) \right]^{(n-j+1)}$ are unchanged and similarly for the term

$$
\sum_{j=((n+1)/2)+1}^{n+1} \pi_j(n) u \left(\left| x^*_{n+1-\psi(\alpha n)} - x^*_i \right| \right).
$$

The magnitude of this change is larger for $u_{\beta'}$ than u_β by Jensen's inequality, and thus the result follows. ∎

6.2   Calibration

For tractability, the calculations assume that voter preferences in each of the three bodies are uniformly distributed. Investigations of other distribution, which may more closely proxy actual preferences (such as the double gamma), confirm, however, that our results are in fact fairly insensitive to the choice of distribution.

The other free parameter is β (which can be thought of as equivalent to the coefficient of absolute risk aversion given the utility function). We solve for the value of β which made the 1789 rules optimal. It turns out that the requirement of 3/4 of the 13 states and 2/3 of

the 26 senators were both optimal voting rule assuming $\beta = 10$ and a uniform distribution. Furthermore, the requirement of 2/3 of the 65 members of the house was very close to optimal – the optimum being 62%. Therefore we use that parameter value throughout our exercise. Our approach is to explicitly calculate the expected utility under each possible voting rule and determine which is optimal in the sense of maximizing the sum of expected utilities.

Note that the number of events to be considered expands with the number of voters. Let event 1 be the even $x_1^* \leq ... \leq x_n^* \leq \hat{\theta} = 1/2$, where x_i^* is the ith order statistic. Let event 2 be the event that $x_1^* \leq ... \leq x_{n-1}^* \leq 1/2 \leq x_n^*$, and so on up to event $n+1$. The probabilities of these events follow straightforwardly from the Binomial Theorem. If s is the required number of voters in a super-majority, the expected utility of a particular voting rule is then simply

$$E\left[U^s\right] = \sum_{i=1}^{l} \Pr(Ev.l) \cdot E\left[U^s\right] | Ev.l.$$

Table 1: Amendments Voted On Since 93rd Congress

Congress	Resolution	Introduced	Sponsor	Official Title	House Vote	Senate Vote	% Yea
93rd	None						
94th	None						
95th	H.J.RES.554	7/25/1977	Rep Edwards	Joint resolution to amend the Constitution to provide representation of the District of Columbia in the Congress			
96th	H.J.RES.74	1/15/1979	Rep Mottl	Joint resolution to amend the Constitution of the United States to prohibit compelling the attendance of a student in a public school other than the public school nearest the residence of such student	209-216		49.2%
	S.J.RES.28	1/25/1979	Sen Bayh	Joint resolution to amend the Constitution to provide for the direct popular election of the President and Vice President of the United States		51-48	51.5%
97th							
98th	**H.J.RES.1**	**1/3/1983**	**Rep Rodino**	**Joint resolution proposing to amend the Constitution of the United States relative to equal rights for men and women**	**278-147**		**65.4%**
	S.J.RES.73	3/24/1983	Sen Thurmond	A joint resolution proposing an amendment to the Constitution of the United States authorizing the Congress and the States relating to voluntary school prayer		56-44	56.0%
99th	None						
100th	None						
101st	**H.RES.417**	**5/11/1989**	**Rep Stenholm**	**Proposing an amendment to the Constitution to provide for a balanced budget for the United States Government and for greater accountability in the enactment of tax legislation**	**279-150**		**65.0%**
	H.J.RES.350	**6/29/1989**	**Rep Michel**	**Proposing an amendment to the Constitution of the United States authorizing the Congress and the States to prohibit the physical desecration of the flag of the United States**	**254-177**		**58.9%**
	S.J.RES.180	7/18/1989	Sen Dole	A joint resolution proposing an amendment to the Constitution of the United States authorizing the Congress and the States to prohibit the physical desecration of the flag of the United States		51-48	51.5%
	S.J.RES.332	5/25/1990	Sen Dole	A joint resolution proposing an amendment to the Constitution of the United States authorizing the Congress and the States to prohibit the physical desecration of the flag of the United States		58-42	58.0%

Congress	Resolution	Introduced	Sponsor	Official Title	House Vote	Senate Vote	% Yea
102nd							
	H.J.RES.290	**6/26/1991**	**Rep Sandholm**	**Proposing an amendment to the Constitution to provide for a balanced budget for the United States Government and for greater accountability in the enactment of tax legislation**	**280-153**		**64.7%**
	S.RES.298	5/19/1992	Sem Byrd	A resolution declaring an article of amendment to be the Twenty-seventh Amendment to the Constitution of the United States		99-0	100.0%
103rd							
	H.J.RES.103	**2/4/1993**	**Rep Sandholm**	**Proposing an amendment to the Constitution to provide for a balanced budget for the United States Government and for greater accountability in the enactment of tax legislation**	**271-153**		**63.9%**
	S.J.RES.41	**2/4/1993**	**Sen Simon**	**Proposing an amendment to the Constitution to provide for a balanced budget**		**63-37**	**63.0%**
104th							
	H.J.RES.1	**1/4/1995**	**Rep Barton**	**Proposing a balanced budget amendment to the Constitution of the United States**		**64-35**	**64.6%**
	H.J.RES.73	3/12/1995	Rep McCollum	Proposing an amendment to the Constitution of the United States with respect to the number of terms of office of Members of the Senate and the House of Representatives	227-204		52.7%
105th							
	H.J.RES.2	1/7/1997	Rep McCollum	Proposing an amendment to the Constitution of the United States with respect to the number of terms of office of Members of the Senate and the House of Representatives	217-211		50.7%
	H.J.RES.78	5/18/1997	Rep Istook	Proposing an amendment to the Constitution of the United States restoring religious freedom	224-203		52.5%
	H.J.RES.119	5/14/1998	Rep DeLay	Proposing an amendment to the Constitution of the United States to limit campaign spending	29-345		7.8%
	S.J.RES.1	**1/21/1997**	**Sen Hatch**	**Proposing an amendment to the Constitution of the United States to require a balanced budget**		**66-34**	**66.0%**
106th							
	H.J.RES.94	**4/6/2000**	**Sen Sessions**	**Proposing an amendment to the Constitution of the United States with respect to tax limitations**	**234-192**		**54.9%**
107th							
	H.J.RES.41	**3/22/2001**	**Sen Sessions**	**Proposing an amendment to the Constitution of the United States with respect to tax limitations**	**232-189**		**55.1%**
	S.J.RES.4	2/7/2001	Sen Hollings	A joint resolution proposing an amendment to the Constitution of the United States relating to contributions and expenditures intended to affect elections		40-56	41.7%
108th							
	H.J.RES.83	6/2/2004	Rep Baird	Proposing an amendment to the Constitution of the United States regarding the appointment of individuals to fill vacancies in the House of Representatives.	63-353		15.1%

	H.J.RES.106	**9/30/2004**	**Rep Musgrave**	**Proposing an amendment to the Constitution of the United States relating to marriage.**	**227-186**		**55.0%**
109th							
	H.J.RES.10	6/22/2005	Rep Cunningham	Proposing an amendment to the Constitution of the United States authorizing the Congress to prohibit the physical desecration of the flag of the United States.	286-130*		68.8%
	S.J.RES.12	**6/27/2006**	**Sen Hatch**	**A joint resolution proposing an amendment to the Constitution of the United States authorizing Congress to prohibit the physical desecration of the flag of the United States.**		**66-34**	**66.0%**
	H.J.RES.88	**7/18/2006**	**Rep Musgrave**	**Proposing an amendment to the Constitution of the United States relating to marriage.**	**236-187**		**55.8%**
110th							
	None						