Appendix for "Zeros, quality and space: Trade theory and trade evidence" by Richard Baldwin and James Harrigan, June 2007.

This appendix provides a more complete treatment of the models discussed in the paper.

1.1. Comparative advantage: Eaton-Kortum

The slightly simplified version of the Eaton-Kortum (EK) model that we work with has C nations each of which is endowed with a single factor of production (labor) used to produce a continuum of goods under conditions of perfect competition and constant returns. The transport costs between a typical origin nation (nation-o) and a typical destination nation (nation-d) are assumed to be of the iceberg type and captured by the parameters $\tau_{od} \geq 1$ where τ_{od} is the amount of the good that must be shipped from o to sell one unit in d. The double-subscript notation follows the standard 'from-to' convention, so $\tau_{oo} = 1$ for all nations (intra-nation trade costs are zero). Consumer preferences are identical across nations and defined over the continuum of goods. They are described by a CES utility function, and expenditure on any given variety by a typical destination nation (nation-d) is

$$p_d(j)c_d(j) = \left(p_d(j)\right)^{1-\sigma} B_d; \qquad B_d \equiv \frac{E_d}{P_d^{1-\sigma}}, \quad P_d \equiv \left(\int_{j\in\Theta} \left(p_d(j)\right)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \tag{A1}$$

where $c_d(j)$ and $p_d(j)$ are nation-d's consumption and consumer price of good-j, P_d is the ideal CES price index, E_d is total expenditure (GDP in equilibrium), and σ is the elasticity of substitution among varieties. Without loss of generality, we order product indices such that the set of available goods Θ equals the unit interval.

Each nation's manufacturing technology – its' vector of unit labor input coefficients – comes from a stochastic technology-generation process much like the one used later by Melitz (2003). In the EK model, this exogenous process is costless and realizations are drawn before the analysis opens. Denoting nation-o's unit labor coefficient for good-j as $a_o(j)$, the model assumes that each $a_o(j)$ is an independent draw from the nation-specific cumulative distribution function (cdf)[1]

$$F_o[a] = 1 - e^{-T_o a^{\theta}}, \quad a \geq 0 \tag{A2}$$

[1] EK work with firm productivity as the random variable rather than the standard Ricardian labour input coefficient, namely $z = 1/a$, so their cdf is $\exp(-T/z^{\theta})$.

where $T_o > 0$ is a technology parameter that differs across countries. The expectation of $a_o(j)$ is $T^{-1/\theta}\Gamma\left[1+\frac{1}{\theta}\right]$, where Γ is the gamma function, so T_o can be thought of as nation-*o*'s average absolute advantage parameter, i.e. its technology level. Importantly, the draws are independent across goods and nations.

Although all nations can make all goods, perfect competition means only the lowest cost supplier actually sells in destination *d*. The price that each nation-*o* could offer for good-*j* in destination nation-*d* is:

$$p_{od}(j) = \tau_{od} w_o a_o(j) \tag{A3}$$

where w_o and $a_o(j)$ are nation-*o*'s wage and unit labor coefficient in good-*j*, respectively. Perfect competition implies that the equilibrium price for good-*j* in nation-*d* satisfies:

$$p_d(j) = \min_{o=1...C} \tau_{od} w_o a_o(j) \tag{A4}$$

Finding comparative advantage

The next step is to find the probability that a particular nation is the lowest cost supplier in a particular good in a particular market. This task involves a series of probability calculations that use two implications of (A2) and (A3). First, the cdf of $p_{cd}(j)$ is $G_{cd}[p] = 1-\exp(-p^{\theta}T_{cd})$ where $T_{cd} \equiv T_c(w_c\tau_{cd})^{-\theta}$.[2] Thus the probability that $p_{cd}(j) > k$ equals $\exp(-k^{\theta}T_{cd})$. Second, $p_{od}(j)$ is lower than the offer price of all other nations with probability 1 minus the probability that all other prices are higher. Since all draws of the *a*'s are independent across nations, the probability that all other prices are higher is $\prod_{c\neq o}\exp\left(-p_{cd}{}^{\theta}T_{cd}\right)$, which simplifies to $\exp\left(-p_{od}^{\theta}\left(\sum_{c\neq o}T_{cd}\right)\right)$. Since $p_{od}(j)$ is just one of many offer-prices that nation-*o* may have drawn, we must integrate over all possible $p_{od}(j)$, weighting each by

[2] Dropping subscripts where clarity permits, $\Pr\left(p\le k\right)=\Pr\left(aw\tau\le k\right)=\Pr\left(a\le\frac{k}{w\tau}\right)=1-\exp\left(-T\left(\frac{k}{w\tau}\right)^{\theta}\right)$, where *k* is an arbitrary price level. Noting that this holds for all *k* and the supports of *p* and *a* are identical, we get the result in the text.

its probability. Thus the probability of nation-*o* having a comparative advantage in good-*j* in market *d* is $\int_0^\infty \exp\left(-p_{od}^\theta\left(\sum_{c\neq o} T_{cd}\right)\right) dG_{od}[p]$ where $G_{od}[p]=1-\exp(-p^\theta T_{od})$. Solving the integral,

$$\pi_{od}=\frac{T_{od}}{\Delta_d}; \qquad \Delta_d \equiv \sum_{c=1}^{C} T_{cd} \tag{A5}$$

Here π_{od} is the probability that nation-*o* exports any given good-*j* to nation-*d*. Since the technology draws are independent across goods, π_{od} applies to each of the continuum of goods $j\in[0,1]$. Notice that Δ_d is akin to the inverse of the remoteness variable in standard gravity equations, i.e. it is an inverse index of the distance between nation-*d* and its trade partners, assuming that trade costs rise with distance.

Given the complexity of the model, it is remarkable that the expression for 'stochastic comparative advantage', π_{od}, is so simple and intuitive. Thinking of the T_{id}'s as the expected competitiveness of nation-*i*'s goods in nation-*d*'s market, the probability that nation-*o* is the most competitive in any given good is just the ratio of nation-*o*'s expected competitiveness to that of the sum of all nations. Notice that the probability π_{od} falls as the bilateral trade costs rises but rises with nation-*o*'s average absolute advantage parameter, T_o. As we shall see, expression (A5) is the key to characterizing the spatial pattern of zeros in the EK model.

Finding the equilibrium prices

To characterize the predictions for the spatial pattern of prices, we draw on two further implications of (A2) and (A3). First, with all draws independent across nations, the probability that we have $p_{cd}(j) > k$ for all origin-nations equals $\prod_{c=1}^{C}\exp\left(-k^\theta T_{cd}\right)$, which simplifies to $\exp\left(-k^\theta \Delta_d\right)$. Second, the probability of at least one nation having a $p_{cd}(j) < k$ is 1 minus the probability that $p_{cd}(j) > k$ for all nations, or $1-\exp\left(-k^\theta \Delta_d\right)$.[3]

[3] $\Pr\left(p_{cd} > k\right) = 1-\left(1-\exp\left(\frac{-T_c k^\theta}{w_c \tau_{cd}}\right)\right) = \exp\left(\frac{-T_c k^\theta}{w_c \tau_{cd}}\right)$. Since the draws are independent, the joint probability that they are all higher is $\prod_{c=1}^{C}\exp\left(\frac{-T_c k^\theta}{w_c \tau_{cd}}\right)$. Simplification yields the result in the text.

Since we do not know each nation's actual *a*'s, we cannot determine the price for any given good. Rather we find the distribution of the prices nation-*d* pays for a typical good. Due to competition, the price paid – i.e. the equilibrium price – is the lowest offer price as described by (A4). By definition, the cdf that describes the equilibrium price gives the probability that the equilibrium price is less than or equal to any particular level. To find the distribution that describes this 'lowest price', we use $F_{cd}[p]=1-\exp(-p^{\theta}T_{cd})$ and the independence of prices across goods and suppliers. Specifically, the probability that all $p_{od}(j)$'s are greater than an arbitrary level p_d is $\exp\left(-p_d{}^{\theta}\Delta_d\right)$, so the probability that at least one $p_{od}(j)$ is below p_d is $1-\exp\left(-p_d{}^{\theta}\Delta_d\right)$. This holds for all possible p_d and for any good-*j* so the nation-specific distribution that describes the equilibrium price for any good is

$$G_d[p]=1-\exp\left(-p^{\theta}\Delta_d\right) \tag{A6}$$

Because each good's *a* is identically and independently distributed, $G_d[p]$ describes the price distribution for any nation-*d*, $d=1,\ldots,C$ for any good $j \in [0,1]$.

Using (A1), (A6) and switching the variable of integration, it is easy to find the equilibrium CES price index for nation-*d*, namely $P_d{}^{1-\sigma}$ which is defined as $\int_{i\in\theta}\left(p_d(i)\right)^{1-\sigma}di$. As noted, we cannot determine the equilibrium price of any given good-*j*, but we know its cdf to be (A6). Moreover, with a continuum of varieties (which implies an infinite number of draws from $G_d[p]$), we know that the distribution of equilibrium prices across all varieties is identical to the underlying distribution $G_d[p]$ for any given variety. This means that $P_d{}^{1-\sigma}$ equals $\int_0^{\infty}p^{1-\sigma}dG_d[p]$. Solving the integral

$$P_d=\Delta_d^{-1/\theta}\left(\Gamma\left[\frac{1-\sigma+\theta}{\theta}\right]\right)^{1/(1-\sigma)} \tag{A7}$$

where the term in large parenthesis is the gamma function. This makes sense assuming the regularity condition 1 - σ+ θ > 1 holds.

The final task is to determine the distribution of prices for the goods that nation-*o* exports to nation-*d*. Since the probability of nation-*o* exporting any particular good to nation-*d* is π_{od} for all goods, the goods that nation-*o* actually exports to *d* is a random sample of all the goods that *d* buys. Thus, $G_d[p]$ also describes the cross-good distribution of the prices for the exports from every origin nation to nation-*d*. This elegant and somewhat surprising result follows from the fact that it is competition inside nation-*d* that determines prices, not the characteristics of any particular exporting nation. Successful exporting countries sell a large number of goods but do not on average charge lower prices. As we shall see, this result is the key to characterizing the spatial price implications in the EK model.

Linking export probability to observables

It is impossible to explicitly solve the EK model for general trade costs. The reason is that all nations' wages enter the system non-linearly, so we cannot use market clearing conditions to determine what wage a nation must have in order to sell all its output. More specifically, every T_{cd} contains the inverse of the wage of nation-*c*. This, together with the form of π_{od} means that each π_{od} is of order C in each wage. While solutions exist for $C \leq 5$, in practice the solution even for a pair of quadratic equations is typically too complicated to be useful. One can, however, easily find the wages in the case of autarky and free trade, as EK show. Without explicit solutions for the *w*'s, we cannot find a closed form solution for π_{od} and thus we cannot solve the precise pattern of zeros predicted by the model. Although this is a major drawback for a theoretical investigation, it poses no problems for our empirical work. We use data from a single exporting nation for a single year so all identification comes from the spatial variation in the data which occurs regardless of the level of wages.

We can link the T_{od}'s and thus the π_{od}'s to observable variables that allow estimation of the impact of distance and destination market-size on the probability of observing a zero. To this end, we specify the market-clearing condition for each origin nation. The share of nation-*d*'s total expenditure on manufactures from nation-*o* is π_{od} times E_d, where E_d is *d*'s total expenditure on manufactures. Rearranging yields a version of EK's expression 10, namely

$$V_{od} = \pi_{od} E_d \tag{A8}$$

where V_{od} is the value of all exports from nation-*o* to nation-*d*, and E_d is nation-*d*'s expenditure.[4] Nation-*o*'s market clearing condition is the summation of (A8) over all destination nations. Using (A5),

[4] Note that this is the expected expenditure of nation-*d* on nation-*o* goods, but since *o* exports an infinite number of goods to *d*, the realisation will be identical to the expectation by the law of large numbers.

the total sales of nation-*o* to all markets (including its own) equals the value of its total output, Y_o i.e. GDP, when[5]

$$Y_o = \frac{T_o}{w_o^{\theta}} \left(\sum_{d=1}^{N} \tau_{od}^{-\theta} \Delta_d^{-\theta} E_d \right) \tag{A9}$$

Solving (A9) for T_o/w_o^{θ}, using the definitions in (A5), and substituting out the Δ's using (A7), noting that the gamma functions cancel, we get

$$\pi_{od} = \frac{Y_o \tau_{od}^{-\theta} P_d^{\theta}}{\tau_{od}^{-\theta} E_d P_d^{\theta} + \sum_{c \neq d} \tau_{cd}^{-\theta} E_c P_c^{\theta}} \tag{A10}$$

1.2. Monopolistic competition

Our version of the monopolistic competition model has *C* countries and a single primary factor *L* that is used in the production of differentiated goods (manufactures) whose trade is subject to iceberg trade costs. Preferences are CES, so expenditure on manufactured good-*j* in typical nation-*d* is given by (A1).

Manufactured goods are produced under conditions of increasing returns and Dixit-Stiglitz monopolistic competition. Unlike the EK model, all firms in all countries face the same unit labor requirement, *a*. According to well-known properties of Dixit-Stiglitz monopolistic competition, nation-*o* firms charge consumer (i.e. c.i.f.) prices in nation-*d* equal to $p_{od} = \frac{\sigma}{\sigma - 1} w_o a \tau_{od}$. Consequently, the shipping (f.o.b.) price for any good is the same for every bilateral trade flow. The CES price index for typical nation-*d* involves the integral over all prices

$$P_d^{1-\sigma} = \sum_{c=1}^{C} \phi_{cd} n_c w_c^{1-\sigma}, \qquad \phi_{od} \equiv \tau_{od}^{1-\sigma} \in [0,1]$$

where we have, without loss of generality, chosen units such that $a = 1\text{-}1/\sigma$. The parameter ϕ_{od} reflects the 'freeness' of bilateral trade (ϕ ranges from zero when τ is prohibitive to unity under costless trade, i.e. $\tau = 1$). n_c is the number of goods produced in *c*.

1.2.1. Free entry conditions

With Dixit-Stiglitz competition, a typical nation-*o* firm's operating profit from selling in market-*d* is[6]

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[5] This is related to EK's unnumbered expression between their expressions 10 and 11.

$$\phi_{od} w_o^{1-\sigma} \frac{B_d}{\sigma} \quad \text{(A11)}$$

Summing across all *C* markets, total operating profit of a typical firm in nation-*o* is $\frac{w_o^{1-\sigma}}{\sigma} \sum_{c=1}^{C} \phi_{oc} B_c$.

Developing a new variety involves a fixed set-up cost, namely an amount of labor F_I (*I* for innovation).[7] In equilibrium, free entry ensures that the benefit and cost of developing a new variety match, so the free-entry condition for nation-*o* is

$$w_o^{1-\sigma} \sum_{c=1}^{C} \phi_{oc} B_c = w_o \sigma F_I \quad \text{(A12)}$$

for all *o* = 1,…,*C*. The equilibrating variables here are the per-firm demand shifters B_d and the wage.

1.2.2. **Employment condition and National budget constraint**

In equilibrium, all labor must be employed. The amount of labor used per variety is $ax + F_I$, where *x* is production of a typical good. Using the demand function, iceberg trade costs and equilibrium prices, the total production of a typical variety produced in *o* is $\sum_{c=1}^{C} (\tau_{oc} w_o)^{-\sigma} B_c$. Solving the integral and using the expression for *P*, the full employment condition for typical nation-*o* is:

$$n_o \left(\sum_{c=1}^{C} (\tau_{oc} w_o)^{-\sigma} B_c + F_I \right) = L_o \quad \text{(A13)}$$

The final equilibrium expression requires that expenditure equals income. Since free entry eliminates pure profits, all income comes from labor income, and so equals $w_o L_o$. The national budget constraint is thus:

$$E_o = w_o L_o \quad \text{(A14)}$$

1.2.3. **Equilibrium**

There are three endogenous variable here for each nation, *w*, *n* and *E* and three equilibrium conditions, the free entry, employment and national budget constraint conditions. As usual, the three equilibrium

[6] Operating profit is proportional to firm revenue since the first order condition $p(1-1/\sigma)=a$ implies $(p-a)c$, equals pc/σ.
[7] To relate this model to the previous one and the next, it is as if a firm must pay F_I to take a draw from the technology-generating distribution, but the distribution is degenerate, always yielding $a=1-1/\sigma$.

conditions – the free entry, employment and national budget constraint conditions – are not independent since we derived the demand equations imposing individual budget constraints. This redundancy allows us to drop one equilibrium condition and choose the labor of one nation as the numeraire.

Unfortunately, it is impossible to solve the model analytically for the same reason the EK model could not be solved – the wages enter the model in a highly non-linear manner. Specifically, we can use (A14) to eliminate the *E*'s and our expression for the price index to get the free entry condition in terms of the *n*'s and *w*'s only. Then we can use mill pricing to express the free-entry condition as $x_o = w_o^{-\sigma} F_I \sigma$, where x_o is the output of a typical firm in nation-*o*, so that the employment condition becomes $n_o = L_o / \{F_I (w_o^{-\sigma}(\sigma - 1) + 1)\}$. This gives us two equations per nation in the *n*'s and *w*'s. However, the *w*'s enter these equations with non-integer powers and this renders analytic solutions impossible.

As before, this lack of tractability is not a problem for our empirics since we work with a single exporter and a single year of data. The key is that given the CES demand structure, the choke-point price is infinity so every importing nation will buy some of every variety produced by every nation. Moreover, given Dixit-Stiglitz monopolistic competition, mill pricing is optimal so the export (i.e. f.o.b.) price should be the same for every destination regardless of transportation costs.

1.2.4. **Aside: MC with an 'outside' sector**

A standard theoretical artifice yields analytic solutions pinning down the wage in all nations. The trick is to introduce a Walrasian sector whose output is costlessly traded. Assuming nations are similar enough in size for all nations to produce some of this 'outside' good, free trade equalizes wages globally. Choosing the outside good as numeraire and choosing its units such that its prices equals the wage, free trade equalizes all wages to unity worldwide. Under this artifice, the free entry condition for nation-*o* is

$$\sum_{c=1}^{C} \phi_{oc} B_c = \sigma F_I \qquad \text{(A15)}$$

The equilibrating variables here are the per-firm demand shifters B_d. The C free-entry conditions are linear in the B_d's and so easily solved.[8] In matrix notation

$$\mathbf{B} = \mathbf{\Phi}^{-1}\sigma F_I \tag{A16}$$

where $\mathbf{\Phi}$ is an $C \times C$ matrix of bilateral trade freeness parameters (e.g., the first row of $\mathbf{\Phi}$ is $\phi_{11},\ldots,\phi_{1C}$), and $\mathbf{B}$ is the $C \times 1$ vector of B_d's. This shows that the equilibrium B's depend upon bilateral trade freeness in a complex manner; all the ϕ's affect every B. The complexity can be eliminated by making strong assumptions on trade freeness, e.g. imposing $\phi_{od}=\phi$ for all trade partners, but we retain arbitrary ϕ_{od}'s. Importantly, the equilibrium B's are completely unrelated to market size; they depend only upon the parameters of bilateral trade freeness. The deep economic logic of this has to do with the Home Market Effect; big markets have many firms since firms enter until the per-firm demand is unrelated to market size.[9]

We can characterize the equilibrium without decomposing the $\mathbf{B}$ into their components (E's and n's) but doing so is awkward because the B's do not map cleanly into real world variables. The natural equilibrating variable – the mass of firms in each nation, n_c – can be extracted from the B's. Using the definition of the CES price index, Dixit-Stiglitz mark-up pricing and nation-wise symmetry of varieties, $P_d^{1-\sigma} = \sum_{c=1}^{C} n_c \phi_{cd}$. Using this, along with the definition of B_d in (A1), we write the C definitions of the B's (with a slight abuse of matrix notation) as $\mathbf{\Phi}'\mathbf{n} = \mathbf{E}/\mathbf{B}$, where $\mathbf{n}$ is the $C \times 1$ vector of n_c's and $\mathbf{E}/\mathbf{B}$ is defined as ($E_1/B_1, \ldots, E_C/B_C$). Solving the linear system

$$\mathbf{n} = \mathbf{\Phi}'^{-1}\left[\frac{E_1}{B_1},\ldots,\frac{E_C}{B_C}\right] \tag{A17}$$

Each n_o directly involves all the ϕ's, all the E's, and all the B's (each of which involves all the ϕ's). Solutions for special cases are readily available, but plainly the equilibrium n's are difficult to characterize for general size and trade cost asymmetries. The complexity of (A17) is the heart of the difficulties the profession has in specifying the Home Market Effect in multi-country models (see Behrens, Lamorgese, Ottaviano and Tabuchi 2004).

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[8] This solution strategy follows Behrens, Lamorgese, Ottaviano and Tabuchi (2004).

[9] In the terminology of Chamberlinian competition, the extent of competition rises until the residual demand curve facing each firm (i.e. $p^{-\sigma}B$) shifts in to the point where each firm is indifferent to entry. Since entry costs are identical in all markets, the residual demand-curve must be in the same position in every market.

Notice that under this artifice, an increase in a nation's *L* is fully offset by a rise in its *n*, so it *B* remains unaffected. This can happen since labor can be drawn from the outside sector at a constant wage rate. In the baseline model without the outside good, the rise in *L* results partly in a rise in *n* and partly in a rise in *w*. Or, to put it differently, the Home Market Effect is much stronger in the model with the outside good since a rising wage does not dampen the profits of local firms.

1.3. A multi-nation asymmetric HFT model

Our HFT model embraces all of the demand, market-structure and trade cost features of the MC model above but adds in two new elements – beachhead costs (i.e. fixed market-entry costs) and hetero-generous marginal costs at the firm level. Firm-level heterogeneity is introduced – as in the EK model – via a stochastic technology-generation process. When a firm pays its standard Dixit-Stiglitz cost of developing the 'blueprint' for a new variety, F_I, it simultaneously draws a unit labor coefficient '*a*' associated with the blueprint from the Pareto cdf[10]

$$G[a] = \left(\frac{a}{a_0}\right)^{\kappa}, \qquad 0 \le a \le a_0 \tag{A18}$$

After seeing its *a*, the firm decides how many markets to enter. Due to the assumed Dixit-Stiglitz market structure, the firm's optimal price is proportional to its marginal cost, its operating profit is proportional to its revenue, and its revenue in a particular market is inversely proportional to its relative price in the market under consideration.

1.3.1. Cut-off conditions

Thus, the cut-off conditions that define the maximum-marginal-cost thresholds for market-entry are

$$\phi_{od} B_d w_o^{1-\sigma} a_{od}{}^{1-\sigma} = w_o f; \qquad f \equiv \sigma F (1-1/\sigma)^{1-\sigma} \tag{A19}$$

for all *o*, *d* = 1,…,*N*, where *F* is the beachhead cost (identical all firms in all nations for notational simplicity). Here B_d is defined as in (A1), and the endogenous a_{od}'s are the cut-off levels of marginal costs for selling from nation-*o* to nation-*d*.

[10] The EK and HFT models work well with a broad family of distributions, but the analytics are more transparent with an explicit distribution, e.g. either the Pareto or exponential distributions; the Pareto is traditional in HFT models. This formulation of the randomness differs trivially from Melitz, who, like EK, works firm-level efficiency (i.e. 1/*a*).

1.3.2. **The free entry conditions**

From the cut-off conditions, we know that not all blueprints will be produced. Thus the mass of blueprints in typical nation *o* – what we call m_o – exceeds the mass of produced varieties – what we call n_o in line with standard MC model notation. Usual Dixit-Stiglitz results imply that the mass of blueprints rises to the point where potential entrants are just indifferent to sinking the development costs w_oF_I and taking a draw from the technology-generating distribution (A18).

A potential entrant in *o* knows the various *a*'s that may be drawn will result in different levels of operating profit. Before paying w_oF_I to take a draw from (A18), the firm forms an expectation over all possible draws using its knowledge of the thresholds defined by (A19). The expected value of drawing a random *a* is $\sum_{d=1}^{C}\int_0^{a_{od}}(\phi_{od}B_d w_o^{1-\sigma}a^{1-\sigma}-w_o f)dG[a]/\left(\sigma\left(1-1/\sigma\right)^{1-\sigma}\right)$. Here each term in the sum reflects the expected operating profit from selling to a particular market (net of the beachhead cost) taking account of the fact that the firm only finds it profitable to sell to the market if it draws a marginal cost below the market-specific threshold marginal cost, a_{od}. Potential entrants are indifferent to taking a draw when this expectation just equals the set-up cost, w_oF_I, so the free-entry conditions hold when $\sum_{d=1}^{C}\int_0^{a_{od}}(\phi_{od}B_d w_o^{1-\sigma}a^{1-\sigma}-w_o f)dG[a]=w_oF_I\sigma(1-1/\sigma)^{1-\sigma}$ for each nation *o*. Solving the integrals (assuming the regularity condition $1 - \sigma + \kappa > 0$ so the integrals converge), the free-entry condition for nation-*o* is

$$\sum_{d=1}^{C}\left(\frac{\phi_{od}B_d w_o^{1-\sigma}a_{od}^{1-\sigma}}{1-1/\beta}-w_o f\right)a_{od}^{\kappa}=w_o f_I; \qquad f_I\equiv F_I\sigma(1-1/\sigma)^{1-\sigma}, \quad \beta\equiv\frac{\kappa}{\sigma-1}>1$$

We use the cut-off conditions to write the free entry condition more simply as

$$f\sum\nolimits_{d=1}^{N}a_{od}^{k}=(\beta-1)f_I \tag{A20}$$

Here we have, without loss of generality, chosen units such that a_0 is unity.

1.3.3. Employment condition and National budget constraint

The labor demand arising from the sale of produced varieties in market-*d* is

$\int_0^{a_{od}} \left[\left(\frac{w_o}{1-1/\sigma} \right)^{-\sigma} a^{1-\sigma} B_d + F \right] m_o dG[a]$. Solving the integral yields

$a_{od}^{\kappa} m_o \left[\left(\frac{1-1/\sigma}{1-1/\beta} \frac{1}{w_o} \right) \left(\frac{w_o a_{od}}{1-1/\sigma} \right)^{1-\sigma} B_d + F \right]$. Using the cut-off condition, this simplifies to

$m_o F \left(\frac{\sigma\beta - 1}{\beta - 1} \right) a_{od}^{\kappa}$. Summing over the labor demand from sales to all markets, adding in the labor demand from developing new blueprints and setting this equal to the labor supply in nation *o*, the full employment condition is

$$m_o F \left(\frac{\sigma\beta - 1}{\beta - 1} \right) \sum_{d=1}^{C} a_{od}^{\kappa} + m_o F_I = L_o$$

Using the free entry condition this simplifies even further to

$$m_o = \frac{L_o}{\sigma\beta F_I}, \qquad o = 1, \ldots C \qquad \text{(A21)}$$

Finally, the national budget constraint is just $E_o = w_o L_o$ since there are no pure profits in equilibrium (the pure profits earned by active firms just pays for the pure losses incurred by firms that abandon their blueprints and never produce).

1.3.4. Equilibrium

There are C^2 threshold a_{od}'s, and *C m*'s, *E*'s and *w*'s; these are determined by the C^2 cut-off conditions, *C* free entry conditions, employment conditions and national budget constraints. We can eliminate the *E*'s with the national budget constraints and lack of pure profit, and the *m*'s with (A21). This leaves C^2 cut-off thresholds and the *C w*'s to be determined from the C^2 cut-off conditions and the *C* free entry conditions. Since the *w*'s enter the cut-off and free-entry conditions with different non-integer powers, there is no analytic solution to the system. Numerical solutions, however, are readily available. Simulation results (available upon request) demonstrate that the *B*'s for big nations (i.e. nations with high *L*'s) are larger than the *B*'s for small nations. Thus a nation's real GDP can be used as a proxy for its *B*.

1.3.5. Aside: Asymmetric HFT with an 'outside' sector

In earlier drafts of this paper, we worked with an outside sector. The result, as in the MC-with-outside-sector model considered above, was that the *B*'s are completely unrelated to market size. This implies that the threshold marginal costs are independent of market size and thus the number of export zeros should be independent of market size. Since this is clearly counterfactual (see Table 4), we decided to eliminate the theoretical artifice of an outside sector despite the fact that this modeling choice implies a lack of analytic solutions.

1.3.6. HFT's spatial pattern of zeros and prices

The spatial pattern of zeros comes from the cut-off thresholds. For a typical nation's export matrix, there should be more zeros with more distant partners. More formally, consider the firm that produces variety-*j* with marginal costs *a*(*j*). The probability of this firm exporting to nation-*d* is the probability that its marginal cost is less than the threshold defined in (A19), namely

$$\Pr\left\{a(j) < \frac{B_d^{\ 1/(\sigma-1)}}{\tau_{od} w_o^{\sigma} f^{1/(\sigma-1)}}\right\} = \frac{B_d^{\beta}}{\tau_{od}^{\kappa} w_o^{\sigma\kappa} f^{\beta} a_0^{\kappa}} \tag{A22}$$

where we used the Pareto distribution to evaluate the probability. In our empirics, we only have data on products that are actually exported to at least one market so it is useful to derive the expression for the conditional probability, i.e. the probability that a firm exports to market *j* given that it exports to at least one market. This conditional probability of exports from *o* to *d* by typical firm *j* is

$$\frac{\tau_{od}^{-\kappa} B_d^{\beta}}{\min_{c\neq o} \tau_{oc}^{-\kappa} B_c^{\beta}} \tag{A23}$$

The wage drops out since we work with data for a single exporting nation. Again, for a typical exporting nation-*o*, the denominator is the same for all destination markets. As discussed in the previous subsection, market size in *d* will be positively related to GDP in *d*. Equation (A23) thus illustrates that the probability of a good being exported from nation-*o* depends positively on the destination nation's GDP and negatively on trade costs between *o* and *d*.

The spatial pattern of prices in the HFT model is also simple to derive. We consider both the export (f.o.b.) price for a particular good exported to several markets, and the average export (f.o.b.) price for all varieties exported by a particular nation. As the HFT model relies on Dixit-Stiglitz monopolistic competition, mill pricing is optimal for every firm, so the f.o.b. export price each good exported should be identical for all destinations. For example, export prices should be unrelated to bilateral distance and unrelated to the destination-nation's size. When it comes to the average export price – i.e. the weighted average of the f.o.b. prices of all varieties exported from nation-*o* to nation-*d* – the cut-off conditions imply $\bar{p}_{od} = \int_0^{a_{od}} \left(\frac{\tau_{od} w_o a}{1-1/\sigma} \right)^{1-\sigma} dG\left(a_{od} \mid a \le a_{oo}\right)$, where $\bar{p}_{od}$ is the average f.o.b. price. Solving the integral,

$$\bar{p}_{od} = \delta_o \tau_{od}^{-\kappa} \left(\frac{B_d}{f} \right)^{\frac{1+\kappa-\sigma}{\sigma-1}} \tag{A24}$$

where δ_o is a function of parameters and country *o* variables only. Since the maximum marginal cost falls a_{od} with bilateral distance, the average export price of nation-*o* varieties in nation-*d* should be lower for more distant trade partners.

1.3.7. The Melitz-Ottaviano model

Melitz and Ottaviano (2005) work with the Ottaviano, Tabuchi and Thisse (2002) monopolistic competition framework and assume *C* nations, a single factor of production, *L*, and iceberg trade costs. They do not allow for beachhead costs. Adopting the standard outside-good artifice to pin down wages, they assume that there are two types of goods: a costlessly traded Walrasian good that equalizes wages internationally, and differentiated goods produced under conditions of monopolistic competition and increasing returns. Nations can be asymmetric in terms of size (i.e. their *L* endowment) and location (i.e. the bilateral iceberg trade costs faced by their firms).

The Ottaviano *et al* framework assumes quasi-linear preferences and this generates a linear demand system where income effects have been eliminated. As usual in the monopolistic competition tradition, there are many firms each producing a single differentiated variety. Since the firms are small, they ignore the impact of their sales on industry-wide variables. Practically, this means that the producer of each differentiated variety acts as a monopolist on a linear residual demand curve. Indirectly, however, firms face competition since the demand curve's intercept declines as the number of competing varieties rises. Specifically, the residual demand curve in market-*d* facing a typical firm is:[11]

$$c_d(i) = \frac{L_d}{\gamma}\left(B_d - p_d(i)\right); \qquad B_d \equiv \frac{\alpha\gamma + P_d}{n_d^c + \gamma}; \quad P_d \equiv \int_{\Theta_d} p_d(j)dj \tag{A25}$$

where L_d is the number of consumers in *d* (and thus nation's labor supply since each person has one unit of labor), B_d is the endogenous *y*-axis intercept (the per-firm demand shifter as in the HFT model), and n_d^c is the mass of varieties consumed in *d* (since not all varieties are traded, we need a separate notation for the number of varieties produced and consumed). Finally, P_d is the price index and Θ_d is the set of varieties sold in market-*d*. Inspection of (A25) reveals two channels thorough which a typical firm faces indirect competition: 1) a *ceteris paribus* increase in the number of varieties consumed, n^c, lowers the intercept *B*, and 2) a decrease in the price index *P* lowers the intercept.

The linear demand system makes this model extremely simple to work with. Atomistic firms take B_d as given and act as monopolists on their linear residual demand curve. A monopolist facing a linear demand curve sets its price halfway between marginal cost and the intercept. Thus optimal prices are linked to heterogeneous marginal costs via

$$p_{od}[a] = \frac{B_d + a\,\tau_{od}}{2} \tag{A26}$$

Here p_{od} is the consumer (i.e. c.i.f.) price and $\tau_{oo} = 1$ for all nations *o*. The operating profit earned by a firm that sells to market-*d* is then

$$\pi_{od}[a] = \frac{L_d}{4\gamma}\left(B_d - a\tau_{od}\right)^2 \tag{A27}$$

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[11] The utility function for the representative consumer is $U = c_0 + \alpha\int_\Theta c(j)dj - \frac{\gamma}{2}\int_\Theta c(j)^2 dj - \frac{\eta}{2}\left(\int_\Theta c(j)dj\right)^2$ where c_0 is consumption of the numeraire and c_j is consumption of variety *j*. We assume that each economy is large enough so that some numeraire is made and consumed in both nations regardless of trade barriers. To reduce notational clutter, we normalise $\eta = 1$ by choice of units (and thus without loss of generality).

Cut-off and free-entry conditions

It is immediately obvious from (A25) that firms with marginal costs above the demand curve intercept B_d find it optimal to sell nothing to market-*d*. This fact defines the C^2 cut-off conditions

$$a_{od} = \frac{B_d}{\tau_{od}}, \qquad \forall o,d = 1,...,C \tag{A28}$$

Note that (A28) implies that export cutoffs into market *d* are just a fraction of the domestic survival cutoff, $a_{od} = \frac{a_{dd}}{\tau_{od}}$. The expected operating profit in all markets to be earned from a random draw from $G[a]$ is $\frac{1}{4\gamma}\sum_{d=1}^{C} L_d \int_0^{a_{od}} \left(B_d - a\tau_{od}\right)^2 dG[a]$. The free entry condition is that this expected profit equals the entry cost F_I. Using (A28) to eliminate B_d, assuming the Pareto distribution (A18) for $G[a]$ and solving the integrals, and finally substituting $a_{od} = \frac{a_{dd}}{\tau_{od}}$, the free-entry condition is

$$\sum_{d=1}^{C} L_d \phi_{od} a_{dd}^{2+\kappa} = f_I, \qquad \phi_{od} \equiv \tau_{od}^{-\kappa}, \qquad f_I \equiv F_I 2\gamma(2+\kappa)(1+\kappa) \tag{A29}$$

for all $o = 1,\ldots C$. This is a system of C equations in the C domestic survival cutoffs a_{dd}. The ϕ's reflect the 'freeness' of bilateral trade, i.e. $\phi_{od} = 0$ corresponds to infinite trade costs ($\tau_{od} = \infty$) and $\phi_{od} = 1$ corresponds to free trade ($\tau_{od} = 1$). The system can be written in matrix notation as $\mathbf{L\Phi\tilde{A}} = f_I \boldsymbol{\iota}$, where $\mathbf{L} = diag\left[L_1,\ldots,L_C\right]$ is a $C \times C$ matrix with country sizes along the diagonal and zeros elsewhere, $\mathbf{\Phi}$ is a $C \times C$ symmetric positive definite[12] matrix with typical element ϕ_{od}, $\mathbf{\tilde{A}} \equiv \left[a_{11}^{2+k},...,a_{CC}^{2+k}\right]^T$ is a $C \times 1$ vector of transformed domestic survival cutoffs, and $\boldsymbol{\iota}$ is a $C \times 1$ vector of ones. The solution for the $a_{dd}^{2+\kappa}$ terms is $\mathbf{\tilde{A}} = \mathbf{\Phi}^{-1}\mathbf{L}^{-1} f_I$. Denoting the *d*-th diagonal element of $\mathbf{\Phi}^{-1}$ as $\tilde{\phi}_d$, the equilibrium cut-offs are

$$a_{od} = \frac{1}{\tau_{od}}\left(\frac{\tilde{\phi}_d}{L_d}\right)^{\frac{1}{2+k}}, \qquad \forall o,d = 1,...,C \tag{A30}$$

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[12] Symmetry follows by $\tau_{od} = \tau_{do}$. A sufficient condition for non-singularity is that trade costs depend on distance, and that no two countries occupy the same point. Positive definiteness follows because the diagonal elements are 1 and the off-diagonal elements are between 0 and 1.

Using this and the optimal pricing rule in (A26) with the cutoff condition (A28), the equilibrium cif import prices are

$$p_{od}[a] = \frac{1}{2}\left(\left(\frac{\tilde{\phi}_d}{L_d} \right)^{\frac{1}{2+k}} + a\tau_{od} \right)$$

Weighted average f.o.b. export prices are computed by dividing $p_{od}[a]$ by bilateral trade costs, and integrating over the density of *a* conditional on exporting from *o* to *d*:

$$\overline{p}_{od} = \int_0^{a_{od}} \frac{p_{od}[a]}{\tau_{od}} dG(a \mid a \le a_{od}) = \frac{1+2\kappa}{(2+2\kappa)\tau} \left(\frac{\tilde{\phi}_d}{L_d} \right)^{\frac{1}{2+\kappa}} \qquad \text{(A31)}$$

MO's spatial pattern of zeros and prices

Inspection of (A30) and (A31) yield the predictions for zeros and prices. Expression (A30) shows that the threshold marginal cost falls with bilateral trade costs and with the size of the destination market[13]. Using these facts with the distribution of *a*'s, we see that zeros are more likely with partners that are distant and large. The counter-intuitive (and counter-factual) prediction for market size on zeros is an implication of the Home Market Effect; large markets have many local firms which implies more severe competition for foreign firms (i.e. a lower P_d and thus lower B_d). Given this intuition for the cutoffs, expression (A31)is not surprising: average f.o.b. export prices are falling with bilateral distance and will be lower for partners with big markets.

1.4. The Quality HFT model

Here we lay out all the assumptions and solve the quality-based heterogeneous-firms trade model that was introduced in the text.

As usual, we assume a world with *C* nations and a single factor of production *L*. The goods produced consist of a continuum of goods that we refer to as manufactures. All goods are traded; labor is internationally immobile and inelastically supplied. CES preferences are as usual with one major difference, which is that consumers value "quality". The utility function is

[13] The bilateral thresholds for exporting to *d* also depend in a complex way on the full distribution of world transport costs through the term $\tilde{\phi}_d$. In a world where all countries are equidistant, $\tilde{\phi}_d$ will not vary across countries.

$$U = \left(\int_{i \in \Theta} (c_i q_i)^{1-1/\sigma} di \right)^{1/(1-1/\sigma)}; \qquad \sigma > 1 \tag{A32}$$

where c and q are the consumption and quality of a typical variety and Θ is the set of consumed varieties. The corresponding expenditure function for nation-*d* is

$$p_d(j)c_d(j) = \left(\frac{p_d(j)}{q(j)} \right)^{1-\sigma} B_d; \qquad B_d \equiv \frac{E_d}{P_d^{1-\sigma}}, \quad P_d \equiv \left(\int_{j \in \Theta} \left(\frac{p_d(j)}{q(j)} \right)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \tag{A33}$$

where $\frac{p_d(j)}{q(j)}$ is the quality-adjusted price of good-*j*, *E* is expenditure, and *P* the CES index of quality-adjusted prices.

Manufacturing firms have constant marginal production costs and three types of fixed costs. The first fixed cost, F_I, is the standard Dixit-Stiglitz cost of developing a new variety. The second and third fixed costs are beachhead costs that reflect the one-time expense of introducing a new variety into a market. Its cost *F* units of *L* to introduce a variety into any market and potential manufacturing firms pay F_I to take a draw from the random distribution of unit labor coefficients, the *a's*. By assumption, quality is linked to marginal cost (the *a*'s) by

$$q(j) = (a(j))^{1+\theta}, \qquad \theta > -1 \tag{A34}$$

where $1+\theta$ is the elasticity of quality with respect to *a*. We could easily generalize the model by allowing a positive correlation between costs and quality, but doing so would raise the level of complexity without providing any compensating insight. The assumed distribution of the *a*'s is

$$G[a] = 1 - \left(\frac{a_0}{a} \right)^{\kappa}, \quad a_0 \le a \tag{A35}$$

(This *G* is distinct from the one in the baseline HFT model.) Notice that it is necessary to flip the usual Pareto distribution for *a*'s to ensure that there are fewer high quality (i.e. high *a*) firms than low quality firms. Without loss of generality, we choose units of manufactures such that $a_0 = 1$.

At the time it chooses prices, the typical firm takes its quality and marginal cost as given, so it faces a demand that can be written as $\left(p(j)/q(j) \right)^{-\sigma} B_d$ where *q(j)* is its quality. Since *p* enters this in the standard way, the standard Dixit-Stiglitz results therefore obtain; mill-pricing with a constant mark-up, $\sigma/(\sigma-1)$, is optimal for all firms in all markets and operating profit is a constant fraction, $1/\sigma$, of firm revenue market by market. Using these facts, operating profit for a typical nation-*o* firm selling in nation-*d* is

$$\left(\frac{a^{-\theta}w_o}{1-1/\sigma}\right)^{1-\sigma}\frac{B_d}{\sigma} \qquad \text{(A36)}$$

The only substantial difference between this and the corresponding expression for profits without quality differences is the θ in the exponent.

Plainly, the properties of this model depend crucially on how elastic quality is with respect to the unit input coefficient. For $\theta \in [-1,0)$, quality increases slowly with cost and the optimal quality-adjusted consumer price increases with cost. In this case, a firm's revenue and operating profit fall with its marginal cost. For $\theta > 0$, by contrast, quality increases quickly enough with marginal cost to ensure that the quality-adjusted price falls as *a* rises. The means that higher *a*'s are associated with higher operating profit. Henceforth we focus on the $\theta > 0$ case.

1.4.1. Cut-off conditions

The cut-off condition for selling to typical market-d is

$$\phi_{od} w_o^{1-\sigma} a_{od}{}^{\theta(\sigma-1)} B_d = w_o f \; ; \qquad f \equiv F\sigma(1-1/\sigma)^{1-\sigma} \qquad \text{(A37)}$$

(This *f* is distinct from the *f* in the HFT model.) With $\theta > 0$, this tells us that only firms with sufficiently high-price/high-quality goods find it worthwhile to sell in a given market. Moreover, controlling for the per-firm demand, the threshold quality rises for more distant markets (since ϕ falls with distance). Notice that the $a_{od}(j)$'s here are minimum cost thresholds rather than maximums as in the standard HFT model.

1.4.2. Free-entry conditions

Turning to the free-entry conditions, a potential entrant pays F_I to develop a new variety with a randomly assigned *a* and associated quality $a^{1+\theta}$. After observing its *a*, the potential entrant decides which markets to enter. In equilibrium, free entry drives expected pure profits to zero. The free entry condition for typical nation-*o* is

$$\sum_{d=1}^{N}\int_{a_{od}}^{\infty}(\phi_{od}w_o^{1-\sigma}a^{\theta(\sigma-1)}B_d - w_o f)dG[a] = w_o f_I \ ; \qquad f_I \equiv F_I\sigma(1-1/\sigma)^{1-\sigma}$$

Assuming the regularity condition $\theta(\sigma\text{-}1)\text{-}k < 0$, this solves to[14]

$$\sum_{d=1}^{N}\left(\frac{\phi_{od}w_o^{1-\sigma}B_d a_{od}{}^{\theta(\sigma-1)}}{1-1/\beta} - w_o f\right)a_{od}^{-\kappa} = w_o f_I \ ; \qquad f_I \equiv F_I\sigma(1-1/\sigma)^{1-\sigma}\ , \quad \beta \equiv \frac{k/\theta}{\sigma-1} > 1$$

Using the cut-off conditions as in the HFT model, the free entry condition is

$$f\sum_{d=1}^{N}a_{od}^{-\kappa} = (\beta-1)f_I \qquad \text{(A38)}$$

Inspection of the $N(N\text{-}1)$ equilibrium conditions defined by (A38) reveals that the QHFT model is isomorphic to the HFT model apart from the definition of the constants, powers and the fact that the a_{od}'s are minimums rather than maximums. Thus our analysis of the HFT model applies here directly and so need not be repeated.

One point that bares some study is the spatial implications for average prices. As in the HFT model, distance acts as selection device on varieties, but the highest priced variety are the most competitive, the basket of varieties sold in distant markets (controlling for B_d of course) will have a higher average price than the basket for a near-by market. The impact of distance and market size on zeros, however, will be identical to that of the HFT model.

Appendix Reference

Behrens, Kristian, Andrea Lamorgese, Gianmarco Ottaviano, and Takatoshi Tabuchi, 2004, "Testing the Home Market Effect in a Multi-Country World: The Theory," CEPR Discussion Papers 4468.

[14] The typical integral is $\int_{a_{od}}^{\infty}(\phi_{od}w_o^{1-\sigma}B_d a^{\theta(\sigma-1)} - w_o f)a_0^{-\kappa}\kappa a^{-\kappa-1}a_0^{\kappa}da$. As long as $\theta(\sigma\text{-}1)\text{-}k<0$, this solves to $\left(\frac{\kappa\phi_{od}w_o^{1-\sigma}B_d a_{od}^{\theta(\sigma-1)}}{\kappa-\theta(\sigma-1)} - w_o f\right)a_{od}^{-\kappa}$. Using the definition of β yields (A38).