Appendix to 'Explaining the Effects of Government Spending Shocks on Consumption and the Real Exchange Rate'

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This appendix presents the firms optimality conditions and the complete set of competitive equilibrium conditions.

1 Firm's Optimality Conditions

Write the Lagrangian of the firm as:

$$
\begin{aligned}
\mathcal{L} \;=\; & E_0 \sum_{t=0}^{\infty} r_{0,t} \Big\{ P_{i,a,t}(c_{i,a,t} + g_{i,a,t}) + P^*_{i,a,t}(c^*_{i,a,t} + g^*_{i,a,t}) - W_t(c_{i,a,t} + g_{i,a,t} + c^*_{i,a,t} + g^*_{i,a,t}) \\
& + P_{a,t}\nu^c_{i,a,t}\left[\left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x^c_{a,t} + \theta^c_a s^c_{i,a,t-1} - c_{i,a,t}\right] \\
& + P_{a,t}\nu^g_{i,a,t}\left[\left(\frac{P_{i,a,t}}{P_{a,t}}\right)^{-\eta} x^g_{a,t} + \theta^g_a s^g_{i,a,t-1} - g_{i,a,t}\right] \\
& + P^*_{a,t}\nu^{c*}_{i,a,t}\left[\left(\frac{P^*_{i,a,t}}{P^*_{a,t}}\right)^{-\eta} x^{c*}_{a,t} + \theta^{c*}_a s^{c*}_{i,a,t-1} - c^*_{i,a,t}\right] \\
& + P^*_{a,t}\nu^{g*}_{i,a,t}\left[\left(\frac{P^*_{i,a,t}}{P^*_{a,t}}\right)^{-\eta} x^{g*}_{a,t} + \theta^{g*}_a s^{g*}_{i,a,t-1} - g^*_{i,a,t}\right] \\
& + P_{a,t}\lambda^c_{i,a,t}\left[s^c_{i,a,t} - \rho s^c_{i,a,t-1} - (1-\rho)c_{i,a,t}\right] \\
& + P_{a,t}\lambda^g_{i,a,t}\left[s^g_{i,a,t} - \rho s^g_{i,a,t-1} - (1-\rho)g_{i,a,t}\right] \\
& + P^*_{a,t}\lambda^{c*}_{i,a,t}\left[s^{c*}_{i,a,t} - \rho s^{c*}_{i,a,t-1} - (1-\rho)c^*_{i,a,t}\right] \\
& + P^*_{a,t}\lambda^{g*}_{i,a,t}\left[s^{g*}_{i,a,t} - \rho s^{g*}_{i,a,t-1} - (1-\rho)g^*_{i,a,t}\right] \Big\}
\end{aligned}
$$

The optimality conditions associated with the firm's problem with respect to $P_{i,a,t}$, $P^*_{i,a,t}$, $c_{i,a,t}$,

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$g_{i,a,t}$, $c^*_{i,a,t}$, $g^*_{i,a,t}$, $s^c_{i,a,t}$, $s^g_{i,a,t}$, $s^{c*}_{i,a,t}$, $s^{g*}_{i,a,t}$, respectively, are

$$c_{i,a,t} + g_{i,a,t} = \eta \frac{P_{a,t}}{P_{i,a,t}} \left[\nu^c_{i,a,t}(c_{i,a,t} - \theta^c_a s^c_{i,a,t-1}) + \nu^g_{i,a,t}(g_{i,a,t} - \theta^g_a s^g_{i,a,t-1}) \right]$$

$$c^*_{i,a,t} + g^*_{i,a,t} = \eta \frac{P^*_{a,t}}{P^*_{i,a,t}} \left[\nu^{c*}_{i,a,t}(c^*_{i,a,t} - \theta^{c*}_a s^{c*}_{i,a,t-1}) + \nu^{g*}_{i,a,t}(g^*_{i,a,t} - \theta^{g*}_a s^{g*}_{i,a,t-1}) \right]$$

$$-P_{a,t}\nu^c_{i,a,t} + P_{i,a,t} - W_t - (1-\rho)P_{a,t}\lambda^c_{i,a,t} = 0$$

$$-P_{a,t}\nu^g_{i,a,t} + P_{i,a,t} - W_t - (1-\rho)P_{a,t}\lambda^g_{i,a,t} = 0$$

$$-P^*_{a,t}\nu^{c*}_{i,a,t} + P^*_{i,a,t} - W_t - (1-\rho)P^*_{a,t}\lambda^{c*}_{i,a,t} = 0$$

$$-P^*_{a,t}\nu^{g*}_{i,a,t} + P^*_{i,a,t} - W_t - (1-\rho)P^*_{a,t}\lambda^{g*}_{i,a,t} = 0$$

$$\theta^c_a E_t r_{t,t+1} P_{a,t+1} \nu^c_{i,a,t+1} + P_{a,t}\lambda^c_{i,a,t} - \rho E_t r_{t,t+1} P_{a,t+1}\lambda^c_{i,a,t+1} = 0$$

$$\theta^g_a E_t r_{t,t+1} P_{a,t+1} \nu^g_{i,a,t+1} + P_{a,t}\lambda^g_{i,a,t} - \rho E_t r_{t,t+1} P_{a,t+1}\lambda^g_{i,a,t+1} = 0$$

$$\theta^{c*}_a E_t r_{t,t+1} P^*_{a,t+1} \nu^{c*}_{i,a,t+1} + P^*_{a,t}\lambda^{c*}_{i,a,t} - \rho E_t r_{t,t+1} P^*_{a,t+1}\lambda^{c*}_{i,a,t+1} = 0$$

$$\theta^{g*}_a E_t r_{t,t+1} P^*_{a,t+1} \nu^{g*}_{i,a,t+1} + P^*_{a,t}\lambda^{g*}_{i,a,t} - \rho E_t r_{t,t+1} P^*_{a,t+1}\lambda^{g*}_{i,a,t+1} = 0$$

Similar optimality conditions can be derived for foreign firms producing varieties of good b.

2 Competitive Equilibrium

A competitive equilibrium is a set of processes $x^c_{a,t}$, $x^g_{a,t}$, $x^{c*}_{a,t}$, $x^{g*}_{a,t}$, $x^c_{b,t}$, $x^g_{b,t}$, $x^{c*}_{b,t}$, $x^{g*}_{b,t}$, $s^c_{a,t}$, $s^g_{a,t}$, $s^{c*}_{a,t}$, $s^{g*}_{a,t}$, $s^c_{b,t}$, $s^g_{b,t}$, $s^{c*}_{b,t}$, $s^{g*}_{b,t}$, $\lambda^c_{a,t}$, $\lambda^g_{a,t}$, $\lambda^{c*}_{a,t}$, $\lambda^{g*}_{a,t}$, $\lambda^c_{b,t}$, $\lambda^g_{b,t}$, $\lambda^{c*}_{b,t}$, $\lambda^{g*}_{b,t}$, $\nu^c_{a,t}$, $\nu^g_{a,t}$, $\nu^{c*}_{a,t}$, $\nu^{g*}_{a,t}$, $\nu^c_{b,t}$, $\nu^g_{b,t}$, $\nu^{c*}_{b,t}$, $\nu^{g*}_{b,t}$, $c_{a,t}$, $c_{b,t}$, $g_{a,t}$, $g_{b,t}$, $c^*_{a,t}$, $c^*_{b,t}$, $g^*_{a,t}$, $g^*_{b,t}$, w_t, w^*_t, h_t, h^*_t, x^c_t, x^{c*}_t, $\mu_{a,t}$, $\mu^*_{a,t}$, $\mu_{b,t}$, $\mu^*_{b,t}$, $e_{a,t}$, $e_{b,t}$, τ_t, g_t, e_t, y_t, tby_t, and c_t satisfying the complete asset market condition, the conditions defining the domestic block, and the conditions defining the foreign block, given initial conditions $s^c_{a,-1}$, $s^g_{a,-1}$, $s^{c*}_{a,-1}$, $s^{g*}_{a,-1}$, $s^c_{b,-1}$, $s^g_{b,-1}$, $s^{c*}_{b,-1}$, $s^{g*}_{b,-1}$, g_{-i}, e_{-i}, y_{-i}, tby_{-i}, and c_{-i} for $i = 1, 2, 3, 4$, and the exogenous processes ϵ^1_t, and g^*_t.

2.1 Complete Asset Market Condition

$$e_{a,t} = \frac{U_{x^*}(x^{c*}_t, h^*_t)\chi^*_a(x^{c*}_{a,t}, x^{c*}_{b,t})}{U_x(x^c_t, h_t)\chi_a(x^c_{a,t}, x^c_{b,t})}. \tag{1}$$

2.2 Domestic Block

We introduce the parameter Θ^j_k for $j = c, g$ and $k = a, b$ to allow for the implementation of the superficial-habit model as a special case of the deep-habit model. Specifically, the deep-habit model emerges by setting $\Theta^j_k = \theta^j_k$, and the superficial-habit model obtains by setting $\theta^j_k = 0$.

$$\frac{\chi_b(x^c_{a,t}, x^c_{b,t})}{\chi_a(x^c_{a,t}, x^c_{b,t})} = \tau_t \tag{2}$$

$$x^c_t = \chi(x^c_{a,t}, x^c_{b,t}) \tag{3}$$

$$-\frac{U_h(x^c_t, h_t)}{U_x(x^c_t, h_t)\chi_a(x_{a,t}, x_{b,t})} = w_t \tag{4}$$

$$c_{a,t} + c^*_{a,t} + g_{a,t} + g^*_{a,t} = h_t \tag{5}$$

$$x^g_{a,t} = g_{a,t} - \Theta^g_a s^g_{a,t-1} \tag{6}$$

$$x^c_{a,t} = c_{a,t} - \Theta^c_a s^c_{a,t-1} \tag{7}$$

$$x^g_{b,t} = g_{b,t} - \Theta^g_b s^g_{b,t-1} \tag{8}$$

$$x^c_{b,t} = c_{b,t} - \Theta^c_b s^c_{b,t-1} \tag{9}$$

$$1 - \frac{1}{\mu_{a,t}} = \nu^c_{a,t} + (1-\rho)\lambda^c_{a,t} \tag{10}$$

$$1 - \frac{1}{\mu_{a,t}} = \nu^g_{a,t} + (1-\rho)\lambda^g_{a,t} \tag{11}$$

$$1 - \frac{1}{\mu^*_{a,t}} = \nu^{c*}_{a,t} + (1-\rho)\lambda^{c*}_{a,t} \tag{12}$$

$$1 - \frac{1}{\mu^*_{a,t}} = \nu^{g*}_{a,t} + (1-\rho)\lambda^{g*}_{a,t} \tag{13}$$

$$c_{a,t} + g_{a,t} = \eta[\nu^c_{a,t}(c_{a,t} - \theta^c_a s^c_{a,t-1}) + \nu^g_{a,t}(g_{a,t} - \theta^g_a s^g_{a,t-1})] \tag{14}$$

$$c^*_{a,t} + g^*_{a,t} = \eta[\nu^{c*}_{a,t}(c^*_{a,t} - \theta^{c*}_a s^{c*}_{a,t-1}) + \nu^{g*}_{a,t}(g^*_{a,t} - \theta^{g*}_a s^{g*}_{a,t-1})] \tag{15}$$

$$\theta^c_a \beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\nu^c_{a,t+1} + \lambda^c_{a,t} = \rho\beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\lambda^c_{a,t+1} \tag{16}$$

$$\theta^g_a \beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\nu^g_{a,t+1} + \lambda^g_{a,t} = \rho\beta E_t \frac{U_x(t+1)\chi_a(t+1)}{U_x(t)\chi_a(t)}\lambda^g_{a,t+1} \tag{17}$$

$$\theta^{c*}_a \beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\nu^{c*}_{a,t+1} + \lambda^{c*}_{a,t} = \rho\beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\lambda^{c*}_{a,t+1} \tag{18}$$

$$\theta^{g*}_a \beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\nu^{g*}_{a,t+1} + \lambda^{g*}_{a,t} = \rho\beta E_t \frac{U^*_x(t+1)\chi^*_a(t+1)}{U^*_x(t)\chi^*_a(t)}\lambda^{g*}_{a,t+1} \tag{19}$$

$$s^c_{a,t} = \rho s^c_{a,t-1} + (1-\rho)c_{a,t} \tag{20}$$

$$s^g_{a,t} = \rho s^g_{a,t-1} + (1-\rho)g_{a,t} \tag{21}$$

$$s^{c*}_{a,t} = \rho s^{c*}_{a,t-1} + (1-\rho)c^*_{a,t} \tag{22}$$

$$s^{g*}_{a,t} = \rho s^{g*}_{a,t-1} + (1-\rho)g^*_{a,t} \tag{23}$$

$$\mu_{a,t} = \frac{1}{w_t} \tag{24}$$

$$\frac{\mu^*_{a,t}}{\mu_{a,t}} = e_{a,t} \tag{25}$$

$$\frac{c_{a,t} + g_{a,t} + e_{a,t}(c^*_{a,t} + g^*_{a,t})}{c_{a,t} + g_{a,t} + c^*_{a,t} + g^*_{a,t}} g_t = g_{a,t} + \tau_t g_{b,t} \tag{26}$$

$$\frac{x^c_{a,t}}{x^c_{b,t}} = \frac{x^g_{a,t}}{x^g_{b,t}} \tag{27}$$

$$\ln(g_t/g) = B^1(L) \begin{bmatrix} \ln(g_{t-1}/g) \\ \ln(y_{t-1}/y) \\ \ln(c_{t-1}/c) \\ tby_{t-1} - tby \\ \ln(e_{t-1}/e) \end{bmatrix} + \epsilon^1_t, \tag{28}$$

$$y_t = h_t \tag{29}$$

$$c_t = \frac{c_{a,t} + \tau_t c_{b,t}}{\gamma + (1-\gamma)\tau_t} \tag{30}$$

$$tby_t = \frac{e_{a,t}(c^*_{a,t} + g^*_{a,t}) - \tau_t(c_{b,t} + g_{b,t})}{(c_{a,t} + g_{a,t}) + e_{a,t}(c^*_{a,t} + g^*_{a,t})} \tag{31}$$

$$e_t = \frac{\gamma^* e_{a,t} + (1-\gamma^*)e_{b,t}\tau_t}{\gamma + (1-\gamma)\tau_t} \tag{32}$$

where $w_t \equiv W_t/P_{a,t}$ denotes the real wage in terms of domestic goods.

2.3 Foreign Block

$$\frac{\chi^*_b(x^{c*}_{a,t}, x^{c*}_{b,t})}{\chi^*_a(x^{c*}_{a,t}, x^{c*}_{b,t})} = \tau_t \frac{e_{b,t}}{e_{a,t}} \tag{33}$$

$$x^{c*}_t = \chi(x^{c*}_{a,t}, x^{c*}_{b,t}) \tag{34}$$

$$-\frac{U_h(x^{c*}_t, h^*_t)}{U_x(x^{c*}_t, h^*_t)\chi^*_b(x^{c*}_{a,t}, x^{c*}_{b,t})} = w^*_t \tag{35}$$

$$c_{b,t} + g_{b,t} + c^*_{b,t} + g^*_{b,t} = z^*_t h^*_t \tag{36}$$

$$x^{c*}_{a,t} = c^*_{a,t} - \Theta^{c*}_a s^{c*}_{a,t-1} \tag{37}$$

$$x^{g*}_{a,t} = g^*_{a,t} - \Theta^{g*}_a s^{g*}_{a,t-1} \tag{38}$$

$$x^{c*}_{b,t} = c^*_{b,t} - \Theta^{c*}_b s^{c*}_{b,t-1} \tag{39}$$

$$x^{g*}_{b,t} = g^*_{b,t} - \Theta^{g*}_b s^{g*}_{b,t-1} \tag{40}$$

$$\frac{\mu_{b,t} - 1}{\mu_{b,t}} = \nu^c_{b,t} + (1-\rho)\lambda^c_{b,t} \tag{41}$$

$$\frac{\mu_{b,t}-1}{\mu_{b,t}} = \nu^{g}_{b,t} + (1-\rho)\lambda^{g}_{b,t} \tag{42}$$

$$\frac{\mu^*_{b,t}-1}{\mu^*_{b,t}} = \nu^{c*}_{b,t} + (1-\rho)\lambda^{c*}_{b,t} \tag{43}$$

$$\frac{\mu^*_{b,t}-1}{\mu^*_{b,t}} = \nu^{g*}_{b,t} + (1-\rho)\lambda^{g*}_{b,t} \tag{44}$$

$$c_{b,t} + g_{b,t} = \eta[v^{c}_{b,t}(c_{b,t} - \theta^{c}_{b} s^{c}_{b,t-1}) + v^{g}_{b,t}(g_{b,t} - \theta^{g}_{b} s^{g}_{b,t-1})] \tag{45}$$

$$c^*_{b,t} + g^*_{b,t} = \eta[v^{c*}_{b,t}(c^*_{b,t} - \theta^{c*}_{b} s^{c*}_{b,t-1}) + v^{g*}_{b,t}(g^*_{b,t} - \theta^{g*}_{b} s^{g*}_{b,t-1})] \tag{46}$$

$$\theta^{c}_{b}\beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)}\nu^{c}_{b,t+1} + \lambda^{c}_{b,t} = \rho\beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)}\lambda^{c}_{b,t+1} \tag{47}$$

$$\theta^{g}_{b}\beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)}\nu^{g}_{b,t+1} + \lambda^{g}_{b,t} = \rho\beta E_t \frac{U_x(t+1)\chi_b(t+1)}{U_x(t)\chi_b(t)}\lambda^{g}_{b,t+1} \tag{48}$$

$$\theta^{c*}_{b}\beta E_t \frac{U^*_x(t+1)\chi^*_b(t+1)}{U^*_x(t)\chi^*_b(t)}\nu^{c*}_{b,t+1} + \lambda^{c*}_{b,t} = \rho\beta E_t \frac{U^*_x(t+1)\chi^*_b(t+1)}{U^*_x(t)\chi^*_b(t)}\lambda^{c*}_{b,t+1} \tag{49}$$

$$\theta^{g*}_{b}\beta E_t \frac{U^*_x(t+1)\chi^*_b(t+1)}{U^*_x(t)\chi^*_b(t)}\nu^{g*}_{b,t+1} + \lambda^{g*}_{b,t} = \rho\beta E_t \frac{U^*_x(t+1)\chi^*_b(t+1)}{U^*_x(t)\chi^*_b(t)}\lambda^{g*}_{b,t+1} \tag{50}$$

$$s^{c}_{b,t} = \rho s^{c}_{b,t-1} + (1-\rho)c_{b,t} \tag{51}$$

$$s^{g}_{b,t} = \rho s^{g}_{b,t-1} + (1-\rho)g_{b,t} \tag{52}$$

$$s^{c*}_{b,t} = \rho s^{c*}_{b,t-1} + (1-\rho)c^*_{b,t} \tag{53}$$

$$s^{g*}_{b,t} = \rho s^{g*}_{b,t-1} + (1-\rho)g^*_{b,t} \tag{54}$$

$$\mu^*_{b,t} = \frac{z^*_t}{w^*_t} \tag{55}$$

$$\frac{\mu^*_{b,t}}{\mu_{b,t}} = e_{b,t} \tag{56}$$

$$g^*_{a,t} + \tau_t \frac{e_{b,t}}{e_{a,t}} g^*_{b,t} = \frac{\tau_t/e_{a,t}(c_{b,t}+g_{b,t}) + e_{b,t}\tau_t/e_{a,t}(c^*_{b,t}+g^*_{b,t})}{c_{b,t}+g_{b,t}+c^*_{b,t}+g^*_{b,t}}\, g^*_t \tag{57}$$

$$\frac{x^{c*}_{a,t}}{x^{c*}_{b,t}} = \frac{x^{g*}_{a,t}}{x^{g*}_{b,t}} \tag{58}$$

where $w^*_t \equiv W^*_t/P^*_{b,t}$.