Leadership, Coordination and Mission-Driven Management
Technical Appendix

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This technical appendix explores alternative formulations of the followers', leader's and firm's objective functions to clarify what assumptions are and are not crucial to our results.

The utility function we currently use is:

$$\Pi_i = -(a_i - a_L)^2 - \int_j (a_j - \bar{a})^2 dj - (a_L - \theta)^2$$

Aligning the leader's and firm's objectives

For simplicity, we assumed that leaders and followers have the same form of objective. However, since the leader is always well-aligned with himself, the first term in the utility $(a_i - a_L)^2 = 0$ for the leader. For the firm, whose utility is an integral over all followers' utility, this term is $\int_i (a_i - a_L)^2 di \neq 0$. If we use this average alignment cost as the first term of the leader's utility function, the quantitative results change, but the qualitative conclusions remain.

The leader's optimal action becomes $a_L = \tilde{\lambda}\theta_L + (1-\tilde{\lambda})S_L$ where

$$\tilde{\lambda} = 1 - \left[2 \left(\frac{\sigma_p^{-1} + \sigma_L^{-2}}{\sigma_L^{-2}} - (1-\phi) \right) \right]^{-1}.$$

The firm's objective is then identical, except that $\tilde{\lambda}$ replaces λ. Since the firm's expected utility is expressed in terms of λ, that expression and the marginal utility for the firm of a higher λ are identical as well. (See appendix A.1.) What changes is that the λ that the rational leader chooses is now higher. We need to verify whether resoluteness is still optimal. Substituting this higher level of $\tilde{\lambda}$ into $\partial E\Pi / \partial \tilde{\lambda} > 0$ and setting $\sigma_p = 1$, yields the following condition

$$\phi + 2\sigma_L^2 + (1-\phi)^2\sigma_F^2 + 1 > 2(\sigma_L^2 + 1) - (1-\phi),$$

which simplifies to

$$\phi(1-\phi) > 0.$$

Since ϕ is less than one by construction, this condition always holds and resoluteness is always optimal.

Followers want to align actions with the true state, not the leader

If we change the first term in the utility function to make alignment a function of the distance of a follower's action from the true state, rather than of the distance from the leader's action, such a utility function would take the form

$$\Pi_i = -(\theta - a_i)^2 - \int_j (a_j - \bar{a})^2 dj - (a_L - \theta)^2.$$

Since the leader's action does not show up in agents' objective, his action does not affect followers. There is no scope here for a leader to lead. The only thing the leader does that influences the followers is to make his announcement.

If we make another change to the model so that followers no not know that the leader is overconfident and believe that his initial signal has variance $\sigma_p < 1$, then this type of resoluteness can be valuable. The reason is that when public signals are perceived to be more precise, followers put more weight on them, relative to their public signals and coordinate better.

In this alternative model, followers' optimal actions are still given by (3), but the weight they put on the leader's announcement when forecasting the true state differs. It becomes $\phi = \sigma_p^{-2}/(\sigma_p^{-2} + \sigma_F^{-2})$. Substituting this value of ϕ into the firm's objective and taking the derivative with respect to σ_p^{-2} yields

$$\frac{\partial \Pi}{\partial \sigma_p^{-2}} = -\frac{\sigma_F^{-2}(\sigma_p^{-2} - 2)}{(\sigma_F^{-2} + \sigma_p^{-2})^3} - \frac{\sigma_L^{-2}(\sigma_p^{-2} - 1)}{(\sigma_L^{-2} + \sigma_p^{-2})^3}.$$

Evaluating this partial derivative at the rational level of signal precision $\sigma_p^{-2} = 1$ yields $\sigma_F^{-2}/(1 + \sigma_F^{-2})^3$, which is always positive. Therefore, resoluteness is optimal.

Internalizing the coordination externality

Only private costs to mis-coordination Suppose the utility function has the property that there is no positive externality from coordination that is not internalized by the agent. Poor coordination results in the variance of actions being high, but it does not change the mean $\bar{a}$. Thus, when other agents choose very different actions from each other, this does not impose a cost on agent i. Consider payoffs

$$\Pi_i = -(a_i - a_L)^2 - (\bar{a} - a_i)^2 - (a_L - \theta)^2.$$

Resoluteness is always costly here. This is an example of a more general point: If agents choose the socially optimal (optimal for the firm) degree of coordination on there own, then there is no coordination problem for the leader to resolve. The only thing the leader choose be concerned with is to choose the best-adapted mission. Rational leaders perform this task best.

The problem with this as a theory of good leadership is that it does not square with the management literature on leadership that identifies coordination and maintaining credibility as key challenges a leader faces. This is a model with an uninteresting role for a leader that does not incorporate these challenges.

Private and social costs to mis-coordination This version is the same as our current model, except that the coordination externality takes a different form. In our model, agents have no private cost from mis-coordination. It was a pure externality. In this version, there is both a private and a social cost. Agents will weight the public signal θ_L more heavily. This formulation has the advantage that mis-coordination with the leader and with other agents takes the same form. The leader just gets a larger weight.

$$U = -(a_i - a_L)^2 - \int_j (a_i - a_j)^2 dj - (a_L - \theta)^2$$

The magnitude of optimal resoluteness is less in this model because agents have more private incentive to coordinate themselves. The optimal perceived precision for the manager is

$$\sigma_p^{-2} = \sigma_L^{-2} \left(3 - \frac{\sigma_F^{-2}(\sigma_L^{-2} + \sigma_F^{-2})}{(2\sigma_L^{-2} + \sigma_F^{-2})^2} \right)$$

Weighting the three pieces of the objective unequally

The results from weighting the three pieces of the objective differently are very intuitive. When coordination matters relatively more, a higher degree of resoluteness is more valuable.

Consider the following objective function for the followers and the leader

$$U_i = -\omega_i (a_i - a_L)^2 - \omega_j \int_j (a_j - \bar{a})^2 dj - (a_L - \theta)^2.$$

As before, the leader's first-order condition delivers $a_L = E[\theta]$, while the follower's delivers $a_i = E[a_L]$. Thus, the weights do not change any of the optimal actions. They only change expected utility.

The firm's utility is

$$\Pi = -(1-\lambda)^2 \left[\omega_i(\phi + \sigma_L^2) + \omega_j (1-\phi)^2 \sigma_F^2 + \sigma_L^2 \right] - \lambda^2.$$

The partial derivative of this expected utility with respect to the leader's weight on the first period signal is

$$\frac{\partial EU_w}{\partial \lambda} = 2(1-\lambda) \left[\omega_i(\phi + \sigma_L^2) + \omega_j (1-\phi)^2 \sigma_F^2 + \sigma_L^2 \right] - 2\lambda.$$

Resoluteness is valuable if this partial derivative is positive for a rational leader. The rational leader has $\lambda = 1/(1 + \sigma_L^{-2})$. Substituting this λ into the expression above, we find that resoluteness is optimal if

$$\omega_i(\phi + \sigma_L^2) + \omega_j (1-\phi)^2 \sigma_F^2 > 0.$$

Thus as long as the weights on alignment and coordination in the firm's objective function are positive ($\omega_i > 0$ and $\omega_j > 0$), resoluteness is always optimal.

Notice that as alignment and coordination become more important, relative to the benefit of alignment, the marginal value of more resoluteness rises. For firms where alignment is crucial (ω_i and ω_j small), the optimal level of resoluteness will still be positive, but small.

Only the leader bears the commitment cost

In our formulation, commitment costs are borne by both the leader and the firm. If only the leader bears this cost, then resoluteness is even more valuable. It induces the leader to choose a higher commitment cost. Since a higher commitment costs benefits the firm and imposes costs on the leader, the firm always wants the leader to choose a higher commitment cost than a rational leader would choose for himself. The results below show that resoluteness continues to be optimal with this change in firm utility.

The leader's and the followers' optimal actions are the same as in the original model because the leader's objective has not changed and the change in the cost of commitment does not affect the followers' first order conditions. The firm's utility becomes

$$E\Pi = -\left(\frac{1-\lambda}{1+c}\right)^2 \left[2\sigma_L^2 + \phi(2-\phi)\right] - \left(\frac{\lambda+c}{1+c}\right)^2.$$

Substitute for c and λ in (23), and note that $\lambda = \sigma_L^2/(\sigma_p^2 + \sigma_L^2)$.

$$\begin{aligned} E\Pi &= -\left(\frac{\sigma_p^2}{2\phi(1-\phi)}\right)^2 \left[2\sigma_L^2 + \phi(2-\phi)\right] - \left(1 - \frac{\sigma_p^2}{2\phi(1-\phi)}\right)^2 \\ &= -\left(\frac{\sigma_p^2}{2\phi(1-\phi)}\right)^2 \left[2\sigma_L^2 + \phi(2-\phi) + 1\right] - 1 + \frac{\sigma_p^2}{\phi(1-\phi)} \end{aligned}$$

The partial derivative of this expected utility with respect to σ_p^2 is

$$\frac{\partial E\Pi}{\partial \sigma_p^2} = \frac{-2\sigma_p^2}{4\phi^2(1-\phi)^2} \left[2\sigma_L^2 + \phi(2-\phi) + 1\right] + \frac{1}{\phi(1-\phi)}.$$

Evaluating this derivative at the rational level of σ_p,

$$\begin{aligned} \left.\frac{\partial E\Pi}{\partial \sigma_p^2}\right|_{\sigma_p=1} &= \frac{-1}{2\phi^2(1-\phi)^2} \left[2\sigma_L^2 + \phi(2-\phi) + 1 - 2\phi(1-\phi)\right] \\ &= \frac{-1}{2\phi^2(1-\phi)^2} \left[2\sigma_L^2 + 1 + \phi^2\right]. \end{aligned}$$

Resoluteness benefits the firm if lowering σ_p from its rational level increases expected utility. In other words, $\partial E\Pi/\partial \sigma_p^2\big|_{\sigma_p=1}$ must be negative. Since the previous expression is always negative, resoluteness is optimal.