# Appendix to An Intertemporal CAPM with Stochastic Volatility

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First draft: October 2011
This Version: June 2015

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# 1 Additional Literature Review

Our model is an example of an affine stochastic volatility model. Affine stochastic volatility models date back at least to Heston (1993) in continuous time, and have been developed and discussed by Ghysels, Harvey, and Renault (1996), Meddahi and Renault (2004), and Darolles, Gourieroux, and Jasiak (2006) among others. Similar models have been applied in the long-run risk literature by Eraker (2008), Eraker and Shaliastovich (2008), and Hansen (2012), but much of this literature uses volatility specifications that are not guaranteed to remain positive.

Two precursors to our work are unpublished papers by Chen (2003) and Sohn (2010). Both papers explore the effects of stochastic volatility on asset prices in an ICAPM setting but make strong assumptions about the covariance structure of various news terms when deriving their pricing equations. Chen (2003) assumes constant covariances between shocks to the market return (and powers of those shocks) and news about future expected market return variance. Sohn (2010) makes two strong assumptions about asset returns and consumption growth, specifically that all assets have zero covariance with news about future consumption growth volatility and that the conditional contemporaneous correlation between the market return and consumption growth is constant through time. Duffee (2005) presents evidence against the latter assumption. It is in any case unattractive to make assumptions about consumption growth in an ICAPM that does not require accurate measurement of consumption.

Chen estimates a VAR with a GARCH model to allow for time variation in the volatility of return shocks, restricting market volatility to depend only on its past realizations and not those of the other state variables. His empirical analysis has little success in explaining the cross-section of stock returns. Sohn uses a similar but more sophisticated GARCH model for market volatility and tests how well short-run and long-run risk components from the GARCH estimation can explain the returns of various stock portfolios, comparing the results to factors previously shown to be empirically successful. In contrast, our paper incorporates the volatility process directly in the ICAPM, allowing heteroskedasticity to affect and to be predicted by all state variables, and showing how the price of volatility risk is pinned down by the time-series structure of the model along with the investor's coefficient of risk aversion.

Stochastic volatility has been explored in other branches of the finance literature. For example, Chacko and Viceira (2005) and Liu (2007) show how stochastic volatility affects the optimal portfolio choice of long-term investors. Chacko and Viceira assume an AR(1) process for volatility and argue that movements in volatility are not persistent enough to generate large intertemporal hedging demands. Our more flexible multivariate process does allow us to detect persistent long-run variation in volatility. Campbell and Hentschel (1992), Calvet and Fisher (2007), and Eraker and Wang (2011) argue that volatility shocks will lower aggregate stock prices by increasing expected returns, if they do not affect cash flows.

The strength of this volatility feedback effect depends on the persistence of the volatility process. Coval and Shumway (2001), Ang, Hodrick, Xing, and Zhang (2006), and Adrian and Rosenberg (2008) present evidence that shocks to market volatility are priced risk factors in the cross-section of stock returns, but they do not develop any theory to explain the risk prices for these factors.

Time-varying volatility is a prime concern of the field of financial econometrics. Since Engle's (1982) seminal paper on ARCH, much of the financial econometrics literature has focused on variants of the univariate GARCH model (Bollerslev 1986), in which return volatility is modeled as a function of past shocks to returns and of its own lags (see Poon and Granger (2003) and Andersen et al. (2006) for recent surveys). More recently, realized volatility from high-frequency data has been used to estimate stochastic volatility processes (Barndorff-Nielsen and Shephard 2002, Andersen et al. 2003). The use of realized volatility has improved the modeling and forecasting of volatility, including its long-run component; however, this literature has primarily focused on the information content of high-frequency intra-daily return data. This allows very precise measurement of volatility, but at the same time, given data availability constraints, limits the potential to use long time series to learn about long-run movements in volatility. In our paper, we measure realized volatility only with daily data, but augment this information with other financial time series that reveal information investors have about underlying volatility components.

A much smaller literature has, like us, looked directly at the information in other variables concerning future volatility. In early work, Schwert (1989) links movements in stock market volatility to various indicators of economic activity, particularly the price-earnings ratio and the default spread, but finds relatively weak connections. Engle, Ghysels and Sohn (2013) study the effect of inflation and industrial production growth on volatility, finding a significant link between the two, especially at long horizons. Campbell and Taksler (2003) look at the cross-sectional link between corporate bond yields and equity volatility, emphasizing that bond yields respond to idiosyncratic firm-level volatility as well as aggregate volatility. Two recent papers, Paye (2012) and Christiansen et al. (2012), look at larger sets of potential volatility predictors, including the default spread and valuation ratios, to find those that have predictive power for quarterly realized variance. The former paper, in a standard regression framework, finds that the commercial paper to Treasury spread and the default spread, among other variables, contain useful information for predicting volatility. The latter uses Bayesian Model Averaging to find the most successful predictors, and documents the importance of the default spread and valuation ratios in forecasting short-run volatility.

# 2 Model Derivation

In this section we derive an expression for the log stochastic discount factor (SDF) of the intertemporal CAPM model, and the corresponding pricing equations, when we allow for stochastic volatility. The SDF is based on Epstein–Zin utility, but imposes additional assumptions that allow us to express the SDF as a function of news about future cash flows, discount rates, and volatility, and obtain empirically testable implications.

## 2.1 The stochastic discount factor

### 2.1.1 Preferences

We begin by assuming a representative agent with Epstein–Zin preferences. We write the value function as

$$V_t = \left[ (1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left( \mathrm{E}_t \left[ V_{t+1}^{1-\gamma} \right] \right)^{1/\theta} \right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where $C_t$ is consumption and the preference parameters are the discount factor $\delta$, risk aversion $\gamma$, and the elasticity of intertemporal substitution (EIS) $\psi$. For convenience, we define $\theta = (1-\gamma)/(1-1/\psi)$.

The corresponding stochastic discount factor can be written as

$$M_{t+1} = \left( \delta \left( \frac{C_t}{C_{t+1}} \right)^{1/\psi} \right)^{\theta} \left( \frac{W_t - C_t}{W_{t+1}} \right)^{1-\theta}, \tag{2}$$

where $W_t$ is the market value of the consumption stream owned by the agent, including current consumption $C_t$. The log return on wealth is $r_{t+1} = \ln \left( W_{t+1} / \left( W_t - C_t \right) \right)$, the log value of wealth tomorrow divided by reinvested wealth today. The log SDF is therefore

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1) \, r_{t+1}. \tag{3}$$

The log SDF is a function of 1) consumption growth $\Delta c_{t+1}$, and 2) the log return on wealth $r_{t+1}$. In the remainder of this section, we show how to re-express the log SDF substituting consumption out, in a manner analogous to Campbell (1993) but allowing explicitly for time-varying volatility. We then discuss the implications of the model and its testable restrictions.

### 2.1.2 First step: A convenient identity

The gross return to wealth can be written

$$1 + R_{t+1} = \frac{W_{t+1}}{W_t - C_t} = \left(\frac{C_t}{W_t - C_t}\right)\left(\frac{C_{t+1}}{C_t}\right)\left(\frac{W_{t+1}}{C_{t+1}}\right), \tag{4}$$

expressing it as the product of the current consumption payout, the growth in consumption, and the future price of a unit of consumption.

We find it convenient to work in logs. We define the log value of reinvested wealth per unit of consumption as $z_t = \ln\left(\left(W_t - C_t\right)/C_t\right)$, and the future value of a consumption claim as $h_{t+1} = \ln\left(W_{t+1}/C_{t+1}\right)$, so that the log return is:

$$r_{t+1} = -z_t + \Delta c_{t+1} + h_{t+1}. \tag{5}$$

Heuristically, the return on wealth is negatively related to the current value of reinvested wealth and positively related to consumption growth and the future value of wealth. The last term in equation (5) will capture the effects of intertemporal hedging on asset prices, hence the choice of the notation $h_{t+1}$ for this term.

The convenient identity (5) can therefore be used to write the log SDF (3) without reference to consumption growth:

$$m_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} z_t + \frac{\theta}{\psi} h_{t+1} - \gamma r_{t+1}. \tag{6}$$

Given that the focus of our paper will be cross-sectional risk premia, it is useful to write the one-period innovation in the SDF:

$$m_{t+1} - \mathrm{E}_t m_{t+1} = \frac{\theta}{\psi}\left[h_{t+1} - \mathrm{E}_t h_{t+1}\right] - \gamma\left[r_{t+1} - \mathrm{E}_t r_{t+1}\right]. \tag{7}$$

As noted in Campbell (1993), consumption growth does not appear in this expression for the log SDF. Instead, the equation illustrates the dependence of the innovations in the SDF (which determine risk premia) on the one-period innovations in the wealth-consumption ratio and on the log return on the wealth portfolio. Next, we impose the asset pricing equation for the wealth portfolio and re-express the innovations in the SDF as a function of news about future cash flows, discount rates, and risk.

### 2.1.3 Second step: imposing the general pricing equation and lognormality to solve the SDF forward

We now add the assumption that asset returns and all state variables in the model are jointly conditionally lognormal. Since we allow for changing conditional volatility, we are careful to write second moments with time subscripts to indicate that they can vary over time. Under this standard assumption, the return on the wealth portfolio must satisfy:

$$0 = \ln \mathrm{E}_t \exp\{m_{t+1} + r_{t+1}\} = \mathrm{E}_t \left[m_{t+1} + r_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t \left[m_{t+1} + r_{t+1}\right], \tag{8}$$

We can then substitute our log SDF (6) into the asset pricing equation (8) and multiply by $\frac{\psi}{\theta}$ to find an equation for $z_t$ :

$$z_t = \psi \ln \delta + (\psi - 1)\mathrm{E}_t r_{t+1} + \mathrm{E}_t h_{t+1} + \frac{\psi}{\theta}\frac{1}{2}\mathrm{Var}_t \left[m_{t+1} + r_{t+1}\right]. \tag{9}$$

Next, we approximate the relationship of $h_{t+1}$ and $z_{t+1}$ by taking a loglinear approximation about $\bar{z}$:

$$h_{t+1} \approx \kappa + \rho z_{t+1} \tag{10}$$

where the loglinearization parameter $\rho = \exp(\bar{z})/(1+\exp(\bar{z})) \approx 1 - C/W$. The two variables $h_{t+1}$ and $z_{t+1}$ are closely related: the former is the log ratio of wealth to consumption, $log(W_{t+1}/C_{t+1})$, the latter is the ratio of reinvested wealth to consumption, $log((W_{t+1} - C_{t+1})/C_{t+1})$. In fact, when the EIS, $\psi$, is 1, the loglinear relationship between the two variables holds exactly.

Combining the two equations (9) and (10) we then obtain an expression for the innovation in $h_{t+1}$:

$$\begin{aligned} h_{t+1} - \mathrm{E}_t h_{t+1} &= \rho(z_{t+1} - \mathrm{E}_t z_{t+1}) \\ &= (\mathrm{E}_{t+1} - \mathrm{E}_t)\rho \left( (\psi - 1) r_{t+2} + h_{t+2} + \frac{\psi}{\theta}\frac{1}{2}\mathrm{Var}_{t+1}\left[m_{t+2} + r_{t+2}\right] \right). \end{aligned} \tag{11}$$

Solving forward to an infinite horizon,

$$\begin{aligned} h_{t+1} - \mathrm{E}_t h_{t+1} &= (\psi - 1)(\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j} \\ &\quad + \frac{1}{2}\frac{\psi}{\theta}(\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j \mathrm{Var}_{t+j}\left[m_{t+1+j} + r_{t+1+j}\right] \\ &= (\psi - 1) N_{DR,t+1} + \frac{1}{2}\frac{\psi}{\theta} N_{RISK,t+1}. \end{aligned} \tag{12}$$

The second equality follows Campbell and Vuolteenaho (2004) and uses the notation $N_{DR}$ ("news about discount rates") for revisions in expected future returns. In a similar spirit, we write revisions in expectations of future risk (the variance of the future log return plus the log stochastic discount factor) as $N_{RISK}$.

Finally, we substitute back into the equation for the innovations in the log SDF (7), and simplify to obtain:

$$\begin{aligned} m_{t+1} - \mathrm{E}_t m_{t+1} &= -\gamma\left[r_{t+1} - \mathrm{E}_t r_{t+1}\right] - (\gamma - 1) N_{DR,t+1} + \frac{1}{2} N_{RISK,t+1} \\ &= -\gamma N_{CF,t+1} - \left[-N_{DR,t+1}\right] + \frac{1}{2} N_{RISK,t+1} \end{aligned} \tag{13}$$

Equation (13) expresses the log SDF in terms of the market return and news about future variables. In particular, it identifies three priced factors: the market return (with a price of risk $\gamma$), discount rate news (with price of risk $(\gamma - 1)$), and news about future risk (with price of risk of $-\frac{1}{2}$). This is an extension of the ICAPM as derived by Campbell (1993), with no reference to consumption or the elasticity of intertemporal substitution $\psi$. When the investor's risk aversion is greater than 1, assets which hedge aggregate discount rates (negative covariance with $N_{DR}$) or aggregate risk (positive covariance with $N_{CF}$) will have lower expected returns, all else equal.

The second equation rewrites the model, following Campbell and Vuolteenaho (2004), by breaking the market return into cash-flow news and discount-rate news. Cash-flow news $N_{CF,t+1}$ is defined by $N_{CF,t+1} = r_{t+1} - \mathrm{E}_t r_{t+1} + N_{DR,t+1}$. The price of risk for cash-flow news is $\gamma$ times greater than the price of risk for discount-rate news, hence Campbell and Vuolteenaho call betas with cash-flow news "bad betas" and those with discount-rate news "good betas". The third term in (13) shows the risk price for exposure to news about future risks and did not appear in Campbell and Vuolteenaho's model, which assumed homoskedasticity. Not surprisingly, the coefficient is positive, indicating that an asset providing positive returns when risk expectations increase will offer a lower return on average (the log SDF is high when future volatility is anticipated to be high).

While the elasticity of intertemporal substitution $\psi$ does not affect risk prices (and therefore risk premia) in our model, this parameter does influence the implied behavior of the investor's consumption.

#### 2.1.4 Third step: linking news about risk to news about volatility

The risk news term $N_{RISK,t+1}$ in equation (13) represents news about the conditional volatility of returns plus the stochastic discount factor, $\mathrm{Var}_t\left[m_{t+1} + r_{t+1}\right]$. It therefore depends on the SDF $m$ and its innovations. To close the model and derive its empirical implications, we need to add assumptions on the data generating process for stock returns and the variance terms that will allow to solve for the term $\text{Var}_t\left[m_{t+1}+r_{t+1}\right]$ and compute the news terms. These assumptions will imply that the conditional volatility of returns plus the stochastic discount factor is proportional to the conditional volatility of returns themselves.

We assume that the economy is described by a first-order VAR

$$\mathbf{x}_{t+1}=\bar{\mathbf{x}}+\mathbf{\Gamma}\left(\mathbf{x}_t-\bar{\mathbf{x}}\right)+\sigma_t\mathbf{u}_{t+1}, \tag{14}$$

where $\mathbf{x}_{t+1}$ is an $n\times 1$ vector of state variables that has $r_{t+1}$ as its first element, $\sigma^2_{t+1}$ as its second element, and $n-2$ other variables that help to predict the first and second moments of aggregate returns. $\bar{\mathbf{x}}$ and $\mathbf{\Gamma}$ are an $n\times 1$ vector and an $n\times n$ matrix of constant parameters, and $\mathbf{u}_{t+1}$ is a vector of shocks to the state variables normalized so that its first element has unit variance. We assume that $\mathbf{u}_{t+1}$ has a constant variance-covariance matrix $\mathbf{\Sigma}$, with element $\Sigma_{11}=1$.

The key assumption here is that a scalar random variable, $\sigma^2_t$, equal to the conditional variance of market returns, also governs time-variation in the variance of all shocks to this system. Both market returns and state variables, including volatility itself, have innovations whose variances move in proportion to one another. This assumption makes the stochastic volatility process affine, as in Heston (1993) and related work discussed above in our literature review.

Given this structure, news about discount rates can be written as

$$\begin{aligned} N_{DR,t+1} &= \left(E_{t+1}-E_t\right)\sum_{j=1}^{\infty}\rho^j r_{t+1+j} \\ &= \mathbf{e}_1'\sum_{j=1}^{\infty}\rho^j\mathbf{\Gamma}^j\sigma_t\mathbf{u}_{t+1} \\ &= \mathbf{e}_1'\rho\mathbf{\Gamma}\left(\mathbf{I}-\rho\mathbf{\Gamma}\right)^{-1}\sigma_t\mathbf{u}_{t+1}, \end{aligned} \tag{15}$$

while implied cash flow news is:

$$\begin{aligned} N_{CF,t+1} &= \left(r_{t+1}-E_t r_{t+1}\right)+N_{DR,t+1} \\ &= \left(\mathbf{e}_1'+\mathbf{e}_1'\rho\mathbf{\Gamma}(\mathbf{I}-\rho\mathbf{\Gamma})^{-1}\right)\sigma_t\mathbf{u}_{t+1}. \end{aligned} \tag{16}$$

Furthermore, our log-linear model will make the log SDF, $m_{t+1}$, a linear function of the state variables. Since all shocks to the SDF are then proportional to $\sigma_t$, $\text{Var}_t\left[m_{t+1}+r_{t+1}\right]\propto\sigma^2_t$. As a result, the conditional variance of the scaled variables, $\text{Var}_t\left[\left(m_{t+1}+r_{t+1}\right)/\sigma_t\right]=\omega_t$, will be a constant that does not depend on the state variables: $\omega$. Without knowing the parameters of the utility function, we can write $\text{Var}_t\left[m_{t+1}+r_{t+1}\right]=\omega\sigma^2_t$, so that the news about risk, $N_{RISK}$, is proportional to news about market return variance, $N_V$.

$$\begin{aligned} N_{RISK,t+1} &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \mathrm{Var}_{t+j} \left[ r_{t+1+j} + m_{t+1+j} \right] \\ &= (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j \left( \omega \sigma_{t+j}^2 \right) \\ &= \omega \rho \mathbf{e}_2' \sum_{j=0}^{\infty} \rho^j \mathbf{\Gamma}^j \sigma_t \mathbf{u}_{t+1} \\ &= \omega \rho \mathbf{e}_2' \left( \mathbf{I} - \rho \mathbf{\Gamma} \right)^{-1} \sigma_t \mathbf{u}_{t+1} = \omega N_{V,t+1}. \end{aligned} \tag{17}$$

## 2.2 Solving for $\omega$

We now show how to solve for the unknown parameter $\omega$. From the definition of $\omega$,

$$\begin{aligned} \omega \sigma_t^2 &= \mathrm{Var}_t \left[ m_{t+1} + r_{t+1} \right] \\ &= \mathrm{Var}_t \left[ \frac{\theta}{\psi} h_{t+1} + (1-\gamma) r_{t+1} \right] \\ &= \mathrm{Var}_t \left[ \frac{\theta}{\psi} \left( (\psi - 1) N_{DR,t+1} + \frac{1}{2} \frac{\psi}{\theta} \omega N_{V,t+1} \right) + (1-\gamma) r_{t+1} \right] \\ &= \mathrm{Var}_t \left[ (1-\gamma) N_{DR,t+1} + \frac{1}{2} \omega N_{V,t+1} + (1-\gamma) r_{t+1} \right] \\ &= \mathrm{Var}_t \left[ (1-\gamma) N_{CF,t+1} + \frac{1}{2} \omega N_{V,t+1} \right] \\ &= (1-\gamma)^2 \mathrm{Var}_t \left[ N_{CF_{t+1}} \right] + \omega (1-\gamma) \mathrm{Cov}_t \left[ N_{CF_{t+1}}, N_{V_{t+1},} \right] + \frac{\omega^2}{4} \mathrm{Var}_t \left[ N_{V_{t+1}} \right]. \end{aligned} \tag{18}$$

This equation can also be written directly in terms of the VAR parameters. We define $\mathbf{x}_{CF}$ and $\mathbf{x}_V$ as the error-to-news vectors that map VAR innovations to volatility-scaled news terms:

$$\frac{1}{\sigma_t} N_{CF,t+1} = \mathbf{x}_{CF} \mathbf{u}_{t+1} = \left( \mathbf{e}_1' + \mathbf{e}_1' \rho \mathbf{\Gamma} (\mathbf{I} - \rho \mathbf{\Gamma})^{-1} \right) \mathbf{u}_{t+1} \tag{19}$$

$$\frac{1}{\sigma_t} N_{V,t+1} = \mathbf{x}_V \mathbf{u}_{t+1} = \left( \mathbf{e}_2' \rho (\mathbf{I} - \rho \mathbf{\Gamma})^{-1} \right) \mathbf{u}_{t+1}. \tag{20}$$

Then $\omega$ solves

$$0 = \omega^2 \frac{1}{4} \mathbf{x}_V \mathbf{\Sigma} \mathbf{x}_V' - \omega \left( 1 - (1-\gamma) \mathbf{x}_{CF} \mathbf{\Sigma} \mathbf{x}_V' \right) + (1-\gamma)^2 \mathbf{x}_{CF} \mathbf{\Sigma} \mathbf{x}_{CF}' \tag{21}$$

We can see two main channels through which $\gamma$ affects $\omega$. First, a higher risk aversion—given the underlying volatilities of all shocks—implies a more volatile stochastic discount factor $m$, and therefore a higher risk. This effect is proportional to $(1-\gamma)^2$, so it increases rapidly with $\gamma$. Second, there is a feedback effect on current risk through future risk: $\omega$ appears on the right-hand side of the equation as well. Given that in our estimation we find $\text{Cov}_t\left[N_{CF,t+1}, N_{V,t+1}\right] < 0$, this second effect makes $\omega$ increase even faster with $\gamma$.

### 2.2.1 Selecting the correct root of the quadratic equation

The equation defining $\omega$ will generally have two solutions

$$\omega = \frac{1-(1-\gamma)\, x_{CF}\mathbf{\Sigma}x_V' \pm \sqrt{\left(1-(1-\gamma)\, x_{CF}\mathbf{\Sigma}x_V'\right)^2 - (1-\gamma)^2\left(x_V\mathbf{\Sigma}x_V'\right)\left(x_{CF}\mathbf{\Sigma}x_{CF}'\right)}}{\frac{1}{2}x_V\mathbf{\Sigma}x_V'}. \tag{22}$$

While the (approximate) Euler equation holds for both solutions, the correct solution is the one with the negative sign on the radical. This result can be confirmed from numerical computation, and it can also be easily seen by observing the behavior of the solutions in the limit as volatility news goes to zero and the model become homoskedastic. With the false solution, $\omega$ becomes infinitely large as $x_V \to 0$. This false solution corresponds to the log value of invested wealth going to negative infinity. On the other hand, we can exploit that the correct solution for $\omega$ converges to $(1-\gamma)^2\, x_{CF}\mathbf{\Sigma}x_{CF}'$. This is what we would expect, since in that case $\omega = \text{Var}_t\left[(1-\gamma)\, N_{CF,t+1}/\sigma_t\right]$.

### 2.2.2 Simplifying the existence condition for a real root

Appendix Figure 1 plots $\omega$ as a function of $\gamma$, conditional on our VAR parameter estimates. The upper bound of 7.2 for $\gamma$ is the value of $\gamma$ above which a real solution to the quadratic equation ceases to exist.

The existence condition for a solution for $\omega$ corresponds to the following inequality:

$$\left[1-(1-\gamma)(x_{CF}'\Sigma x_V)\right]^2 - (1-\gamma)^2\left(x_V'\Sigma x_V\right)\left(x_{CF}'\Sigma x_{CF}\right) \geq 0 \tag{23}$$

We show here that this condition can be simplified to a set of bounds on $\gamma$ of the form:

$$1 - \frac{1}{(\rho_n+1)\sigma_{cf}\sigma_v} \leq \gamma \leq 1 - \frac{1}{(\rho_n-1)\sigma_{cf}\sigma_v} \tag{24}$$

where $\rho_n$ is the correlation of the news terms, $\sigma_{cf}$ is the scaled standard deviation of cash flow news, and $\sigma_v$ is the scaled standard deviation of volatility news. Note that since $-1 \leq \rho_n \leq 1$, the lower bound on $\gamma$ is always (weakly) below 1, and the upper bound is always (weakly) above 1. We also note that empirically, the lower bound is often below zero, and therefore not actually binding. For example, in our case depicted in Appendix Figure 1, only the upper bound on $\gamma$ is binding, as the lower bound from equation (24) lies below zero.

As evident from equation (23), the existence condition is itself a simple quadratic inequality in $(1-\gamma)$. We can rewrite it as:

$$(1-\gamma)^2(x'_{CF}\Sigma x_V)^2 + 1 - 2(1-\gamma)(x'_{CF}\Sigma x_V) - (1-\gamma)^2\left(x'_{CF}\Sigma x_{CF}\right)\left(x'_V\Sigma x_V\right) \geq 0$$

or:

$$(1-\gamma)^2\left[(x'_{CF}\Sigma x_V)^2 - \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)\right] - 2(1-\gamma)(x'_{CF}\Sigma x_V) + 1 \geq 0$$

The two roots of this equation can be found as:

$$\begin{aligned}(1-\gamma) &= \frac{2(x'_{CF}\Sigma x_V) \pm \sqrt{4(x'_{CF}\Sigma x_V)^2 - 4\left[(x'_{CF}\Sigma x_V)^2 - \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)\right]}}{2\left[(x'_{CF}\Sigma x_V)^2 - \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)\right]} \\ &= \frac{(x'_{CF}\Sigma x_V) \pm \sqrt{\left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)}}{\left[(x'_{CF}\Sigma x_V)^2 - \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)\right]}\end{aligned}$$

Note that this equation always has two real solutions, since $\left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right) > 0$. The denominator can be written as:

$$\begin{aligned}\left[(x'_{CF}\Sigma x_V)^2 - \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)\right] &= \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)(\rho_n^2 - 1) \\ = \left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)(\rho_n+1)(\rho_n-1) &= \sigma_v^2\sigma_{cf}^2(\rho_n+1)(\rho_n-1)\end{aligned}$$

while the numerator can be written as:

$$(x'_{CF}\Sigma x_V) \pm \sqrt{\left(x'_V\Sigma x_V\right)\left(x'_{CF}\Sigma x_{CF}\right)} = \sigma_v\sigma_{cf}\rho_n \pm \sigma_v\sigma_{cf} = \sigma_v\sigma_{cf}(\rho_n \pm 1)$$

Therefore, the two roots can be found as:

$$(1-\gamma) = \frac{\sigma_v\sigma_{cf}(\rho_n \pm 1)}{\sigma_v^2\sigma_{cf}^2(\rho_n+1)(\rho_n-1)} = \frac{(\rho_n \pm 1)}{\sigma_v\sigma_{cf}(\rho_n+1)(\rho_n-1)}$$

Or:

$$\overline{(1-\gamma)} = \frac{(\rho_n - 1)}{\sigma_v\sigma_{cf}(\rho_n+1)(\rho_n-1)} = \frac{1}{\sigma_v\sigma_{cf}(\rho_n+1)} \geq 0$$

and

$$\underline{(1-\gamma)} = \frac{(\rho_n + 1)}{\sigma_v\sigma_{cf}(\rho_n+1)(\rho_n-1)} = \frac{1}{\sigma_v\sigma_{cf}(\rho_n-1)} \leq 0$$

Finally, we note that since $[(x'_{CF}\Sigma x_V)^2 - (x'_V\Sigma x_V)\,(x'_{CF}\Sigma x_{CF})] = \sigma_v^2\sigma_{cf}^2(\rho_n+1)(\rho_n-1) \leq 0$, the quadratic inequality (23) will have solutions between the two roots, for

$$1-\gamma \leq \overline{(1-\gamma)} = \frac{1}{\sigma_v\sigma_{cf}(\rho_n+1)}$$

and for

$$1-\gamma \geq \underline{(1-\gamma)} = \frac{1}{\sigma_v\sigma_{cf}(\rho_n-1)}$$

or equivalently:

$$\gamma \geq 1 - \frac{1}{\sigma_v\sigma_{cf}(\rho_n+1)}$$
$$\gamma \leq 1 - \frac{1}{\sigma_v\sigma_{cf}(\rho_n-1)}$$

## 2.3 Derivation of the moment conditions

After solving for $\omega$, we can rewrite the stochastic discount factor as:

$$m_{t+1} - \mathrm{E}_t m_{t+1} = -\gamma N_{CF,t+1} - [-N_{DR,t+1}] + \frac{1}{2}\omega N_{V,t+1}$$

To derive the moment conditions of the model, we go back to the general asset pricing equation under lognormality

$$0 = \ln \mathrm{E}_t \exp\{m_{t+1} + r_{i,t+1}\} = \mathrm{E}_t\left[m_{t+1} + r_{i,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[m_{t+1} + r_{i,t+1}\right]. \qquad (25)$$

The same equation can be rewritten as:

$$0 = \mathrm{E}_t\left[m_{t+1}\right] + \mathrm{E}_t\left[r_{i,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[r_{i,t+1}\right] + Cov_t(r_{i,t+1}, m_{t+1} - \mathrm{E}_t m_{t+1}) \quad (26)$$

We now rearrange this equation and make two substitutions. First, we note that the conditional mean of the log SDF innovation is zero, so that

$$Cov_t(r_{i,t+1}, m_{t+1} - \mathrm{E}_t m_{t+1}) = \mathrm{E}_t\left[r_{i,t+1}(m_{t+1} - \mathrm{E}_t m_{t+1})\right]$$

Second, we note that

$$E_t r_{i,t+1} + \frac{1}{2}\sigma_{it}^2 \simeq (E_t R_{i,t+1} - 1)$$

which links the expected log returns (adjusted by their variance) to the expected gross level returns $r_{i,t+1}$.[2]

After these two substitutions, we can rearrange (26) to yield:

$$E_t R_{i,t+1} - 1 = -\mathrm{E}_t\left[m_{t+1}\right] - \frac{1}{2}\mathrm{Var}_t\left[m_{t+1}\right] - \mathrm{E}_t\left[r_{i,t+1}(m_{t+1} - \mathrm{E}_t m_{t+1})\right] \tag{27}$$

Given any reference asset $j$ (which could be but does not need to be the risk-free rate), we can write the relative risk premium of $i$ relative to $j$ as:

$$E_t\left[R_{i,t+1} - R_{j,t+1}\right] = -\mathrm{E}_t\left[(r_{i,t+1} - r_{j,t+1})(m_{t+1} - \mathrm{E}_t m_{t+1})\right] \tag{28}$$

by taking the difference of equation (27) between $i$ and $j$. We can then substitute the expression for the innovations in the SDF and write:

$$E_t\left[R_{i,t+1} - R_{j,t+1}\right] = \mathrm{E}_t\left[(r_{i,t+1} - r_{j,t+1})(\gamma N_{CF,t+1} + [-N_{DR,t+1}] - \frac{1}{2}\omega N_{V,t+1})\right] \tag{29}$$

## 2.4 A simple fully-solved example with $\psi = 1$

In this section we solve analytically for a simple model with $\psi = 1$ and $\gamma > 1$. We show that for the value function to exist the parameters of the model must satisfy a quadratic equation, and we show that in this model the equation corresponds to equation (12) in Campbell, Giglio, Polk, and Turley (2015), i.e. the upper bound on $\gamma$ that ensures existence of a real solution for the price of volatility risk $\omega$ (only the upper bound matters here, since we are looking at the case $\gamma > 1$). For tractability purposes, we assume that consumption growth is *iid*, so the only state variable will be volatility. Finally, we consider separately the existence conditions for the case of a homoskedastic volatility process.

Since $\psi = 1$, we can write the log value function relative to consumption, $v_t = \ln(V_t/C_t)$, recursively as (see Hansen, Heaton and Li 2008):

$$v_t = \frac{\delta}{1-\gamma}\ln \mathrm{E}_t \exp\left\{(1-\gamma)(v_{t+1} + \Delta c_{t+1})\right\} \tag{30}$$

Assume that volatility and consumption growth follow the process

$$\sigma_{t+1}^2 = s + d\sigma_t^2 + x\sigma_t\epsilon_{t+1} \tag{31}$$

$$\Delta c_{t+1} = k\sigma_t\eta_{t+1} \tag{32}$$

with $\epsilon_t$ and $\eta_t$ normal with unit standard deviation, and correlation $\theta$. $d$ captures the

[2] By lognormality, we have: $E_t[r_{i,t+1}] + \frac{\sigma_{i,t}^2}{2} = lnE_t[R_{i,t+1}]$. Now, for the expected gross return $E_t[R_{i,t+1}]$ close to 1, we will have: $lnE_t[R_{i,t+1}] \simeq E_t[R_{i,t+1}] - 1$, from which the result follows.

persistence of volatility, while $x$ scales the volatility of volatility.

### 2.4.1 Existence of a solution in the simple model

We conjecture that $v$ and $\Delta c$ are jointly lognormal, and write:

$$\begin{aligned} v_t &= \frac{\delta}{1-\gamma} \ln \mathrm{E}_t \exp\left\{(1-\gamma)(v_{t+1}+\Delta c_{t+1})\right\} \\ &= \frac{\delta}{1-\gamma}\left[\mathrm{E}_t\left\{(1-\gamma)(v_{t+1}+\Delta c_{t+1})\right\} + 0.5\mathrm{Var}_t\left\{(1-\gamma)(v_{t+1}+\Delta c_{t+1})\right\}\right] \\ &= \delta \mathrm{E}_t\left\{v_{t+1}+\Delta c_{t+1}\right\} + \delta(1-\gamma)0.5\mathrm{Var}_t\left\{v_{t+1}+\Delta c_{t+1}\right\}. \end{aligned} \tag{33}$$

Since consumption has mean zero in the simple model,

$$v_t = \delta \mathrm{E}_t\left\{v_{t+1}\right\} + \delta(1-\gamma)0.5\mathrm{Var}_t\left\{v_{t+1}+\Delta c_{t+1}\right\} \tag{34}$$

We now guess that the log value function is linear in $\sigma_t^2$:

$$v_t = a + b\sigma_t^2 \tag{35}$$

and obtain:

$$\begin{aligned} a + b\sigma_t^2 &= \delta(a + b\mathrm{E}_t\left\{\sigma_{t+1}^2\right\}) + \delta(1-\gamma)0.5\mathrm{Var}_t\left\{a + b\sigma_{t+1}^2 + k\sigma_t\eta_{t+1}\right\} \\ &= \delta(a + b\mathrm{E}_t\left\{s + d\sigma_t^2\right\}) + \delta(1-\gamma)0.5\mathrm{Var}_t\left\{a + b\sigma_{t+1}^2 + k\sigma_t\eta_{t+1}\right\} \\ &= \delta(a + b\mathrm{E}_t\left\{s + d\sigma_t^2\right\}) + \delta(1-\gamma)0.5\mathrm{Var}_t\left\{bx\sigma_t\epsilon_{t+1} + k\sigma_t\eta_{t+1}\right\} \\ &= \delta(a + bs + bd\sigma_t^2) + \delta(1-\gamma)0.5[b^2x^2 + k^2 + 2bxk\theta]\sigma_t^2. \end{aligned} \tag{36}$$

Matching coefficients on $\sigma_t^2$:

$$b = \delta bd + \delta(1-\gamma)0.5(b^2x^2 + k^2 + 2bxk\theta) \tag{37}$$

or:

$$\left(\delta(1-\gamma)0.5x^2\right)b^2 + \left(\delta d - 1 + \delta(1-\gamma)xk\theta\right)b + \left(\delta(1-\gamma)0.5k^2\right) = 0 \tag{38}$$

This is a quadratic equation which may not have a solution. For the solution to exist, we need:

$$\left(\delta d - 1 + \delta(1-\gamma)xk\theta\right)^2 > \left(\delta(1-\gamma)x^2\right)\left(\delta(1-\gamma)k^2\right) \tag{39}$$

Given the signs of these variables, this equation can be rewritten as:

$$1 - \delta d - \delta(1-\gamma)xk\theta > \delta(\gamma-1)xk \tag{40}$$

Rearranging, the existence condition for the value function in this model is given by:

$$(\gamma - 1) \leq \frac{1 - \delta d}{k\delta x(1 - \theta)}. \tag{41}$$

### 2.4.2 Comparison with the existence of a real solution to $\omega$

We can compare this equation with the condition for having a real solution for the price of volatility risk $\omega$ in the general model of Campbell, Giglio, Polk, and Turley (2015). We can rewrite that upper bound on $\gamma$ (from equation 24) as:

$$(\mathrm{Corr}(N_{cf}, N_V) - 1)(1 - \gamma)\sigma_{cf}\sigma_v \leq 1. \tag{42}$$

We now apply this condition to the fully solved model presented above. In this model we have:

$$N_{CF,t+1} = \Delta c_{t+1} = k\sigma_t \eta_{t+1} \tag{43}$$

$$\begin{aligned} N_{V,t+1} &= (\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \delta^j \sigma_{t+j}^2 = (\mathrm{E}_{t+1} - \mathrm{E}_t)\delta \sum_{j=0}^{\infty} \delta^j \sigma_{t+j+1}^2 \\ &= \delta \sum_{j=0}^{\infty} \delta^j d^j (x\sigma_t \epsilon_{t+1}) = \frac{\delta}{1 - \delta d} x\sigma_t \epsilon_{t+1}. \end{aligned} \tag{44}$$

$$\mathrm{Corr}(N_{CF}, N_{DR}) = \theta \tag{45}$$

Substituting:

$$(\theta - 1)(1 - \gamma)\frac{k\delta x}{1 - \delta d} \leq 1 \tag{46}$$

or

$$(\gamma - 1) \leq \frac{1 - \delta d}{k\delta x(1 - \theta)} \tag{47}$$

which precisely coincides with the existence condition for $v_t$ shown in the previous subsection.

# 3 A Homoskedastic Stochastic Volatility Model

It is interesting to explore the alternative hypothesis of a homoskedastic process for $\sigma_t^2$ (as in Bansal, Kiku, Shaliastovich, and Yaron, BKSY 2014). We show in the paper that under the assumption that $\sigma_t^2$ scales *all* the shocks of the VAR, we obtain the result that $\text{Var}_t(RISK_{t+1}) \equiv \text{Var}_t(m_{t+1} + r_{t+1}) = \omega\sigma_t^2$, so that $N_{RISK} = \omega N_V$. Given this proportionality, in our empirical analysis we can use $N_V$ as a pricing factor, with a price of risk of $\omega$. We now explore whether this proportionality holds under the assumption of homoskedasticity of the variance process.

For $N_{RISK}$ to be proportional to $N_V$, a sufficient condition is that $\text{Var}_t(RISK_{t+1})$ is proportional (as in our case) or at least affine (as in BKSY) in $\sigma_t^2$. If this is not the case, the news terms will not generally be proportional to each other, and it will not generally be appropriate to use $N_V$ as a pricing factor.

When considering the homoskedastic volatility case, it is important to define which shocks are actually homoskedastic. When the volatility process $\sigma_t^2$ is modeled as part of a VAR, the fact that its own innovation has constant variance does *not* imply that $N_V$ will also have constant variance. To see this, call $\eta_{t+1}$ the *unscaled* vector of VAR innovations. If $\sigma_t^2$ is the $v-th$ element of the VAR and its shock has constant variance, but the other shocks are scaled by $\sigma_t^2$, then we will have: $\text{Var}_t(e_v'\eta_{t+1})$ equal to a constant but $\text{Var}_t(e_{i\neq v}'\eta_{t+1}) \propto \sigma_t^2$ (where $e_i$ is a vector of zeros with 1 at the i-th element). Now consider that the volatility news term $N_V$ can be expressed as $\lambda_v'\eta_{t+1}$, where $\lambda_v = e_v'^{-1}$. In general $N_V$ will *not* have constant variance, but rather its variance will be a linear function of $\sigma_t$ and $\sigma_t^2$. With a simple example, suppose that $\sigma_t^2$ is the second element of a 2-variable VAR, and $\lambda_v = [\lambda_1\, \lambda_2]'$. Then,

$$\text{Var}_t(N_{V,t+1}) = \text{Var}_t(\lambda_1\sigma_t u_{1,t+1} + \lambda_2 u_{2,t+1}) = a_1 + a_2\sigma + a_3\sigma_t^2 \tag{48}$$

for some $a_1, a_2, a_3$, and for $u_{t+1}$ being a vector with constant variance-covariance matrix. Similarly, the covariance between $N_V$ and $N_{CF}$ will be a function of $\sigma_t$ and $\sigma_t^2$. The general intuition for this result is that news about long-run volatility is driven by *all* the shocks in the VAR, not just by the innovation to the volatility equation, and therefore the term $N_V$ will generally have time-varying second moments even when the volatility equation is homoskedastic.

What does this imply? Remember that for $\text{Var}_t(m_{t+1} + r_{t+1})$ to be affine in $\sigma_t^2$ we need

$$\text{Var}_t(m_{t+1}+r_{t+1}) = (1-\gamma)^2\text{Var}_t(N_{CF,t+1}) + \omega(1-\gamma)\text{Cov}_t(N_{CF,t+1}, N_{V,t+1}) + \frac{\omega^2}{4}\text{Var}_t(N_{V,t+1}) = f + \omega\sigma_t^2 \tag{49}$$

for some coefficients $f$ and $\omega$ (this is analogous to equation (18) with the addition of a constant, $f$). Under the case considered above, the left-hand side will depend on $\sigma_t$ in addition to $\sigma_t^2$ and a constant. Setting the $\sigma_t$ term to zero requires additional restrictions on the parameters $f$ and $\omega$ and their relation with the news terms; otherwise the proportionality of $N_{RISK}$ and $N_V$ is violated. We consider these restrictions below in greater detail.

Suppose, instead, that by homoskedasticity we mean that $N_V$ itself has constant variance: i.e., $\text{Var}_t(\lambda_v'\eta_{t+1}) = c$, a constant. To obtain this, the vector $\lambda_v$ must be loading *only* on VAR innovations that are homoskedastic. Even in this case, we can show that a $\sigma_t$ term will appear on the left-hand side of eq. (49). To see why, note that $N_{CF} = \lambda'_{CF}\eta_{t+1}$, and at least some of the elements of $\eta$ must depend on $\sigma_t$ (otherwise, the whole model would be homoskedastic and time-varying volatility would be irrelevant). For simplicity, consider the case $N_{CF} = \sigma_t \lambda'_{CF} u_{t+1}$, which is also the case considered in BKSY. We will have

$$\text{Cov}_t(N_{V,t+1}, N_{CF,t+1}) = \text{Cov}_t(\lambda'_{CF}\sigma_t u_{t+1}, \lambda'_V u_{t+1}) = h\sigma_t \tag{50}$$

for a scalar $h = \text{Cov}(\lambda'_{CF}u_{t+1}, \lambda'_V u_{t+1})$. Eq. (49) then reduces to

$$(1-\gamma)^2 \lambda'_{CF}\Sigma\lambda_{CF}\sigma_t^2 + \omega(1-\gamma)h\sigma_t + \frac{\omega^2}{4}c = f + \omega\sigma_t^2 \tag{51}$$

Matching coefficients requires that $\omega(1-\gamma)h = 0$, which can be possible only if either the price of volatility risk is 0 ($\omega = 0$), or if $N_{CF}$ and $N_V$ are uncorrelated. We note that the latter assumption is counterfactual since these news series are negatively correlated in the data.

We conclude that under the assumption of homoskedasticity it will not generally be possible to write $V_t(m_{t+1} + r_{t+1})$ as an affine function of $\sigma_t^2$, and therefore generally it will not be the case that $N_{RISK}$ is proportional to $N_V$.

A similar intuition can be obtained by looking at the conditions for the existence of the value function, in the special case with $\psi = 1$ analyzed above. Suppose that

$$\sigma_{t+1}^2 = s + d\sigma_t^2 + x\epsilon_{t+1} \tag{52}$$

$$\Delta c_{t+1} = k\sigma_t\eta_{t+1} \tag{53}$$

so that $\sigma_t$ scales the volatility of consumption growth but not its own. Conjecturing that $v_t = a + b\sigma_t^2$ and substituting, we find:

$$a + b\sigma_t^2 = \delta(a + bc + bd\sigma_t^2) + \delta(1-\gamma)0.5[b^2x^2 + k^2\sigma_t^2 + 2bxk\theta\sigma_t]. \tag{54}$$

In the right-hand side we now have a term proportional to $\sigma_t$ (and not only $\sigma_t^2$): $bxk\theta\sigma_t$. For the coefficients on the two sides to match, we need to have:

$$bxk\theta = 0. \tag{55}$$

Therefore, for the value function to have a solution of the form $a + b\sigma_t^2$ we need that either the value function does not depend at all on volatility ($b = 0$), one of the shocks has zero variance ($x = 0$ or $k = 0$), or shocks to volatility and shocks to consumption growth are *uncorrelated* ($\theta = 0$). The latter assumption would imply, counterfactually, that $N_V$ and $N_{CF}$ should be uncorrelated, while they are clearly strongly negatively correlated in the data. Unless one of these conditions is met, the value function cannot be written as an affine function of $\sigma_t^2$. And in this case $\text{Var}_t(m_{t+1}+r_{t+1})$, which is proportional to $\text{Var}_t(v_{t+1}+\Delta c_{t+1})$, will not be proportional to $\sigma_t^2$, which again implies that the terms $N_V$ and $N_{RISK}$ will not be proportional to each other.

# 4 Construction of the Test Portfolios

Our primary cross section consists of the excess returns on the 25 ME- and BE/ME-sorted portfolios, studied in Fama and French (1993), extended in Davis, Fama, and French (2000), and made available by Professor Kenneth French on his web site.[3]

We consider two main subsamples: early (1931:3-1963:3) and modern (1963:4-2011:4) due to the findings in Campbell and Vuolteenaho (2004) of dramatic differences in the risks of these portfolios between the early and modern period. The first subsample is shorter than that in Campbell and Vuolteenaho (2004) as we require each of the 25 portfolios to have at least one stock as of the time of formation in June.

We also follow the advice of Daniel and Titman (1997, 2012) and Lewellen, Nagel, and Shanken (2010) and construct a second set of six portfolios double-sorted on past risk loadings to market and variance risk. First, we run a loading-estimation regression for each stock in the CRSP database where $r_{i,t}$ is the log stock return on stock $i$ for month $t$.

$$\sum_{j=1}^{3} r_{i,t+j} = b_0 + b_{r_M} \sum_{j=1}^{3} r_{M,t+j} + b_{\Delta VAR} \sum_{j=1}^{3} \Delta VAR_{t+j} + \varepsilon_{i,t+3} \tag{56}$$

We calculate $\Delta VAR$ as a weighted sum of changes in the VAR state variables. The weight on each change is the corresponding value in the linear combination of VAR shocks that defines news about market variance. We choose to work with changes rather than shocks as this allows us to generate pre-formation loading estimates at a frequency that is different from our VAR. Namely, though we estimate our VAR using calendar-quarter-end data, our approach allows a stock's loading estimates to be updated at each interim month.

The regression is reestimated from a rolling 36-month window of overlapping observations for each stock at the end of each month. Since these regressions are estimated from stock-level instead of portfolio-level data, we use quarterly data to minimize the impact of infrequent trading. With loading estimates in hand, each month we perform a two-dimensional sequential sort on market beta and $\Delta VAR$ beta. First, we form three groups by sorting stocks on $\widehat{b}_{r_M}$. Then, we further sort stocks in each group to three portfolios on $\widehat{b}_{\Delta VAR}$ and record returns on these nine value-weight portfolios. The final set of risk-sorted portfolios are the two sets of three $\widehat{b}_{r_M}$ portfolios within the extreme $\widehat{b}_{\Delta VAR}$ groups. To ensure that the average returns on these portfolio strategies are not influenced by various market-microstructure

[3] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

issues plaguing the smallest stocks, we exclude the five percent of stocks with the lowest $ME$ from each cross-section and lag the estimated risk loadings by a month in our sorts.

Finally, we consider equity portfolios that are formed based on both characteristics and past risk loadings. One possible explanation for our finding that growth stocks hedge volatility relative to value stocks is that growth firms are more likely to hold real options, which increase in value when volatility increases, all else equal. To test this interpretation, we sort stocks based on two firm characteristics that are often used to proxy for the presence of real options and that are available for a large percentage of firms throughout our sample period: BE/ME and idiosyncratic volatility (*ivol*).

We first sort stocks into tritiles based on BE/ME and then into tritiles based on *ivol*. We follow Ang, Hodrick, Xing, and Zhang (2006) and others and estimate *ivol* as the volatility of the residuals from a Fama and French (1993) three-factor regression using daily returns within each month. Finally, we split each of these nine portfolios into two subsets based on pre-formation estimates of *simple* volatility beta, $\widehat{\beta}_{\Delta VAR}$, estimated as above but in a simple regression that does not control for the market return. One might expect that sorts on simple rather than partial betas will be more effective in establishing a link between pre-formation and post-formation estimate of volatility beta, since the market is correlated with volatility news. As before, we exclude the bottom five percent of stocks based on market capitalization and lag our loadings and idiosyncratic volatility estimates by one month.

# 5 Predicting Long-Run Volatility

The predictability of volatility, and especially of its long-run component, is central to this paper. In the text, we have shown that volatility is strongly predictable, and it is predictable in particular by variables beyond lagged realizations of volatility itself: $PE$ and $DEF$ contain essential information about future volatility. We have also proposed a VAR-based methodology to construct long-horizon forecasts of volatility that incorporate all the information in lagged volatility as well as in the additional predictors like $PE$ and $DEF$.

We now ask how well our proposed long-run volatility forecast captures the long-horizon component of volatility. In Appendix Table 1 we regress realized, discounted, annualized long-run variance up to period $h$,

$$LHRVAR_h = \frac{4\Sigma_{j=1}^{h}\rho^{j-1}RVAR_{t+j}}{\Sigma_{j=1}^{h}\rho^{j-1}},$$

on both our VAR forecast and some alternative forecasts of long-run variance.[4] We focus our discussion on the 10-year horizon ($h = 40$) as longer horizons come at the cost of fewer

[4] Note that we measure $LHRVAR$ in annual units. In particular, we rescale by the sum of the weights $\rho^j$ to maintain the scale of the coefficients in the predictive regressions across different horizons.

independent observations; however, Appendix Table 2 confirms that our results are robust to horizons ranging from one to 15 years.

We estimate two standard GARCH-type models, specifically designed to capture the long-run component of volatility. The first one is the two-component EGARCH model proposed by Adrian and Rosenberg (2008). This model assumes the existence of two separate components of volatility, one of which is more persistent than the other, and therefore will tend to capture the long-run dynamics of the volatility process. The other model we estimate is the FIGARCH model of Baillie, Bollerslev, and Mikkelsen (1996), in which the process for volatility is modeled as a fractionally-integrated process, and whose slow, hyperbolic rate of decay of lagged, squared innovations potentially captures long-run movements in volatility better. We first estimate both GARCH models using the full sample of daily returns and then generate the appropriate forecast of $LHRVAR_{40}$.[5] To these two models, we add the set of variables from our VAR, and compare the forecasting ability of these different models.

Appendix Table 1 Panel A reports the results of forecasting regressions of long run volatility $LHRVAR_{40}$ using different specifications. The first regression presents results using the state variables in our VAR, each included separately. The second regression predicts $LHRVAR_{40}$ with the horizon-specific forecast implied by our VAR ($VAR_{40}$). The third and fourth regressions forecast $LHRVAR_{40}$ with the corresponding forecast from the EGARCH model ($EG_{40}$) and the FIGARCH model ($FIG_{40}$) respectively. The fifth and sixth regressions join the VAR variables with the two GARCH-based forecasts, one at a time. The seventh and eighth regressions conduct a horse race between $VAR_{40}$ and $FIG_{40}$ and between $VAR_{40}$ and $DEF$. Regressions nine through 13 focus on the forecasting ability of our two key state variables, $DEF$ and $PE$; we discuss these specifications in more detail below.

First, note that both the EGARCH and FIGARCH forecasts by themselves capture a significant portion of the variation in long-run realized volatility: both have significant coefficients, and both have nontrivial $R^2$s. Our VAR variables provide as good or better explanatory power, and $RVAR$, $PE$ and $DEF$ are strongly statistically significant. Appendix Table 2 documents that these conclusions are true at all horizons (with the exception of $RVAR$ at $h = 8$ and $h = 20$, i.e. two and five years). Finally, the coefficient on the VAR-implied forecast, $VAR_{40}$, is 1.02. This estimate is not only significantly different from zero but also not significantly different from one. This finding indicates that our VAR is able to produce forecasts of volatility that not only go in the right direction, but are also of the right magnitude, even at the 10-year horizon.

Very interesting results appear once we join our variables to the two GARCH models. Even after controlling for the GARCH-based forecasts (which render $RVAR$ insignificant), $PE$ and $DEF$ significantly predict long-horizon volatility, and the addition of the VAR state variables strongly increases the $R^2$. We further show that when using the VAR-implied

[5] We start our forecasting exercise in January 1930 so that we have a long enough history of past returns to feed the FIGARCH model. Other long-run GARCH models could be estimated in a similar manner, for example the FIEGARCH model of Bollerslev and Mikkelsen (1996).

forecast together with the FIGARCH forecast, the coefficient on $VAR_{40}$ is still very close to one and always statistically significant while the FIGARCH coefficient moves closer to zero (though it remains statistically significant at the 10-year horizon).

We develop an additional test of our VAR-based model of stochastic volatility from the idea that the variables that form the VAR – in particular the strongest of them, $DEF$ – should predict volatility at long horizons only *through* the VAR, not *in addition* to it. In other words, the VAR forecasts should ideally represent the best way to combine the information contained in the state variables concerning long-run volatility. If true, after controlling for the VAR-implied forecast, $DEF$ or other variables that enter the VAR should not significantly predict future long-run volatility. We test this hypothesis by running a regression using *both* the VAR-implied forecast and $DEF$ as right-hand side variables. We find that the coefficient on $VAR_{40}$ is still not significantly different from one, while the coefficient on $DEF$ is essentially measured as zero. The online appendix shows that this finding is true at all horizons we consider.

The bottom part of Appendix Table 1 Panel A examines more carefully the link between $DEF$ and $LHRVAR_{40}$. Regressions nine through 13 in the table forecast $LHRVAR_{40}$ with $PE$, $DEF$, $PEO$ ($PE$ orthogonalized to $DEF$), and $DEFO$ ($DEF$ orthogonalized to $PE$). These regressions show that by itself, $PE$ has no information about low-frequency variation in volatility. In contrast, $DEF$ forecasts nearly 22% of the variation in $LHRVAR_{40}$. And once $DEF$ is orthogonalized to $PE$, the $R^2$ increases to nearly 51%. Adding $PEO$ has little effect on the $R^2$. We argue that this is clear evidence of the strong predictive power of the orthogonalized component of the default spread.

As a further check on the usefulness of our VAR approach, we compare our variance forecasts to option-implied variance forecasts. Specifically, using option data from OptionMetrics for the period 1998–2011, we construct the synthetic prices of variance swaps (claims to the realized variance from inception to the maturity of the contract), replicated using a portfolio of options. We construct these prices for maturities 1 to 12 months: $VIX^2_{n,t}$. Under the assumption that returns follow a diffusion, we will have: $VIX^2_{n,t} = E^Q_t[\int_t^{t+n} \sigma^2_s ds]$. We compute $VIX^2_{n,t}$ using the same methodology used by the CBOE to construct the 30-day VIX, applying it to all maturities. We compare the forecast of long-horizon variance at horizon $h$ from our baseline VAR ($VAR_h$) to the corresponding $VIX^2$ at horizon $h$ ($VIX^2_h$).[6] Since our VAR is quarterly, we study forecasts at the three-month, six-month, nine-month, and twelve-month horizons. The top panels of Appendix Figure 2 plot the time series of these forecasts for the three-month and twelve-month horizons. We find that forecasts from the two quite different methods line up well, though the $VIX^2$ forecasts are generally higher, especially near the end of the sample. Appendix Figure 2 also shows that the $VAR_h$ forecasts become smoother when the horizon is extended, relative to both the shorter-horizon

[6] As the $VIX^2_h$ measures do not discount future volatility, for this portion of the analysis, we do not discount either expectations of future variance when constructing our $VAR_h$ measures or their realized variance counterparts when constructing $LHRVAR_h$.

$VAR_h$ forecasts as well as the $VIX_h^2$ forecasts at the same horizon. Appendix Table 1 Panel B confirms these facts by reporting the mean, standard deviation, and correlation of these forecasts, along with the value for realized variance ($LHRVAR_h$) over the corresponding horizon. The $VIX^2$ forecasts are on average approximately 20% larger than their realized variance counterparts.

Appendix Table 1 Panel C reports regressions forecasting $LHRVAR_h$ using the $VAR_h$ forecast, the $VIX_h^2$ forecast, or both together, at each horizon. Both the VAR and the option-based forecasts are individually statistically significant, though the coefficient on $VAR_h$ is always closer to the predicted value of 1.0 at all horizons except for three months. The bottom panels of Appendix Figure 2 plot $LHRVAR_h$ against the fitted value from the $VAR_h$ forecast and against the fitted value of the $VIX_h^2$ forecast for the three-month and twelve-month horizons. The figure confirms that $VAR_h$ is as informative as $VIX_h^2$, if not more so. Indeed, Appendix Table 1 Panel C shows that when both forecasts are included in the regression, $VAR_h$ subsumes $VIX_h^2$, remaining statistically and economically significant.

Taken together, these results make a strong case that credit spreads and valuation ratios contain information about future volatility not captured by simple univariate models, even those like the FIGARCH model or the two-component EGARCH model that are designed to fit long-run movements in volatility, and that our VAR method for calculating long-horizon forecasts preserves this information.

# 6 Changing Volatility Beta of the Aggregate Stock Market

In the paper we find that the average $\beta_V$ of the 25 size- and book-to-market portfolios changes sign from the early to the modern subperiod. Over the 1931-1963 period, the average $\beta_V$ is -0.10 while over the 1964-2011 period this average becomes 0.06. Of course, given the strong positive link between $PE$ and volatility news documented in the paper, one should not be surprised that the market's $\beta_V$ can be positive. Moreover, in Appendix Table 3 we show that the correlation between $PE$ and some of the key variables driving $EVAR$ changes from one subperiod to the other. Nevertheless, we study this change in sign more carefully.

Appendix Figure 3 shows scatter plots with the early period as blue triangles and the modern period data as red asterisks. The top two plots in this figure emphasize that variance news betas are not the same as $RVAR$ betas. The top left portion of the figure plots the market return against $RVAR$. This plot shows that the market does do poorly when realized variance is high, and that this is the case in both subsamples. In fact, this relation is slightly more negative in the modern period. However, our theory tells us that long-horizon investors care about low frequency movements in volatility. The top right portion of the figure plots the market return against volatility news, $N_V$. Consistent with the estimates in the paper, the relation between the market return and $N_V$ is negative in the early period and positive in the modern period.[7] This plot shows that the estimates are robust and not driven by outliers.

The bottom two plots in this figure illustrate what drives this relation in our VAR. The bottom left of the figure plots $PE$ against $DEFO$, our simple proxy for news about long-horizon variance. It is easy to see that the market's $PE$ is high when $DEFO$ is low in the early period, but this relation reverses in the latter period. The bottom right of the figure plots market returns against the contemporaneous change in $DEFO$, showing a negative relation in the early period and a positive relation in the modern period. In other words, the orthogonalization of $DEF$ to $PE$ that creates $DEFO$ is valid over the whole sample, but conceals negative comovement in the early period and positive comovement in the modern period.

In summary, Appendix Figure 3 highlights the important distinction between single-period realized variance $RVAR$ and long-run volatility news, and confirms that the sign change in the market's volatility beta from the early to the modern period can be seen in simple plots of the market return against the change in our key state variable, the $PE$-adjusted default spread.

# 7 Implications for Consumption Growth

## 7.1 Analytical results

Following Campbell (1993), in this paper we substitute consumption out of the pricing equations using the intertemporal budget constraint. However the model does have interesting implications for the implied consumption process. From equation (4) in the text and the identity $r_{t+1} - \mathrm{E}_t r_{t+1} = (\Delta c_{t+1} - \mathrm{E}_t \Delta c_{t+1}) + (h_{t+1} - \mathrm{E}_t h_{t+1})$, we can derive the expression:

$$\Delta c_{t+1} - E_t \Delta c_{t+1} = (r_{t+1} - E_t r_{t+1}) - (\psi - 1) N_{DR,t+1} - (\psi - 1) \frac{1}{2} \frac{\omega}{1-\gamma} N_{V,t+1}. \tag{57}$$

The first two components of the equation for consumption growth are the same as in the homoskedastic case. An unexpectedly high return of the wealth portfolio has a one-for-one effect on consumption. An increase in expected future returns increases today's consumption if $\psi < 1$, as the low elasticity of intertemporal substitution induces the representative investor to consume today (the income effect dominates). If $\psi > 1$, instead, the same increase induces the agent to reduce consumption to better exploit the improved investment opportunities

[7] Straddle returns are negatively correlated with the return on the market portfolio in the 1986:1-2011:4 sample. This negative correlation is not inconsistent with the positive correlation we find between the market return and $N_V$ in the modern sample as the straddle portfolio consists of one-month maturity options and thus should respond to short-term volatility expectations.

(the substitution effect dominates).

The introduction of time-varying conditional volatility adds an additional term to the equation describing consumption growth. News about high future risk is news about a deterioration of future investment opportunities, which is bad news for a risk-averse investor ($\gamma > 1$). When $\psi < 1$, the representative agent will reduce consumption and save to ensure adequate future consumption. An investor with high elasticity of intertemporal substitution, on the other hand, will increase current consumption and reduce the amount of wealth exposed to the future (worse) investment opportunities.

Using estimates of the news terms from our VAR model, we can explore the implications of the model for consumption growth. As shown in the text, the three shocks that drive innovations in consumption growth ($r_{t+1} - E_t r_{t+1}$, $N_{DR,t+1}$, $N_{V,t+1}$) can all be expressed as functions of the vector of innovations $\sigma_t u_{t+1}$. The conditional variance of consumption growth, $\text{Var}_t(\Delta c_{t+1})$, will then be proportional to the conditional variance of returns, $\text{Var}_t(r_{t+1})$; similarly, the conditional standard deviation of consumption growth will be proportional to the conditional standard deviation of returns. As a consequence, the ratio of the standard deviations,

$$A(\gamma, \psi) \equiv \frac{\sqrt{\text{Var}_t(\Delta c_{t+1})}}{\sqrt{\text{Var}_t(r_{t+1})}}$$

will be a constant that depends on the model parameters $\gamma$ and $\psi$ as well as on the unconditional variances and covariances of the innovation vector $u_{t+1}$, which we obtain by estimating the VAR.

Appendix Figure 4 plots the coefficient $A(\gamma, \psi)$ for different values of $\gamma$ and $\psi$ for the homoskedastic case (left panel), and for the heteroskedastic case (right panel). In each panel, we plot $A(\gamma, \psi)$ as $\gamma$ varies between 0 and the maximum possible value of $\gamma$, for different values of $\psi$. Each line corresponds to a different $\psi$ between 0.5 and 1.5; when $\psi = 1$ the value of $A(\gamma, \psi)$ is always equal to 1 since in that case the volatility of consumption growth is equal to the volatility of returns.

As expected, in the homoskedastic case (left panel), the variance of consumption growth does not depend on $\gamma$ but only on $\psi$. It is rising in $\psi$ because our VAR estimates imply that the return on wealth is negatively correlated with news about future expected returns $N_{DR,t+1}$, that is, wealth returns are mean-reverting. This confirms results reported in Campbell (1996). Once we add stochastic volatility (right panel), as $\gamma$ increases the volatility of consumption growth increases for all values of $\psi$ as long as $\psi > 1$, while for values of $\psi < 1$, the effect depends on $\psi$. The reason for this is that the variance of consumption growth depends on the variances and covariances of the three terms that add up to consumption growth as shown in equation (57). Whether $A(\gamma, \psi)$ increases with $\gamma$ or not depends on the relative magnitude of these variances and covariances, which in turn depends on $\psi$.

Overall, Appendix Figure 4 shows that including stochastic volatility makes little difference to the variance of consumption growth for the range of $\gamma$ in which the model admits a solution. And $\gamma$ has a relatively minor effect on the variance of consumption growth, which continues to depend primarily on $\psi$.

## 7.2 Implied and measured aggregate consumption and cash flows

Next we compare the implied consumption innovations ($\Delta c_{t+1} - E_t \Delta c_{t+1}$) to observed innovations in real log aggregate consumption growth, real log dividend growth and real log earnings growth. We construct aggregate consumption growth using nondurable and services data from the BEA, and obtain long-term dividend and earnings data from Professor Robert Shiller's website. We construct consumption, dividends and earnings innovations by taking the residuals of an AR(1) regression for each series. To maximize the length of our time series and to avoid the issue of seasonality in dividends and earnings, we work here with yearly data, from 1930 to 2011. All the results are robust to using smoothed quarterly data for the available periods.

Panel A of Appendix Table 7 reports the standard deviations of six series: innovations in actual consumption, dividends and earnings growth, and implied innovations in consumption under the three calibration for $\psi$ (0.5, 1, 1.5). The table shows that the implied consumption innovation series are more volatile than actual consumption innovations. The fact that aggregate consumption growth is smooth is well known; as pointed out, for example, in Malloy, Moskowitz and Vissing-Jørgensen (2009), aggregate consumption growth may not be the right benchmark for our asset pricing model because it includes consumption of non-stockholder consumers.[8] However, the table also shows that implied consumption growth is about as volatile as earnings and dividend growth innovations (depending on $\psi$).

Panel B of Appendix Table 7 reports the correlations between the consumption innovation series implied by our model and the observed consumption, dividend and earnings growth innovations. The left panel reports the correlations of the raw series, while the right panel reports correlations of exponentially-weighted moving averages of each series, which are useful to show the low-frequency comovements. The table shows that our implied consumption series are positively correlated with the realizations of consumption, dividend and earnings growth. The strength of the correlations increases in almost all cases when looking at the smoothed series.

To gauge the low-frequency comovements between these series better, Appendix Figure 5 shows the exponentially-weighted moving average of the series of implied consumption innovations versus actual consumption, dividends and earnings growth innovations. Given the quite different standard deviations of these series (as reported above), we have standardized all of them before plotting. We use a value of $\psi = 0.5$ when calibrating implied consumption innovations in this graph, so that the volatility of implied consumption innovation matches

[8] In theory, we could test the model directly using data on stockholder consumption, but the time series available spans 20 years, which is not long enough to be used to estimate the news terms and test our model.

that of dividend growth; results are similar for $\psi = 1.0$ and 1.5. The Appendix Figure shows that the time variation in consumption, dividend, and earnings growth innovations aligns well with our implied consumption series, for example, capturing most of the major booms and busts.

Finally, we turn to the implied $N_{CF}$ series. Panel C of Appendix Table 7 reports correlations between our implied $N_{CF}$ series and future long-run consumption growth (looking at the next five, 10 and 15 years). The left side of the table uses the raw $N_{CF}$ series, while the right side of the table uses the exponentially smoothed $N_{CF}$ as a predictor. The table shows that the implied $N_{CF}$ is indeed positively correlated with future consumption, dividend, and earnings growth in almost all cases. Moreover, the relations are generally stronger for longer horizons and when using the smoothed $N_{CF}$ series. Overall, the table suggests that our $N_{CF}$ news term is indeed related to the actual consumption process.

# 8 Robustness

Appendix Table 8 examines the robustness of our findings. Where appropriate, we include in bold font our baseline model as a benchmark. Panel A shows results using various subsets of variables in our baseline VAR. These results indicate that including both $DEF$ and $PE$ are generally essential for our finding of a negative $\beta_V$ for $HML$, consistent with the importance of these two variables in long-run volatility forecasting. Moreover, successful zero-beta-rate volatility ICAPM pricing in the modern period requires $PE$, $DEF$, and $VS$ in the VAR. The results in Panel A also show that the positive volatility beta of the aggregate stock index in the modern period is due to the inclusion of $PE$ and $DEF$ in the VAR. This finding makes sense once one is convinced (and the long-horizon regressions of Appendix Table 1 make a strong case) that, controlling for $DEF$, high $PE$ forecasts high volatility in the future. Since the market will certainly covary positively (and quite strongly) with the $PE$ shock, one should expect this component of volatility news to be positive and an important determinant of the market's volatility beta.

Panel B presents results based on different estimation methods for the VAR. These methods include an OLS VAR, two different bounds on the maximum ratio of WLS weights, a single-stage approach where the weights are proportional to $RVAR$ rather than $EVAR$, and a partial VAR where we throw out in each regression those variables with $t$-statistics under 1.0 (in an iterative fashion, starting with the weakest $t$-statistic first). These results show that our major findings (a negative $\beta_V$ for $HML$ and successful zero-beta rate ICAPM pricing in both time periods) are generally robust to using different methods.

In Panel C, we vary the way in which we estimate realized variance. In the second, fifth, and sixth columns of the Table, we estimate the VAR using annual data. Thus, our estimate of realized variance reflects information over the entire year. In columns three and five, we compute the realized variance of monthly returns rather than the realized variance of daily returns as in our benchmark specification. In the fourth and six columns, we simply sum squared monthly returns. Across Panel C, the $R^2$s of the zero-beta rate ICAPM remain high in the modern period for quarterly VARs.

In Panel D, we alter the set of variables included in the VAR as a response to the concern of Chen and Zhao (2009) that VAR-based forecasts are sensitive to this choice. (See also Engsted, Pedersen, and Tanggaard 2012 for a clarifying discussion of this issue.) We first explore different ways to measure the market's valuation ratio. In the second column of the Table, we replace $PE$ with $PE_{\mathrm{Re}al}$ where we construct the price-earnings ratio by deflating both the price and the earnings series by the CPI before taking their ratio. In the third column, we use the log price-dividend ratio, $PD$, instead of $PE$. In column four, we replace $PE$ with $PE_{\mathrm{Re}al}$ and the CPI inflation rate, $INFL$. Panel D also explores adding two additional state variables. In column five, we add $CAY$ (Lettau and Ludvigson (2001)) to the VAR as $CAY$ is known to be a strong predictor of future market returns. Column six adds the quarterly $FIGARCH$ forecast to the VAR as Appendix Table 1 Panel B documents that GARCH-based methods are useful predictors of future market return variance.

Column seven adds the volatility of the term spread ($TYVol$) to the list of state variables based on the evidence in Fornari and Mele (2011) that this variable contains information about time-varying expected returns. In particular, following Fornari and Mele (2011), we add the mean absolute monthly change in the term spread over the previous twelve months.[9] In unreported results, we find that $TYVol$ is not incrementally important for forecasting either the first or second moment of the real market return. In fact, even if we exclude some or all of $PE$, $R^{Tbill}$, $DEF$, or $VS$ from the VAR, $TYVol$ never comes in significantly. However, $TYVol$ does help forecast $DEF$. Nevertheless, adding $TYVOL$ to the VAR does not qualitatively change the conclusions of the pricing tests. In total, this Panel confirms that our finding of a negative $\beta_V$ for $HML$ and successful zero-beta rate ICAPM pricing in both time periods is generally robust to these variations.

In Panel E, we study the out-of-sample properties of our model. In particular, we estimate our baseline VAR on an expanding window, using the estimates in each window to generate news terms for the quarter ahead. Since the interesting pricing results are in the modern period, our initial window is the 1926:2 to 1963:2 period, so that the first out-of-sample news realizations occur in 1963:3, corresponding to the first data point in the modern period. We find that the out-of-sample news terms are strongly correlated with their in-sample counterparts. Specifically, the correlations are 0.36, 0.43, and 0.63 for the cash-flow, discount-rate, and volatility news terms respectively over the 1963:3-2011:4 subperiod. Appendix Table 8 Panel E documents that the pricing of the out-of-sample news terms is very consistent with the full-sample results. In fact, the out-of-sample version of both the risk-free and zero-beta rate implementations of the ICAPM have higher $R^2$s, with the risk-free rate $R^2$ increasing dramatically from -37% to 64%.

In Panel F, we explore using alternative proxies for the wealth portfolio. In particular,

[9]Results are robust to measuring the volatility of the monthly term spread over the last year instead.

we replace the market returns with the return on a delevered market portfolio that combines Treasury Bills and the market in various constant proportions. By doing so, we are able to assess how varying the volatility of this central series affects our results. The three specific delevered portfolios we examine have 80%, 60%, or 40% invested in the market. We find that the cross-sectional fit of our model remains high, with $R^2$s in the early period essentially unaffected and $R^2$s in the modern period declining slowly. Perhaps not surprisingly, the estimated risk aversion parameter increases as the degree of delevering increases. In the modern period, delevering the market portfolio by 20% results in a risk aversion estimate of 10.2, delevering by 40% requires risk aversion of 14.5, and delevering by 80% generates a risk aversion estimate of 16.3.

Panel G reports the results when we vary $\rho$ and the excess zero-beta rate. One might argue that our excess zero-beta rate estimate of 80 basis points a quarter is too high to be consistent with equilibrium. Fortunately, we find that $R^2$s remain reasonable (60%) for excess zero-beta rates that are as low as 40 bps/quarter when $\rho$ takes only a slightly lower value, 0.94.

Panels H and I present information to help us better understand the volatility betas we have estimated for the market as a whole, and for value stocks relative to growth stocks. Panel H reports components of $RMRF$ and $HML$'s $\beta_V$ in each period (estimated either with WLS or OLS). Specifically, these results use the elements of the vector defined in equation (17) and the corresponding VAR shock to measure how each shock contributes to the $\beta_V$ in question. Panel H documents, consistent with Panel A, that the excess return on the market has a positive volatility beta in the modern period due in part to the $PE$ state variable. The results in Panel H also show that all of the non-zero components of $HML$'s $\beta_V$ in the modern period are negative. This finding is comforting as it further confirms that our negative $HML$ beta finding is robust. Panel H also reports OLS estimates of simple betas on $RVAR$ and the 15-year horizon $FIGARCH$ forecast ($FIG_{60}$) for $HML$ and the excess market return. The $HML$ betas based on these two simple proxies have the same sign as our more sophisticated and more appropriate measure of volatility news. However, conclusions about the relevance of volatility risk for the value effect clearly depend on measuring the long-run component of volatility well.

Panel I reports time-series regressions of $HML$ on $N_{V,t}$ by itself as well as on all three factors together. We find that $N_{V,t}$ explains over 22% of $HML$'s returns in the modern period. The three news factors together explain slightly over 32%. Thus our model is able to explain not only the cross-sectional variation in average returns of the 25 size- and book-to-market-sorted portfolios of Fama and French (1993) but also a significant amount of time series variation in realized returns on the key factor that they argue is multifactor-minimum-variance (Fama and French, 1996).

Finally, given the difference in results between the risk-free and zero-beta rate versions of the model, we explore here the difference between the two specifications within the framework of our baseline results of Table 4 of the main text. The relatively poor performance of the risk-free rate version of the three-beta ICAPM is due to the derived link between $\gamma$ and $\omega$. To show this, Appendix Figure 6 provides two contour plots (one each for the risk-free and zero-beta rate versions of the model in the top and bottom panels of the figure respectively) of the $R^2$ resulting from combinations of ($\gamma$,$\omega$) ranging from (0,0) to (40,40). On the same figure we also plot the relation between $\gamma$ and $\omega$ derived in equation (22). The top panel of the figure shows that even with the intercept restricted to zero, $R^2$'s are as high as 70% for some combinations of ($\gamma$,$\omega$). Unfortunately, as the plot shows, these combinations do not coincide with the curve implied by equation (22). Once the zero-beta rate is unconstrained, the contours for $R^2$'s greater than 60% cover a much larger area of the plot and coincide nicely with the ICAPM relation of equation (22).

Consistent with the contour plots of Appendix Figure 6, the pricing results in Table 4 in the text based on the partially-constrained factor model further confirm that the link between $\gamma$ and $\omega$ is responsible for the poor fit of the restricted zero-beta rate version of the three-beta ICAPM in the modern period. When removing the constraint linking $\gamma$ and $\omega$ but leaving the constraint on the discount-rate beta premium in place, the $R^2$ increases from -37% to 71%. Moreover, the risk prices for $\gamma$ and $\omega$ remain economically large and of the right sign.

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Table 1: Forecasting Long-Horizon Realized Variance

This Appendix Table studies the estimates of long-run variance implied by the VAR model of the paper. Panel A reports the WLS parameter estimates of constrained regressions forecasting the annualized discounted sum of future $RVAR$ over the next 40 quarters $(4 * \sum_{k=1}^{40} \rho^{(k-1)} RVAR_{t+k} / \sum_{k=1}^{40} \rho^{(k-1)})$. The forecasting variables include the VAR state variables, the corresponding annualized long-horizon forecast implied from estimates of the VAR in the paper ($VAR_{40}$) as well as FIGARCH ($FIG_{40}$) and two-factor EGARCH ($EG_{40}$) models estimated from the full sample of daily returns. $r_M$ is the log real return on the CRSP value-weight index. $RVAR$ is the realized variance of within-quarter daily simple returns on the CRSP value-weight index. $PE$ is the log ratio of the S&P 500's price to the S&P 500's ten-year moving average of earnings. $r_{Tbill}$ is the log three-month Treasury Bill yield. $DEF$ is the default yield spread in percentage points, measured as the difference between the log yield on Moody's BAA bonds and the log yield on Moody's AAA bonds. $VS$ is the small-stock value-spread, the difference in the log book-to-market ratios of small value and small growth stocks. $PEO$ is $PE$ orthogonalized to $DEF$ and $DEFO$ is $DEF$ orthogonalized to $PE$. Initial WLS weights are inversely proportional to the corresponding $FIG_{40}$ long-horizon forecast except in those regressions involving $VAR_{40}$ or $EG_{40}$ forecasts, where the corresponding $VAR_{40}$ or $EG_{40}$ long-horizon forecast is used instead. Newey-West standard errors estimated with lags corresponding to twice the number of overlapping observations are in square brackets. The sample period for the dependent variable is 1930.1-2011.4. Panel B of the Appendix Table reports summary statistics for realized variance ($RVAR_h$), the corresponding forecasts from the VAR ($VAR_h$), and the prices of variance swaps ($VIX_h^2$) at various horizons $h$. Panel C of the Appendix Table shows regressions forecasting $LHRVAR_h$ with $VAR_h$ and $VIX_h^2$. In this Panel, we set $\rho$ to 1 when calculating $LHRVAR_h$ and $VAR_h$. Newey-West $t$-statistics that take into account overlapping observations are in brackets.

Panel A: Forecasting 10-year Realized Variance $(4 * \sum_{h=1}^{40} \rho^{(h-1)} RVAR_{t+h} / \sum_{k=1}^{40} \rho^{(h-1)})$

| | Constant | $r_M$ | $RVAR$ | $PE$ | $r_{Tbill}$ | $DEF$ | $VS$ | $VAR_{40}$ | $EG_{40}$ | $FIG_{40}$ | $PEO$ | $DEFO$ | $R^2$% |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | -0.067 | -0.007 | 0.099 | 0.023 | 0.067 | 0.012 | 0.003 | | | | | | 56.66% |
| | [0.016] | [0.005] | [0.031] | [0.006] | [0.086] | [0.004] | [0.003] | | | | | | |
| 2 | -0.009 | | | | | | | 1.017 | | | | | 49.83% |
| | [0.006] | | | | | | | [0.228] | | | | | |
| 3 | -0.067 | | | | | | | | 1.458 | | | | 40.01% |
| | [0.016] | | | | | | | | [0.269] | | | | |
| 4 | -0.006 | | | | | | | | | 0.987 | | | 37.31% |
| | [0.006] | | | | | | | | | [0.177] | | | |
| 5 | -0.105 | -0.010 | 0.021 | 0.022 | 0.023 | 0.011 | 0.002 | | 0.773 | | | | 58.95% |
| | [0.022] | [0.006] | [0.023] | [0.005] | [0.081] | [0.003] | [0.004] | | [0.277] | | | | |
| 6 | -0.077 | -0.008 | -0.022 | 0.021 | 0.106 | 0.008 | 0.002 | | | 0.827 | | | 60.31% |
| | [0.015] | [0.006] | [0.021] | [0.006] | [0.080] | [0.003] | [0.003] | | | [0.225] | | | |
| 7 | -0.016 | | | | | | | 0.820 | | 0.454 | | | 55.73% |
| | [0.006] | | | | | | | [0.211] | | [0.221] | | | |
| 8 | -0.010 | | | | | 0.001 | | 0.995 | | | | | 50.36% |
| | [0.006] | | | | | [0.003] | | [0.232] | | | | | |
| 9 | -0.006 | | | 0.009 | | | | | | | | | -0.53% |
| | (0.026) | | | (0.009) | | | | | | | | | |
| 10 | 0.012 | | | | | 0.008 | | | | | | | 21.75% |
| | (0.005) | | | | | (0.004) | | | | | | | |
| 11 | -0.052 | | | | | | | | | | 0.025 | | 29.36% |
| | (0.014) | | | | | | | | | | (0.005) | | |
| 12 | 0.002 | | | | | | | | | | | 0.018 | 50.60% |
| | (0.003) | | | | | | | | | | | (0.004) | |
| 13 | -0.070 | | | 0.025 | | 0.017 | | | | | | | 51.42% |
| | (0.019) | | | (0.006) | | (0.004) | | | | | | | |

Panel B: Comparing $VAR_h$ and $VIX_h^2$

| | $h=1$ | $h=2$ | $h=3$ | $h=4$ |
|---|---|---|---|---|
| mean | | | | |
| $RVAR$ | 0.048 | 0.047 | 0.047 | 0.047 |
| $VAR_h$ | 0.046 | 0.046 | 0.045 | 0.045 |
| $VIX_h^2$ | 0.059 | 0.058 | 0.058 | 0.058 |
| standard deviation | | | | |
| $RVAR$ | 0.066 | 0.057 | 0.051 | 0.046 |
| $VAR_h$ | 0.021 | 0.018 | 0.017 | 0.015 |
| $VIX_h^2$ | 0.042 | 0.036 | 0.035 | 0.034 |
| correlation | | | | |
| $(VAR_h, VIX_h^2)$ | 0.75 | 0.72 | 0.71 | 0.70 |

Panel C: Forecasting $LHRVAR_h$ with $VAR_h$ and $VIX_h^2$

| $h=1$ | | | $h=2$ | | | $h=3$ | | | $h=4$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Constant | $VAR_h$ | $VIX_h^2$ | Constant | $VAR_h$ | $VIX_h^2$ | Constant | $VAR_h$ | $VIX_h^2$ | Constant | $VAR_h$ | $VIX_h^2$ |
| -0.024 | 1.552 | | -0.011 | 1.275 | | -0.006 | 1.176 | | -0.001 | 1.062 | |
| [-1.22] | [3.97] | | [-0.66] | [3.04] | | [-0.35] | [2.73] | | [-0.05] | [2.55] | |
| 0.009 | | 0.657 | 0.022 | | 0.435 | 0.026 | | 0.365 | 0.029 | | 0.314 |
| [0.62] | | [3.30] | [2.34] | | [2.20] | [2.47] | | [2.27] | [2.26] | | [1.96] |
| -0.022 | 1.255 | 0.194 | -0.012 | 1.422 | -0.099 | -0.008 | 1.375 | -0.127 | -0.002 | 1.194 | -0.083 |
| [-1.08] | [2.11] | [0.66] | [-0.71] | [3.11] | [-0.69] | [-0.44] | [3.15] | [-0.93] | [-0.11] | [3.31] | [-0.65] |

Table 2: Forecasting Long-Horizon Realized Variance

This Appendix Table reports the WLS parameter estimates of constrained regressions forecasting the annualized discounted sum of future $RVAR$ over the next $h$ quarters $(4 * \sum_{k=1}^{h} \rho^{(k-1)} RVAR_{t+k} / \sum_{k=1}^{h} \rho^{(k-1)})$. The forecasting variables include the VAR state variables, the corresponding annualized long-horizon forecast ($VAR_h$) implied from estimates of the VAR in the paper as well as FIGARCH ($FIG_h$) and two-factor EGARCH ($EG_h$) models estimated from the full sample of daily returns. $r_M$ is the log real return on the CRSP value-weight index. $RVAR$ is the realized variance of within-quarter daily simple returns on the CRSP value-weight index. $PE$ is the log ratio of the S&P 500's price to the S&P 500's ten-year moving average of earnings. $r^{Tbill}$ is the log three-month Treasury Bill yield. $DEF$ is the default yield spread in percentage points, measured as the difference between the log yield on Moody's BAA bonds and the log yield on Moody's AAA bonds. $VS$ is the small-stock value-spread, the difference in the log book-to-market ratios of small value and small growth stocks. Initial WLS weights are inversely proportional to the corresponding $FIG_h$ long-horizon forecast except in those regressions involving $VAR_h$ or $EG_h$ forecasts, where the corresponding $VAR_h$ or $EG_h$ long-horizon forecast is used instead. Newey-West standard errors estimated with lags corresponding to twice the number of overlapping observations are in square brackets. The sample period for the dependent variable is 1930.1-2011.4.

Varying the Horizon $h$ in $(4 * \sum_{k=1}^{h} \rho^{(k-1)} RVAR_{t+k} / \sum_{k=1}^{h} \rho^{(k-1)})$

| Constant | $r_M$ | $RVAR$ | $PE$ | $r_{Tbill}$ | $DEF$ | $VS$ | $VAR_h$ | $EG_h$ | $FIG_h$ | $R^2$% |
|---|---|---|---|---|---|---|---|---|---|---|
| $h = 4$ (**1 year ahead**) | | | | | | | | | | |
| -0.083 | -0.025 | 0.198 | 0.027 | -0.178 | 0.028 | -0.001 | | | | 47.20% |
| [0.024] | [0.023] | [0.101] | [0.009] | [0.195] | [0.010] | [0.010] | | | | |
| -0.001 | | | | | | | 0.980 | | | 44.86% |
| [0.005] | | | | | | | [0.214] | | | |
| -0.007 | | | | | | | | 1.054 | | 47.11% |
| [0.004] | | | | | | | | [0.172] | | |
| -0.001 | | | | | | | | | 0.998 | 43.17% |
| [0.004] | | | | | | | | | [0.185] | |
| -0.064 | -0.031 | -0.141 | 0.020 | -0.268 | 0.018 | -0.003 | | 0.897 | | 54.22% |
| [0.019] | [0.019] | [0.092] | [0.007] | [0.165] | [0.007] | [0.009] | | [0.211] | | |
| -0.083 | -0.029 | -0.015 | 0.026 | -0.150 | 0.022 | -0.002 | | | 0.581 | 50.85% |
| [0.023] | [0.021] | [0.095] | [0.009] | [0.191] | [0.009] | [0.010] | | | [0.210] | |
| -0.007 | | | | | | | 0.708 | | 0.492 | 50.04% |
| [0.004] | | | | | | | [0.206] | | [0.186] | |
| -0.003 | | | | | 0.004 | | 0.851 | | | 45.23% |
| [0.006] | | | | | [0.006] | | [0.205] | | | |
| $h = 8$ (**2 years ahead**) | | | | | | | | | | |
| -0.101 | -0.024 | 0.125 | 0.032 | -0.137 | 0.027 | 0.003 | | | | 44.21% |
| [0.028] | [0.017] | [0.082] | [0.011] | [0.206] | [0.011] | [0.010] | | | | |
| -0.003 | | | | | | | 1.023 | | | 44.86% |
| [0.005] | | | | | | | [0.256] | | | |
| -0.013 | | | | | | | | 1.024 | | 37.58% |
| [0.007] | | | | | | | | [0.217] | | |
| 0.001 | | | | | | | | | 0.936 | 32.24% |
| [0.006] | | | | | | | | | [0.234] | |
| -0.094 | -0.025 | -0.087 | 0.027 | -0.186 | 0.019 | 0.000 | | 0.717 | | 48.07% |
| [0.027] | [0.017] | [0.082] | [0.010] | [0.189] | [0.009] | [0.010] | | [0.223] | | |
| -0.102 | -0.027 | 0.019 | 0.032 | -0.119 | 0.024 | 0.002 | | | 0.352 | 45.43% |
| [0.028] | [0.017] | [0.106] | [0.011] | [0.199] | [0.011] | [0.011] | | | [0.363] | |
| -0.008 | | | | | | | 0.866 | | 0.326 | 44.48% |
| [0.006] | | | | | | | [0.262] | | [0.271] | |
| -0.004 | | | | | 0.002 | | 0.953 | | | 42.64% |
| [0.007] | | | | | [0.007] | | [0.239] | | | |
| $h = 20$ (**5 years ahead**) | | | | | | | | | | |
| -0.078 | -0.006 | 0.091 | 0.028 | -0.120 | 0.020 | -0.002 | | | | 44.33% |
| [0.017] | [0.008] | [0.062] | [0.007] | [0.127] | [0.007] | [0.008] | | | | |
| -0.004 | | | | | | | 0.932 | | | 39.58% |
| [0.005] | | | | | | | [0.243] | | | |
| -0.030 | | | | | | | | 1.037 | | 29.62% |
| [0.015] | | | | | | | | [0.299] | | |
| 0.000 | | | | | | | | | 0.865 | 31.13% |
| [0.006] | | | | | | | | | [0.224] | |
| -0.087 | -0.007 | 0.046 | 0.027 | -0.130 | 0.018 | -0.001 | | 0.309 | | 44.51% |
| [0.022] | [0.008] | [0.054] | [0.007] | [0.109] | [0.006] | [0.007] | | [0.408] | | |
| -0.080 | -0.007 | -0.011 | 0.027 | -0.080 | 0.017 | -0.002 | | | 0.471 | 45.63% |
| [0.007] | [0.008] | [0.043] | [0.007] | [0.129] | [0.007] | [0.008] | | | [0.363] | |
| -0.008 | | | | | | | 0.758 | | 0.342 | 43.34% |
| [0.007] | | | | | | | 0.178 | | 0.283 | |
| -0.005 | | | | | 0.002 | | 0.895 | | | 39.88% |
| [0.006] | | | | | [0.004] | | [0.188] | | | |

Varying the Horizon $h$ in $(4 * \sum_{k=1}^{h} \rho^{(k-1)} RVAR_{t+k} / \sum_{k=1}^{h} \rho^{(k-1)})$

| Constant | $r_M$ | $RVAR$ | $PE$ | $r_{Tbill}$ | $DEF$ | $VS$ | $VAR_h$ | $EG_h$ | $FIG_h$ | $R^2\%$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $h = 60$ (**15 years ahead**) | | | | | |
| -0.060 | -0.005 | 0.075 | 0.022 | 0.090 | 0.011 | 0.001 | | | | 50.76% |
| [0.023] | [0.004] | [0.016] | [0.008] | [0.053] | [0.003] | [0.002] | | | | |
| -0.012 | | | | | | | 1.056 | | | 41.54% |
| [0.006] | | | | | | | [0.224] | | | |
| -0.059 | | | | | | | | 1.254 | | 28.76% |
| [0.008] | | | | | | | | [0.386] | | |
| -0.003 | | | | | | | | | 0.812 | 30.86% |
| [0.041] | | | | | | | | | [0.210] | |
| -0.108 | -0.007 | 0.023 | 0.024 | 0.061 | 0.010 | 0.000 | | 0.765 | | 52.63% |
| [0.041] | [0.004] | [0.022] | [0.008] | [0.056] | [0.002] | [0.002] | | [0.442] | | |
| -0.077 | -0.007 | -0.022 | 0.022 | 0.119 | 0.008 | 0.000 | | | 0.863 | 55.29% |
| [0.024] | [0.004] | [0.015] | [0.008] | [0.043] | [0.002] | [0.002] | | | [0.202] | |
| -0.016 | | | | | | | 0.857 | | 0.343 | 45.71% |
| [0.005] | | | | | | | [0.252] | | [0.264] | |
| -0.011 | | | | | 0.001 | | 1.012 | | | 41.46% |
| [0.007] | | | | | [0.003] | | [0.345] | | | |

Table 3: Summary Statistics

This Appendix Table reports descriptive statistics for quarterly observations of the state variables included in the VAR. $r_M$ is the log real return on the CRSP value-weight index. $RVAR$ is the realized variance of within-quarter daily returns on the CRSP value-weight index. $PE$ is the log ratio of the S&P 500's price to the S&P 500's ten-year moving average of earnings. $r^{Tbill}$ is the log three-month Treasury Bill yield. $DEF$ is the default yield spread in percentage points, measured as the difference between the log yield on Moody's BAA bonds and the log yield on Moody's AAA bonds. $VS$ is the small-stock value-spread, the difference in the log book-to-market ratios of small value and small growth stocks. The small-value and small-growth portfolios are two of the six elementary portfolios constructed by Davis et al. (2000). The paper reports the WLS parameter estimates of a constrained regression forecasting $RVAR$ with lagged values of these state variables; the forecasted values from that regression are the state variable $EVAR$ used in the second stage of the estimation and described below. Panel A reports descriptive statistics of these state variables over the full sample period 1926.2-2011.4, 343 quarterly data points. Panel B reports descriptive statistics of these state variables over the early sample period 1926.2-1963.2, 149 quarterly data points. Panel C reports descriptive statistics of these state variables over the modern sample period 1963.3-2011.4, 194 quarterly data points. "Stdev." denotes standard deviation and "Autocorr." the first-order autocorrelation of the series.

Panel A: Full-Sample Summary Statistics

| Variable | Mean | Median | Stdev. | Min | Max | Autocorr. | |
|---|---|---|---|---|---|---|---|
| $r_M$ | 0.016 | 0.027 | 0.107 | -0.406 | 0.635 | -0.038 | |
| $RVAR$ | 0.007 | 0.003 | 0.012 | 0.000 | 0.113 | 0.524 | |
| $EVAR$ | 0.007 | 0.005 | 0.007 | 0.000 | 0.062 | 0.754 | |
| $PE$ | 2.924 | 2.919 | 0.379 | 1.508 | 3.910 | 0.965 | |
| $r_{Tbill}$ | 0.016 | 0.014 | 0.013 | 0.000 | 0.063 | 0.965 | |
| $DEF$ | 1.072 | 0.852 | 0.671 | 0.324 | 5.167 | 0.901 | |
| $VS$ | 1.640 | 1.517 | 0.356 | 1.183 | 2.685 | 0.969 | |
| Correlations | $r_{M,t+1}$ | $RVAR_{t+1}$ | $EVAR_{t+1}$ | $PE_{t+1}$ | $r_{Tbill,t+1}$ | $DEF_{t+1}$ | $VS_{t+1}$ |
| $r_{M,t+1}$ | 1 | -0.306 | -0.306 | 0.083 | -0.040 | -0.138 | -0.037 |
| $RVAR_{t+1}$ | -0.306 | 1 | 0.922 | -0.214 | -0.189 | 0.600 | 0.336 |
| $EVAR_{t+1}$ | -0.306 | 0.922 | 1 | -0.166 | -0.229 | 0.777 | 0.505 |
| $PE_{t+1}$ | 0.083 | -0.214 | -0.166 | 1 | 0.106 | -0.597 | -0.356 |
| $r_{Tbill,t+1}$ | -0.040 | -0.189 | -0.229 | 0.106 | 1 | -0.133 | -0.482 |
| $DEF_{t+1}$ | -0.138 | 0.600 | 0.777 | -0.597 | -0.133 | 1 | 0.645 |
| $VS_{t+1}$ | -0.037 | 0.336 | 0.505 | -0.356 | -0.482 | 0.645 | 1 |
| $r_{M,t}$ | -0.038 | -0.157 | -0.156 | 0.095 | -0.014 | -0.163 | -0.023 |
| $RVAR_t$ | 0.031 | 0.524 | 0.594 | -0.215 | -0.205 | 0.568 | 0.355 |
| $EVAR_t$ | 0.018 | 0.609 | 0.754 | -0.169 | -0.246 | 0.717 | 0.515 |
| $PE_t$ | -0.156 | -0.113 | -0.067 | 0.965 | 0.113 | -0.541 | -0.350 |
| $r_{Tbill,t}$ | -0.031 | -0.156 | -0.191 | 0.101 | 0.965 | -0.102 | -0.477 |
| $DEF_t$ | 0.077 | 0.520 | 0.661 | -0.582 | -0.150 | 0.901 | 0.640 |
| $VS_t$ | -0.032 | 0.338 | 0.491 | -0.358 | -0.490 | 0.619 | 0.969 |
| Covariances | $r_M$ | $RVAR$ | $EVAR$ | $PE$ | $r_{Tbill}$ | $DEF$ | $VS$ |
| $r_M$ | 0.0115 | -0.0004 | -0.0002 | 0.0032 | -0.0001 | -0.0100 | -0.0015 |
| $RVAR$ | -0.0004 | 0.0001 | 0.0001 | -0.0009 | 0.0000 | 0.0047 | 0.0014 |
| $EVAR$ | -0.0002 | 0.0001 | 0.0001 | -0.0005 | 0.0000 | 0.0038 | 0.0013 |
| $PE$ | 0.0032 | -0.0009 | -0.0005 | 0.1434 | 0.0005 | -0.1512 | -0.0478 |
| $r_{Tbill}$ | -0.0001 | 0.0000 | 0.0000 | 0.0005 | 0.0002 | -0.0012 | -0.0022 |
| $DEF$ | -0.0100 | 0.0047 | 0.0038 | -0.1512 | -0.0012 | 0.4496 | 0.1538 |
| $VS$ | -0.0015 | 0.0014 | 0.0013 | -0.0478 | -0.0022 | 0.1538 | 0.1264 |

Panel B: 1926-1963 Summary Statistics

| Variable | Mean | Median | Stdev. | Min | Max | Autocorr. | |
|---|---|---|---|---|---|---|---|
| $r_M$ | 0.020 | 0.029 | 0.128 | -0.406 | 0.635 | -0.105 | |
| $RVAR$ | 0.008 | 0.003 | 0.013 | 0.001 | 0.091 | 0.568 | |
| $EVAR$ | 0.008 | 0.004 | 0.009 | 0.000 | 0.043 | 0.812 | |
| $PE$ | 2.715 | 2.723 | 0.300 | 1.508 | 3.502 | 0.914 | |
| $RF$ | 0.006 | 0.005 | 0.006 | 0.000 | 0.021 | 0.937 | |
| $DEF$ | 1.214 | 0.820 | 0.879 | 0.435 | 5.167 | 0.910 | |
| $VS$ | 1.838 | 1.730 | 0.441 | 1.236 | 2.685 | 0.981 | |
| Correlations | $r_{M,t+1}$ | $RVAR_{t+1}$ | $EVAR_{t+1}$ | $PE_{t+1}$ | $r_{Tbill,t+1}$ | $DEF_{t+1}$ | $VS_{t+1}$ |
| $r_{M,t+1}$ | 1 | -0.233 | -0.311 | 0.128 | 0.013 | -0.217 | -0.100 |
| $RVAR_{t+1}$ | -0.233 | 1 | 0.918 | -0.458 | -0.123 | 0.684 | 0.411 |
| $EVAR_{t+1}$ | -0.311 | 0.918 | 1 | -0.533 | -0.185 | 0.894 | 0.644 |
| $PE_{t+1}$ | 0.128 | -0.458 | -0.533 | 1 | 0.605 | -0.727 | -0.501 |
| $r_{Tbill,t+1}$ | 0.013 | -0.123 | -0.185 | 0.605 | 1 | -0.340 | -0.549 |
| $DEF_{t+1}$ | -0.217 | 0.684 | 0.894 | -0.727 | -0.340 | 1 | 0.777 |
| $VS_{t+1}$ | -0.100 | 0.411 | 0.644 | -0.501 | -0.549 | 0.777 | 1 |
| $r_{M,t}$ | -0.105 | -0.107 | -0.134 | 0.131 | 0.010 | -0.170 | -0.061 |
| $RVAR_t$ | 0.046 | 0.568 | 0.642 | -0.447 | -0.170 | 0.652 | 0.428 |
| $EVAR_t$ | 0.025 | 0.684 | 0.812 | -0.530 | -0.237 | 0.838 | 0.651 |
| $PE_t$ | -0.239 | -0.332 | -0.383 | 0.914 | 0.605 | -0.615 | -0.480 |
| $r_{Tbill,t}$ | 0.001 | -0.053 | -0.133 | 0.580 | 0.937 | -0.316 | -0.528 |
| $DEF_t$ | 0.068 | 0.667 | 0.812 | -0.704 | -0.383 | 0.910 | 0.771 |
| $VS_t$ | -0.039 | 0.410 | 0.623 | -0.494 | -0.567 | 0.749 | 0.981 |
| Covariances | $r_M$ | $RVAR$ | $EVAR$ | $PE$ | $r_{Tbill}$ | $DEF$ | $VS$ |
| $r_M$ | 0.0163 | -0.0004 | -0.0004 | 0.0048 | 0.0000 | -0.0243 | -0.0058 |
| $RVAR$ | -0.0004 | 0.0002 | 0.0001 | -0.0018 | 0.0000 | 0.0078 | 0.0024 |
| $EVAR$ | -0.0004 | 0.0001 | 0.0001 | -0.0014 | 0.0000 | 0.0069 | 0.0025 |
| $PE$ | 0.0048 | -0.0018 | -0.0014 | 0.0898 | 0.0010 | -0.1911 | -0.0657 |
| $r_{Tbill}$ | 0.0000 | 0.0000 | 0.0000 | 0.0010 | 0.0000 | -0.0017 | -0.0014 |
| $DEF$ | -0.0243 | 0.0078 | 0.0069 | -0.1911 | -0.0017 | 0.7723 | 0.3009 |
| $VS$ | -0.0058 | 0.0024 | 0.0025 | -0.0657 | -0.0014 | 0.3009 | 0.1945 |

Panel C: 1963-2011 Summary Statistics

| Variable | Mean | Median | Stdev. | Min | Max | Autocorr. | |
|---|---|---|---|---|---|---|---|
| $RVAR$ | 0.006 | 0.004 | 0.011 | 0.000 | 0.113 | 0.464 | |
| $EVAR$ | 0.007 | 0.006 | 0.006 | 0.000 | 0.062 | 0.653 | |
| $PE$ | 3.085 | 3.114 | 0.354 | 2.331 | 3.910 | 0.976 | |
| $r_{Tbill}$ | 0.023 | 0.022 | 0.013 | 0.000 | 0.063 | 0.943 | |
| $DEF$ | 0.963 | 0.855 | 0.421 | 0.324 | 3.167 | 0.854 | |
| $VS$ | 1.488 | 1.484 | 0.147 | 1.183 | 2.045 | 0.809 | |
| Correlations | $r_{M,t+1}$ | $RVAR_{t+1}$ | $EVAR_{t+1}$ | $PE_{t+1}$ | $r_{Tbill,t+1}$ | $DEF_{t+1}$ | $VS_{t+1}$ |
| $r_{M,t+1}$ | 1 | -0.414 | -0.301 | 0.102 | -0.053 | 0.008 | 0.060 |
| $RVAR_{t+1}$ | -0.414 | 1 | 0.936 | -0.005 | -0.235 | 0.480 | 0.256 |
| $EVAR_{t+1}$ | -0.301 | 0.936 | 1 | 0.173 | -0.367 | 0.554 | 0.377 |
| $PE_{t+1}$ | 0.102 | -0.005 | 0.173 | 1 | -0.558 | -0.543 | 0.429 |
| $r_{Tbill,t+1}$ | -0.053 | -0.235 | -0.367 | -0.558 | 1 | 0.193 | -0.213 |
| $DEF_{t+1}$ | 0.008 | 0.480 | 0.554 | -0.543 | 0.193 | 1 | 0.060 |
| $VS_{t+1}$ | 0.060 | 0.256 | 0.377 | 0.429 | -0.213 | 0.060 | 1 |
| $r_{M,t}$ | 0.064 | -0.233 | -0.196 | 0.131 | 0.010 | -0.185 | -0.007 |
| $RVAR_t$ | 0.005 | 0.464 | 0.529 | -0.011 | -0.242 | 0.441 | 0.287 |
| $EVAR_t$ | 0.002 | 0.507 | 0.653 | 0.167 | -0.373 | 0.473 | 0.391 |
| $PE_t$ | -0.104 | 0.106 | 0.257 | 0.976 | -0.548 | -0.532 | 0.421 |
| $r_{Tbill,t}$ | -0.028 | -0.199 | -0.309 | -0.567 | 0.943 | 0.270 | -0.201 |
| $DEF_t$ | 0.088 | 0.265 | 0.346 | -0.524 | 0.184 | 0.854 | 0.044 |
| $VS_t$ | -0.114 | 0.269 | 0.370 | 0.407 | -0.231 | 0.015 | 0.809 |
| Covariances | $r_M$ | $RVAR$ | $EVAR$ | $PE$ | $r_{Tbill}$ | $DEF$ | $VS$ |
| $r_M$ | 0.0079 | -0.0004 | -0.0002 | 0.0032 | -0.0001 | 0.0002 | 0.0008 |
| $RVAR$ | -0.0004 | 0.0001 | 0.0001 | 0.0000 | 0.0000 | 0.0021 | 0.0004 |
| $EVAR$ | -0.0002 | 0.0001 | 0.0000 | 0.0004 | 0.0000 | 0.0014 | 0.0003 |
| $PE$ | 0.0032 | 0.0000 | 0.0004 | 0.1255 | -0.0025 | -0.0807 | 0.0222 |
| $r_{Tbill}$ | -0.0001 | 0.0000 | 0.0000 | -0.0025 | 0.0002 | 0.0010 | -0.0004 |
| $DEF$ | 0.0002 | 0.0021 | 0.0014 | -0.0807 | 0.0010 | 0.1768 | 0.0033 |
| $VS$ | 0.0008 | 0.0004 | 0.0003 | 0.0222 | -0.0004 | 0.0033 | 0.0215 |

Table 4: VAR Estimation

This Appendix table reports the correlation ("Corr/std") and autocorrelation ("Autocorr.") matrices of both the unscaled and scaled shocks from the second-stage VAR estimated in Table 1. The sample period for the dependent variables is 1926.3-2011.4, 342 quarterly data points.shows the WLS parameter estimates for a first-order VAR model. The state variables

in the VAR include the log real return on the CRSP value-weight index ($r_M$), the realized variance ($RVAR$) of within-quarter daily simple returns on the CRSP value-weight index, the log ratio of the S&P 500's price to the S&P 500's ten-year moving average of earnings ($PE$), the log three-month Treasury Bill yield ($r_{Tbill}$), the default yield spread ($DEF$) in percentage points, measured as the difference between the log yield on Moody's BAA bonds and the log yield on Moody's AAA bonds, and the small-stock value spread ($VS$), the difference in the log book-to-market ratios of small value and small growth stocks. The small-value and small-growth portfolios are two of the six elementary portfolios constructed by Davis et al. (2000). For the sake of interpretation, we estimate the VAR in two stages.

Autocorrelations of VAR residuals

| Autocorr. | $r_{M,t+1}$ | $EVAR_{t+1}$ | $PE_{t+1}$ | $r_{Tbill,t+1}$ | $DEF_{t+1}$ | $VS_{t+1}$ |
|---|---|---|---|---|---|---|
| | | | unscaled | | | |
| $r_{M,t}$ | -0.064 | 0.090 | -0.058 | -0.041 | 0.085 | 0.045 |
| $EVAR_t$ | 0.073 | -0.157 | 0.086 | 0.114 | -0.188 | -0.080 |
| $PE_t$ | -0.075 | 0.180 | -0.141 | -0.063 | 0.206 | 0.093 |
| $r_{Tbill,t}$ | 0.002 | 0.016 | -0.013 | -0.139 | -0.029 | -0.057 |
| $DEF_t$ | 0.132 | -0.140 | 0.169 | 0.109 | -0.289 | -0.145 |
| $VS_t$ | 0.021 | -0.035 | 0.018 | 0.037 | -0.085 | -0.083 |
| | | | scaled | | | |
| $r_{M,t}$ | 0.005 | 0.042 | 0.001 | -0.003 | -0.001 | -0.008 |
| $EVAR_t$ | 0.061 | -0.105 | 0.074 | 0.077 | -0.124 | -0.053 |
| $PE_t$ | -0.007 | 0.125 | -0.072 | -0.032 | 0.097 | 0.026 |
| $r_{Tbill,t}$ | -0.014 | 0.038 | -0.028 | -0.123 | 0.000 | -0.036 |
| $DEF_t$ | 0.080 | -0.097 | 0.109 | 0.085 | -0.202 | -0.102 |
| $VS_t$ | 0.020 | -0.027 | 0.008 | 0.018 | -0.076 | -0.066 |

Table 5: Average Excess Returns on Test Assets

This Appendix Table shows the average excess returns on the 25 ME- and BE/ME-sorted portfolios (Panel A), six risk-sorted portfolios (Panel B), and 18 BE/ME, IVol, and $\widehat{\beta}_{\Delta VAR}$-sorted portfolios (Panel C). "Growth" denotes the lowest BE/ME, "Value" the highest BE/ME, "Small" the lowest ME, and "Large" the highest ME stocks. $\widehat{b}_{\Delta VAR}$ and $\widehat{b}_{r_M}$ are past return-loadings on the weighted sum of changes in the VAR state variables, where the weights are according to $\lambda_V$ as estimated in Table 1 in Campbell, Giglio, Polk, and Turley (2015), and on the market-return shock. Estimates are based on quarterly data for the 1931:3-1963:2 and 1963:3-2011:4 subperiods.

Panel A: 25 ME- and BE/ME-sorted portfolios

| 1931:3-1963:2 | | | | | |
|---|---|---|---|---|---|
| | Growth | 2 | 3 | 4 | Value |
| Small | 3.45% | 3.80% | 6.13% | 6.61% | 7.35% |
| 2 | 3.76% | 5.36% | 5.30% | 5.62% | 6.18% |
| 3 | 4.42% | 4.05% | 4.79% | 4.79% | 5.73% |
| 4 | 3.03% | 3.87% | 4.33% | 4.51% | 5.82% |
| Large | 2.85% | 2.53% | 3.55% | 3.95% | 4.80% |
| 1963:3-2011:4 | | | | | |
| | Growth | 2 | 3 | 4 | Value |
| Small | 0.94% | 2.52% | 2.60% | 3.12% | 3.58% |
| 2 | 1.49% | 2.21% | 2.88% | 2.86% | 3.12% |
| 3 | 1.47% | 2.31% | 2.37% | 2.65% | 3.23% |
| 4 | 1.76% | 1.69% | 2.08% | 2.54% | 2.53% |
| Large | 1.28% | 1.46% | 1.32% | 1.55% | 1.63% |

Panel B: 6 risk-sorted portfolios

| 1931:3-1963:2 | | | |
|---|---|---|---|
| | Lo $\widehat{b}_{r_M}$ | 2 | Hi $\widehat{b}_{r_M}$ |
| Lo $\widehat{b}_{VAR}$ | 2.74% | 3.56% | 4.48% |
| Hi $\widehat{b}_{VAR}$ | 2.74% | 4.12% | 4.67% |
| 1963:3-2011:4 | | | |
| | Lo $\widehat{b}_{r_M}$ | 2 | Hi $\widehat{b}_{r_M}$ |
| Lo $\widehat{b}_{VAR}$ | 1.87% | 2.19% | 2.48% |
| Hi $\widehat{b}_{VAR}$ | 0.98% | 1.29% | 1.28% |

Panel C: 18 BE/ME, IVol, and $\widehat{\beta}_{\Delta VAR}$-sorted portfolios

| 1931:3-1963:2 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\widehat{\beta}_{CF}$ | Growth | | | 2 | | | Value | | |
| | Low IVol | 2 | High IVol | Low IVol | 2 | High IVol | Low IVol | 2 | High IVol |
| Low $\widehat{\beta}_{\Delta VAR}$ | 2.40% | 2.63% | 3.88% | 3.33% | 4.45% | 4.09% | 4.96% | 5.77% | 6.41% |
| High $\widehat{\beta}_{\Delta VAR}$ | 2.72% | 3.33% | 3.14% | 3.57% | 4.35% | 4.86% | 5.02% | 5.17% | 6.03% |
| | $P1$: 2.54% | | | $P2$: Growth 0.95%, Value 1.22% | | | $P3$: 0.03% | | |

Panel C: 18 BE/ME, IVol, and $\widehat{\beta}_{\Delta VAR}$-sorted portfolios

| 1963:3-2011:4 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\widehat{\beta}_{CF}$ | Growth | | | 2 | | | Value | | |
| | Low IVol | 2 | High IVol | Low IVol | 2 | High IVol | Low IVol | 2 | High IVol |
| Low $\widehat{\beta}_{\Delta VAR}$ | 1.52% | 1.39% | -0.22% | 1.83% | 2.18% | 2.28% | 2.02% | 3.65% | 3.51% |
| High $\widehat{\beta}_{\Delta VAR}$ | 1.06% | 0.91% | -0.44% | 1.47% | 1.52% | 1.18% | 2.02% | 3.03% | 3.29% |
| | $P1$: 2.22% | | | $P2$: Growth -1.62%, Value 1.38% | | | $P3$: -0.46% | | |

Table 6: Cash-flow, Discount-rate, and Variance Betas: BE/ME, IVol, and Risk-sorted Portfolios

The table shows the estimated cash-flow ($\widehat{\beta}_{CF}$), discount-rate ($\widehat{\beta}_{DR}$), and variance betas ($\widehat{\beta}_V$) for the 18 BE/ME, IVol, and $\widehat{\beta}_{\Delta VAR}$-sorted portfolios (Panels A and B) for the early (1931:3-1963:2) and modern (1963:3-2011:4) subsamples respectively. $\widehat{\beta}_{\Delta VAR}$ is the past return-loadings on the weighted sum of changes in the VAR state variables, where the weights are according to $\lambda_V$ as estimated in Campbell, Giglio, Polk, and Turley (2015) Table 2. $P1$ is the composite portfolio that is long the equal-weight average of the value portfolios and is short the equal-weight average of the growth portfolios. $P2$ is the composite portfolio that is long the high idiosyncratic portfolio and short the low idiosyncratic portfolio for *either* the growth subset *or* the value subset. $P3$ is the portfolio that is long the equal-weight average of the high $\widehat{\beta}_{\Delta VAR}$ portfolios and is short the equal-weight average of the low $\widehat{\beta}_{\Delta VAR}$ portfolios. Bootstrapped standard errors [in brackets] are conditional on the estimated news series. Estimates are based on quarterly data using weighted least squares where the weights are the same as those used to estimate the VAR.

18 BE/ME, IVol, and $\widehat{\beta}_{\Delta VAR}$-sorted portfolios

Panel A: Early Period (1931:3-1963:2)

| $\widehat{\beta}_{CF}$ | Growth | | | | | | 2 | | | | | | Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | |
| Low $\widehat{\beta}_{\Delta VAR}$ | 0.24 | [0.08] | 0.31 | [0.10] | 0.38 | [0.11] | 0.25 | [0.08] | 0.39 | [0.11] | 0.43 | [0.13] | 0.37 | [0.09] | 0.47 | [0.12] | 0.47 | [0.12] |
| High $\widehat{\beta}_{\Delta VAR}$ | 0.21 | [0.06] | 0.26 | [0.07] | 0.29 | [0.08] | 0.24 | [0.07] | 0.33 | [0.10] | 0.36 | [0.09] | 0.37 | [0.10] | 0.38 | [0.10] | 0.44 | [0.10] |
| | $P1$: 0.31 [0.07] | | | | | | $P2$: Growth 0.13 [0.03], Value 0.10 [0.03] | | | | | | $P3$: -0.05 [0.02] | | | | | |

| $\widehat{\beta}_{DR}$ | Growth | | | | | | 2 | | | | | | Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | |
| Low $\widehat{\beta}_{\Delta VAR}$ | 0.69 | [0.10] | 0.89 | [0.13] | 1.07 | [0.15] | 0.86 | [0.14] | 1.03 | [0.13] | 1.21 | [0.15] | 0.96 | [0.13] | 1.18 | [0.16] | 1.16 | [0.13] |
| High $\widehat{\beta}_{\Delta VAR}$ | 0.60 | [0.07] | 0.68 | [0.08] | 0.73 | [0.08] | 0.75 | [0.11] | 0.90 | [0.12] | 0.95 | [0.13] | 1.06 | [0.15] | 1.10 | [0.15] | 1.09 | [0.16] |
| | $P1$: 1.05 [0.49] | | | | | | $P2$: Growth 0.31 [0.10], Value 0.15 [0.05] | | | | | | $P3$: -0.15 [0.04] | | | | | |

| $\widehat{\beta}_{V}$ | Growth | | | | | | 2 | | | | | | Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | |
| Low $\widehat{\beta}_{\Delta VAR}$ | -0.04 | [0.02] | -0.08 | [0.03] | -0.11 | [0.03] | -0.09 | [0.04] | -0.10 | [0.04] | -0.12 | [0.03] | -0.10 | [0.03] | -0.14 | [0.05] | -0.13 | [0.04] |
| High $\widehat{\beta}_{\Delta VAR}$ | -0.04 | [0.02] | -0.05 | [0.02] | -0.06 | [0.03] | -0.08 | [0.04] | -0.09 | [0.03] | -0.10 | [0.04] | -0.12 | [0.04] | -0.13 | [0.04] | -0.14 | [0.05] |
| | $P1$: -0.13 [0.06] | | | | | | $P2$: Growth -0.05 [0.02], Value -0.02 [0.01] | | | | | | $P3$: 0.02 [0.01] | | | | | |

Panel B: Modern Period (1963:3-2011:4)

| $\widehat{\beta}_{CF}$ | Growth | | | | | | 2 | | | | | | Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | |
| Low $\widehat{\beta}_{\Delta VAR}$ | 0.17 | [0.03] | 0.19 | [0.05] | 0.22 | [0.06] | 0.22 | [0.04] | 0.26 | [0.05] | 0.29 | [0.06] | 0.23 | [0.04] | 0.30 | [0.06] | 0.27 | [0.06] |
| High $\widehat{\beta}_{\Delta VAR}$ | 0.17 | [0.03] | 0.16 | [0.05] | 0.19 | [0.07] | 0.18 | [0.03] | 0.22 | [0.04] | 0.25 | [0.05] | 0.20 | [0.04] | 0.25 | [0.05] | 0.32 | [0.05] |
| | $P1$: 0.09 [0.02] | | | | | | $P2$: Growth 0.03 [0.04], Value 0.08 [0.03] | | | | | | $P3$: -0.02 [0.02] | | | | | |

| $\widehat{\beta}_{DR}$ | Growth | | | | | | 2 | | | | | | Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | |
| Low $\widehat{\beta}_{\Delta VAR}$ | 0.71 | [0.05] | 1.06 | [0.10] | 1.33 | [0.12] | 0.59 | [0.05] | 0.79 | [0.07] | 1.11 | [0.09] | 0.59 | [0.06] | 0.85 | [0.09] | 0.96 | [0.12] |
| High $\widehat{\beta}_{\Delta VAR}$ | 0.74 | [0.05] | 1.07 | [0.08] | 1.37 | [0.14] | 0.57 | [0.05] | 0.86 | [0.07] | 1.08 | [0.10] | 0.56 | [0.07] | 0.88 | [0.09] | 1.05 | [0.08] |
| | $P1$: -0.20 [0.05] | | | | | | $P2$: Growth 0.58 [0.10], Value 0.44 [0.07] | | | | | | $P3$: 0.00 [0.04] | | | | | |

| $\widehat{\beta}_{V}$ | Growth | | | | | | 2 | | | | | | Value | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | | Low IVol | | 2 | | High IVol | |
| Low $\widehat{\beta}_{\Delta VAR}$ | 0.07 | [0.04] | 0.08 | [0.08] | 0.14 | [0.07] | 0.03 | [0.04] | 0.02 | [0.05] | 0.07 | [0.07] | 0.01 | [0.05] | 0.01 | [0.07] | -0.03 | [0.09] |
| High $\widehat{\beta}_{\Delta VAR}$ | 0.10 | [0.04] | 0.12 | [0.06] | 0.13 | [0.09] | 0.04 | [0.04] | 0.06 | [0.06] | 0.09 | [0.06] | 0.05 | [0.04] | 0.02 | [0.07] | 0.07 | [0.05] |
| | $P1$: -0.09 [0.02] | | | | | | $P2$: Growth 0.05 [0.04], Value 0.00 [0.03] | | | | | | $P3$: 0.03 [0.01] | | | | | |

Table 7: Actual and Implied Consumption

This Appendix Table reports a comparison of the consumption innovations and cash flow news series implied by the model and their data counterparts (obtained using observed consumption, dividends and earnings). We use yearly data between 1930 and 2011. Real consumption growth is constructed from BEA data, real dividend and earnings growth series are obtain from Professor Shiller's website. Innovations in these variables are constructed by taking a residual of an AR(1) regression for each series. Implied cash flow news series are constructed by estimating a yearly version of our baseline VAR. Panel A reports standard deviations of implied consumption innovations (for different values of $\psi$) and actual consumption, dividends, and earnings innovations. Panel B reports the correlations between implied consumption and actual innovations in consumption, dividends and earnings. The left part of the table reports correlations with the innovations of the raw series. The right side of the table reports correlations with smoothed consumption innovations (using the exponential moving average as in the paper, with the same smoothing parameter of 0.08 per quarter, or 0.29 per year). Panel C reports the correlations of the raw (left side) and exponentially-smoothed (right side) $N_{CF}$ series with future cumulative consumption growth, looking 5, 10 and 15 years ahead.

Panel A: Standard Deviations

| | |
|---|---|
| $\Delta c$ (actual) | 0.019 |
| $\Delta d$ (actual) | 0.108 |
| $\Delta e$ (actual) | 0.291 |
| $\Delta c$ (implied, $\psi = 0.5$) | 0.107 |
| $\Delta c$ (implied, $\psi = 1$) | 0.178 |
| $\Delta c$ (implied, $\psi = 1.5$) | 0.277 |

Panel B: Implied vs. actual consumption innovations

| | Raw series | | | Smoothed series | | |
|---|---|---|---|---|---|---|
| | $\Delta c$ (actual) | $\Delta d$ (actual) | $\Delta e$ (actual) | $\Delta c$ (actual) | $\Delta d$ (actual) | $\Delta e$ (actual) |
| $\Delta c$ (implied, $\psi = 0.5$) | 0.32 | 0.29 | 0.15 | 0.34 | 0.47 | 0.21 |
| $\Delta c$ (implied, $\psi = 1$) | 0.33 | 0.27 | 0.23 | 0.36 | 0.40 | 0.25 |
| $\Delta c$ (implied, $\psi = 1.5$) | 0.30 | 0.24 | 0.24 | 0.35 | 0.35 | 0.24 |

Panel C: $N_{CF}$ vs. actual long-run consumption

| | Raw $N_{CF}$ series | | | Smoothed $N_{CF}$ series | | |
|---|---|---|---|---|---|---|
| | 5-year | 10-year | 15-year | 5-year | 10-year | 15-year |
| Long-run $\Delta c$ | 0.03 | -0.01 | 0.09 | -0.04 | -0.03 | 0.19 |
| Long-run $\Delta d$ | 0.07 | 0.10 | 0.09 | 0.22 | 0.22 | 0.13 |
| Long-run $\Delta e$ | -0.04 | 0.05 | 0.17 | 0.09 | 0.24 | 0.29 |

Table 8: Robustness

This Appendix Table provides a variety of robustness tests. When appropriate, the baseline model appears in bold font. Panel A reports the results when only a subset of state variables from the baseline VAR ($D \equiv DEF$, $R \equiv r_{Tbill}$, $V \equiv VS$, $P \equiv PE$) are used to forecast returns and realized variance. Panel B reports the results when different estimation techniques are used. Panel C reports results as we change the estimate of realized variance. Panel D reports the results when other state variables either replace or are added to the VAR. These variables include the log real PE ratio ($PE_{\mathrm{Real}}$), the log price-dividend ratio ($PD$), log inflation ($INFL$), $CAY$, the quarterly $FIGARCH$ variance forecast ($FIG$), and the term spread volatility ($TYVol$). Panel E reports the modern-period results when out-of-sample versions of the model's news terms are used in the pricing tests. Panel F reports results using delevered market portfolios. Panel G reports results when the excess zero-beta rate is varied from 40 to 86 basis points per quarter. Panel H reports the components of $RMRF$ and $HML$'s $\widehat{\beta}_V$ by re-estimating $\widehat{\beta}_V$ using each component of $\mathbf{e2}'\lambda_V$. Panel I also reports simple loadings of $RMRF$ and $HML$ on $RVAR$ and the 15-year $FIGARCH$ variance forecast. Panel J reports time-series regressions explaining $HML$ with the three news terms.

Panel A: Results Using Various Subsets of the Baseline VAR ($r_M$ and $RVAR$ always included)

| | None | D | D/R/V | **ALL** | P/D/V | P/D | P |
|---|---|---|---|---|---|---|---|
| $\widehat{\gamma}^{Max}$ | 4.5 | 3.2 | 3.1 | **7.2** | 6.7 | 8.9 | 14.8 |
| | | | | Early Period | | | |
| | | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | -0.03 | -0.23 | -0.20 | **-0.03** | -0.03 | -0.02 | 0.08 |
| $SMB$ | -0.01 | -0.08 | -0.07 | **-0.02** | -0.02 | -0.02 | 0.03 |
| $HML$ | 0.00 | -0.12 | -0.13 | **-0.06** | -0.06 | -0.04 | 0.06 |
| | | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | 2.1 | 2.1 | 2.3 | **4.8** | 4.6 | 4.7 | 7.7 |
| $\widehat{\omega}$ | 1.9 | 2.0 | 2.4 | **5.0** | 4.2 | 3.3 | 10.9 |
| $\widehat{R^2}$ | 52% | 51% | 52% | **52%** | 54% | 51% | 45% |
| | | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | 0.02% | -0.03% | 0.09% | **0.30%** | 0.21% | 0.08% | 0.57% |
| $\widehat{\gamma}$ | 2.1 | 2.2 | 2.2 | **4.4** | 4.3 | 4.6 | 5.7 |
| $\widehat{\omega}$ | 1.8 | 2.0 | 2.3 | **4.0** | 3.6 | 3.1 | 5.3 |
| $\widehat{R^2}$ | 52% | 51% | 53% | **53%** | 54% | 51% | 46% |
| | | | | Modern Period | | | |
| | | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | -0.11 | -0.15 | -0.07 | **0.10** | 0.11 | 0.07 | 0.00 |
| $SMB$ | -0.03 | -0.05 | -0.02 | **0.03** | 0.03 | 0.01 | 0.00 |
| $HML$ | 0.00 | -0.01 | -0.11 | **-0.11** | -0.11 | -0.05 | -0.02 |
| | | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | 2.4 | 2.4 | 2.7 | **6.8** | 6.2 | 8.9 | 6.2 |
| $\widehat{\omega}$ | 3.1 | 3.1 | 5.4 | **16.0** | 11.6 | 29.4 | 6.5 |
| $\widehat{R^2}$ | -41% | -41% | 4% | **-37%** | -102% | -20% | -49% |
| | | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | 2.13% | 2.10% | -0.74% | **0.80%** | 0.98% | 1.57% | 1.60% |
| $\widehat{\gamma}$ | 0.0 | 0.0 | 3.1 | **7.2** | 6.7 | 0.0 | 0.0 |
| $\widehat{\omega}$ | 1.3 | 1.1 | 14.9 | **24.9** | 20.3 | 0.2 | 0.2 |
| $\widehat{R^2}$ | 1% | 2% | 26% | **63%** | 53% | -3% | -3% |

Panel B: Results Using Different Estimation Methods

| | All OLS | WLS 3 | **WLS 5** | WLS 8 | RVAR Weighted | Partial VAR |
|---|---|---|---|---|---|---|
| $\widehat{\gamma}^{Max}$ | 1.6 | 7.1 | **7.2** | 7.1 | 7.1 | 7.1 |
| | | | Early Period | | | |
| | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | -0.07 | -0.05 | **-0.03** | -0.01 | -0.01 | 0.01 |
| $SMB$ | -0.03 | -0.02 | **-0.02** | -0.02 | -0.02 | -0.01 |
| $HML$ | -0.08 | -0.07 | **-0.06** | -0.05 | -0.05 | -0.03 |
| | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | 1.6 | 4.7 | **4.8** | 4.8 | 4.8 | 3.9 |
| $\widehat{\omega}$ | 2.3 | 5.0 | **5.0** | 5.4 | 5.9 | 15.9 |
| $\widehat{R^2}$ | -251% | 52% | **52%** | 52% | 52% | 41% |
| | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | 2.06% | 0.29% | **0.30%** | 0.32% | 0.31% | 0.85% |
| $\widehat{\gamma}$ | 1.6 | 4.4 | **4.4** | 4.4 | 4.5 | 3.7 |
| $\widehat{\omega}$ | 2.3 | 4.0 | **4.0** | 4.2 | 4.6 | 8.4 |
| $\widehat{R^2}$ | 42% | 53% | **53%** | 52% | 53% | 53% |
| | | | Modern Period | | | |
| | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | 0.08 | 0.09 | **0.10** | 0.11 | 0.10 | 0.11 |
| $SMB$ | 0.02 | 0.02 | **0.03** | 0.03 | 0.03 | 0.03 |
| $HML$ | -0.10 | -0.10 | **-0.11** | -0.11 | -0.13 | -0.12 |
| | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | 1.6 | 6.7 | **6.8** | 6.8 | 5.8 | 2.8 |
| $\widehat{\omega}$ | 2.0 | 15.4 | **16.0** | 16.9 | 12.8 | 2.8 |
| $\widehat{R^2}$ | -356% | -41% | **-37%** | -36% | -51% | -324% |
| | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | 1.33% | 0.81% | **0.80%** | 0.78% | 0.73% | 1.63% |
| $\widehat{\gamma}$ | 1.6 | 7.1 | **7.2** | 7.1 | 5.9 | 3.9 |
| $\widehat{\omega}$ | 2.1 | 24.4 | **24.9** | 25.3 | 15.2 | 15.9 |
| $\widehat{R^2}$ | 4% | 62% | **63%** | 63% | 53% | 56% |

Panel C: Results Using Different Measures of Realized Variance

| | Quarterly Var Daily | Annual Var Daily | Quarterly Var Monthly | Quarterly Sum Monthly | Annual Var Monthly | Annual Sum Monthly |
|---|---|---|---|---|---|---|
| $\widehat{\gamma}^{Max}$ | **7.2** | 6.8 | 4.9 | 5.4 | 5.8 | 5.9 |
| | | | Early Period | | | |
| | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | **-0.03** | 0.16 | -0.19 | -0.20 | 0.11 | 0.14 |
| $SMB$ | **-0.02** | 0.07 | -0.08 | -0.06 | 0.04 | 0.04 |
| $HML$ | **-0.06** | 0.06 | -0.15 | -0.13 | 0.01 | 0.09 |
| | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | **4.8** | 7.2 | 3.6 | 4.1 | 5.4 | 5.7 |
| $\widehat{\omega}$ | **5.0** | 24.9 | 2.9 | 3.0 | 11.3 | 12.3 |
| $\widehat{R^2}$ | **52%** | 9% | 52% | 49% | -128% | 56% |
| | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | **0.30%** | 1.93% | 0.23% | 0.41% | 3.72% | -1.43% |
| $\widehat{\gamma}$ | **4.4** | 7.2 | 3.5 | 3.8 | 5.4 | 5.8 |
| $\widehat{\omega}$ | **4.0** | 24.9 | 2.5 | 2.2 | 11.3 | 15.4 |
| $\widehat{R^2}$ | **53%** | 53% | 52% | 49% | 34% | 60% |
| | | | Modern Period | | | |
| | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | **0.10** | 0.08 | 0.03 | 0.03 | 0.07 | 0.04 |
| $SMB$ | **0.03** | -0.01 | 0.00 | 0.00 | -0.02 | -0.03 |
| $HML$ | **-0.11** | -0.06 | -0.13 | -0.15 | -0.06 | -0.08 |
| | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | **6.8** | 0.0 | 4.9 | 5.4 | 5.4 | 5.8 |
| $\widehat{\omega}$ | **16.0** | 0.3 | 12.7 | 11.3 | 11.3 | 15.4 |
| $\widehat{R^2}$ | **-37%** | -1212% | 24% | 33% | -1156% | -1033% |
| | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | **0.80%** | 4.66% | 0.40% | 0.33% | 4.57% | 4.33% |
| $\widehat{\gamma}$ | **7.2** | 0.0 | 4.9 | 5.4 | 5.4 | 5.8 |
| $\widehat{\omega}$ | **24.9** | 0.3 | 12.7 | 11.3 | 11.3 | 15.4 |
| $\widehat{R^2}$ | **63%** | -7% | 57% | 56% | 4% | 6% |

Panel D: Results Replacing/Adding Other State Variables to the VAR

| | $PE$ | $PE_{\text{Real}}$ | $PD$ | $INFL$ | $CAY$ | $FIG$ | $TYVOL$ |
|---|---|---|---|---|---|---|---|
| $\widehat{\gamma}^{Max}$ | **7.2** | 9.6 | 4.6 | 9.3 | 9.3 | 9.3 | 7.2 |
| | | | | Early Period | | | |
| | | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | **-0.03** | 0.03 | -0.13 | 0.02 | 0.07 | -0.02 | -0.02 |
| $SMB$ | **-0.02** | -0.01 | -0.05 | -0.01 | 0.01 | -0.02 | -0.02 |
| $HML$ | **-0.06** | -0.02 | -0.11 | -0.03 | 0.00 | -0.02 | -0.05 |
| | | | Risk-free Rate ICAPM | | | | |
| $\widehat{\gamma}$ | **4.8** | 5.2 | 2.9 | 5.3 | 11.4 | 4.6 | 4.7 |
| $\widehat{\omega}$ | **5.0** | 4.4 | 2.4 | 4.6 | 23.0 | 5.6 | 5.0 |
| $\widehat{R^2}$ | **52%** | 52% | 54% | 52% | -1601% | 52% | 52% |
| | | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | **0.30%** | 0.10% | 0.11% | 0.07% | 2.41% | 0.36% | 0.36% |
| $\widehat{\gamma}$ | **4.4** | 5.1 | 2.9 | 5.2 | 16.1 | 4.2 | 4.3 |
| $\widehat{\omega}$ | **4.0** | 4.1 | 2.2 | 4.4 | 62.3 | 4.1 | 3.9 |
| $\widehat{R^2}$ | **53%** | 52% | 54% | 52% | 26% | 52% | 52% |
| | | | | Modern Period | | | |
| | | | | $\widehat{\beta}_V$ | | | |
| $RMRF$ | **0.10** | 0.13 | -0.01 | 0.14 | 0.06 | 0.11 | 0.09 |
| $SMB$ | **0.03** | 0.03 | -0.01 | 0.03 | 0.01 | 0.03 | 0.02 |
| $HML$ | **-0.11** | -0.09 | -0.09 | -0.09 | -0.05 | -0.08 | -0.11 |
| | | | | Risk-free Rate ICAPM | | | |
| $\widehat{\gamma}$ | **6.8** | 9.3 | 4.0 | 9.0 | 15.9 | 5.3 | 6.9 |
| $\widehat{\omega}$ | **16.0** | 24.0 | 7.3 | 21.7 | 58.2 | 9.7 | 18.1 |
| $\widehat{R^2}$ | **-37%** | 23% | 0% | 18% | 0% | -115% | -23% |
| | | | | Zero-beta Rate ICAPM | | | |
| $\widehat{R}_{zb}$ less $R_f$ | **0.80%** | 0.43% | -0.52% | 0.48% | 0.34% | 0.96% | 0.72% |
| $\widehat{\gamma}$ | **7.2** | 9.6 | 4.6 | 9.3 | 16.7 | 5.7 | 7.2 |
| $\widehat{\omega}$ | **24.9** | 31.4 | 18.3 | 29.4 | 81.5 | 16.0 | 26.2 |
| $\widehat{R^2}$ | **63%** | 46% | 19% | 47% | 30% | 46% | 64% |

| Panel E: Modern Period Pricing | | |
|---|---|---|
| | **Full Sample** | Out of Sample |
| | $\widehat{\beta}_V$ | |
| $RMRF$ | **0.10** | -0.10 |
| $SMB$ | **0.03** | -0.04 |
| $HML$ | **-0.11** | -0.10 |
| | Risk-free Rate ICAPM | |
| $\widehat{\gamma}$ | **6.8** | 5.2 |
| $\widehat{\omega}$ | **16.0** | 6.5 |
| $\widehat{R^2}$ | **-37%** | 64% |
| | Zero-beta Rate ICAPM | |
| $\widehat{R}_{zb}$ less $R^{Tbill}$ | **0.80%** | -0.71% |
| $\widehat{\gamma}$ | **7.2** | 6.3 |
| $\widehat{\omega}$ | **24.9** | 12.4 |
| $\widehat{R^2}$ | **63%** | 72% |

Panel F: Results Using delevered market portfolios

| Equity % | **100%** | 80% | 60% | 40% |
|---|---|---|---|---|
| $\widehat{\gamma}^{Max}_{Exact}$ | **7.2** | 10.2 | 14.5 | 16.3 |
| | | Early Period | | |
| | | $\widehat{\beta}_V$ | | |
| $RMRF$ | **-0.03** | -0.03 | -0.03 | -0.02 |
| $SMB$ | **-0.02** | -0.02 | -0.02 | -0.02 |
| $HML$ | **-0.06** | -0.06 | -0.05 | -0.04 |
| | | Risk-free Rate ICAPM | | |
| $\widehat{\gamma}$ | **4.8** | 7.1 | 10.8 | 14.5 |
| $\widehat{\omega}$ | **5.0** | 13.0 | 43.4 | 303.7 |
| $\widehat{R^2}$ | **52%** | 53% | 54% | 55% |
| | | Zero-beta Rate ICAPM | | |
| $\widehat{R}_{zb}$ less $R_f$ | **0.30%** | 0.35% | 0.41% | 0.61% |
| $\widehat{\gamma}$ | **4.4** | 6.6 | 10.0 | 13.3 |
| $\widehat{\omega}$ | **4.0** | 10.5 | 35.4 | 235.4 |
| $\widehat{R^2}$ | **52%** | 54% | 55% | 57% |
| | | Modern Period | | |
| | | $\widehat{\beta}_V$ | | |
| $RMRF$ | **0.10** | 0.10 | 0.10 | 0.08 |
| $SMB$ | **0.03** | 0.03 | 0.02 | 0.02 |
| $HML$ | **-0.11** | -0.11 | -0.11 | -0.09 |
| | | Risk-free Rate ICAPM | | |
| $\widehat{\gamma}$ | **6.8** | 9.5 | 13.3 | 13.8 |
| $\widehat{\omega}$ | **16.0** | 32.1 | 79.4 | 259.9 |
| $\widehat{R^2}$ | **-37%** | -74% | -157% | -443% |
| | | Zero-beta Rate ICAPM | | |
| $\widehat{R}_{zb}$ less $R_f$ | **0.80%** | 0.96% | 1.10% | 1.65% |
| $\widehat{\gamma}$ | **7.2** | 10.2 | 14.4 | 16.1 |
| $\widehat{\omega}$ | **24.9** | 50.1 | 116.7 | 458.6 |
| $\widehat{R^2}$ | **63%** | 61% | 56% | 43% |

Panel G: Varying $\rho$ and the Excess Zero-beta Rate in the Modern Period

| | | | $\rho = 0.94$ | | | |
|---|---|---|---|---|---|---|
| $\widehat{R}_{zb}$ less $R_f$ | 0.40% | 0.50% | 0.60% | 0.70% | 0.80% | 0.54% |
| $\widehat{\gamma}$ | 7.1 | 7.2 | 7.2 | 7.2 | 7.2 | 7.2 |
| $\widehat{\omega}$ | 23.3 | 24.9 | 24.9 | 24.9 | 24.9 | 24.9 |
| $\widehat{R^2}$ | 60% | 63% | 63% | 58% | 50% | 63% |
| | | | $\rho = 0.95$ | | | |
| $\widehat{R}_{zb}$ less $R_f$ | 0.40% | 0.50% | 0.60% | 0.70% | 0.80% | **0.80%** |
| $\widehat{\gamma}$ | 6.9 | 7.0 | 7.1 | 7.1 | 7.2 | **7.2** |
| $\widehat{\omega}$ | 18.2 | 19.5 | 21.2 | 23.4 | 24.9 | **24.9** |
| $\widehat{R^2}$ | 41% | 51% | 57% | 61% | 63% | **63%** |
| | | | $\rho = 0.96$ | | | |
| $\widehat{R}_{zb}$ less $R_f$ | 0.40% | 0.50% | 0.60% | 0.70% | 0.80% | 1.08% |
| $\widehat{\gamma}$ | 6.6 | 6.6 | 6.7 | 6.8 | 7.0 | 7.2 |
| $\widehat{\omega}$ | 14.2 | 14.8 | 15.7 | 16.9 | 18.5 | 24.9 |
| $\widehat{R^2}$ | 3% | 21% | 35% | 46% | 54% | 61% |

Panel H: Components of and Proxies for $\widehat{\beta}_V$

| Early Period | | | | |
|---|---|---|---|---|
| | *RMRF* | | *HML* | |
| | WLS | OLS | WLS | OLS |
| $\widehat{\beta}_V$ | -0.03 | -0.06 | -0.06 | -0.07 |
| $\widehat{\beta}_{\lambda_V^1 r_M\ Shock}$ | 0.01 | 0.01 | 0.00 | 0.00 |
| $\widehat{\beta}_{\lambda_V^2 EVAR\ Shock}$ | -0.06 | -0.06 | -0.03 | -0.03 |
| $\widehat{\beta}_{\lambda_V^3 PE\ Shock}$ | 0.14 | 0.14 | 0.06 | 0.07 |
| $\widehat{\beta}_{\lambda_V^4 r^{Tbill}\ Shock}$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $\widehat{\beta}_{\lambda_V^5 DEF\ Shock}$ | -0.13 | -0.12 | -0.09 | -0.08 |
| $\widehat{\beta}_{\lambda_V^6 VS\ Shock}$ | -0.02 | -0.02 | -0.02 | -0.03 |
| $\widehat{\beta}_{RVAR}$ | | -0.01 | | 1.50 |
| $\widehat{\beta}_{FIGARCH}$ | | 0.02 | | 0.04 |

| Modern Period | | | | |
|---|---|---|---|---|
| | RMRF | | HML | |
| | WLS | OLS | WLS | OLS |
| $\widehat{\beta}_V$ | 0.10 | 0.07 | -0.11 | -0.09 |
| $\widehat{\beta}_{\lambda_V^1 r_M\ Shock}$ | 0.01 | 0.01 | 0.00 | 0.00 |
| $\widehat{\beta}_{\lambda_V^2 EVAR\ Shock}$ | -0.08 | -0.07 | -0.01 | -0.01 |
| $\widehat{\beta}_{\lambda_V^3 PE\ Shock}$ | 0.14 | 0.13 | -0.02 | -0.02 |
| $\widehat{\beta}_{\lambda_V^4 r^{Tbill}\ Shock}$ | 0.00 | 0.00 | 0.00 | 0.00 |
| $\widehat{\beta}_{\lambda_V^5 DEF\ Shock}$ | -0.01 | -0.02 | -0.02 | -0.01 |
| $\widehat{\beta}_{\lambda_V^6 VS\ Shock}$ | 0.03 | 0.03 | -0.05 | -0.05 |
| $\widehat{\beta}_{RVAR}$ | | -3.31 | | -0.51 |
| $\widehat{\beta}_{FIGARCH}$ | | -0.08 | | -0.01 |

Panel I: Time-series Regressions explaining $HML$

Early Period

| | (1) | | (2) | |
|---|---|---|---|---|
| Intercept | 0.01 | 1.39 | 0.01 | 2.30 |
| $N_{CF}$ | | | 0.39 | 3.68 |
| $-N_{DR}$ | | | 0.40 | 6.67 |
| $N_V$ | -1.96 | -6.46 | -1.30 | -4.96 |
| $\widehat{R^2}$ | 24% | | 49% | |

Modern Period

| | (1) | | (2) | |
|---|---|---|---|---|
| Intercept | 0.01 | 2.28 | 0.01 | 2.67 |
| $N_{CF}$ | | | 0.24 | 2.70 |
| $-N_{DR}$ | | | -0.23 | -4.92 |
| $N_V$ | -1.05 | -7.34 | -0.82 | -5.84 |
| $\widehat{R^2}$ | 22% | | 32% | |

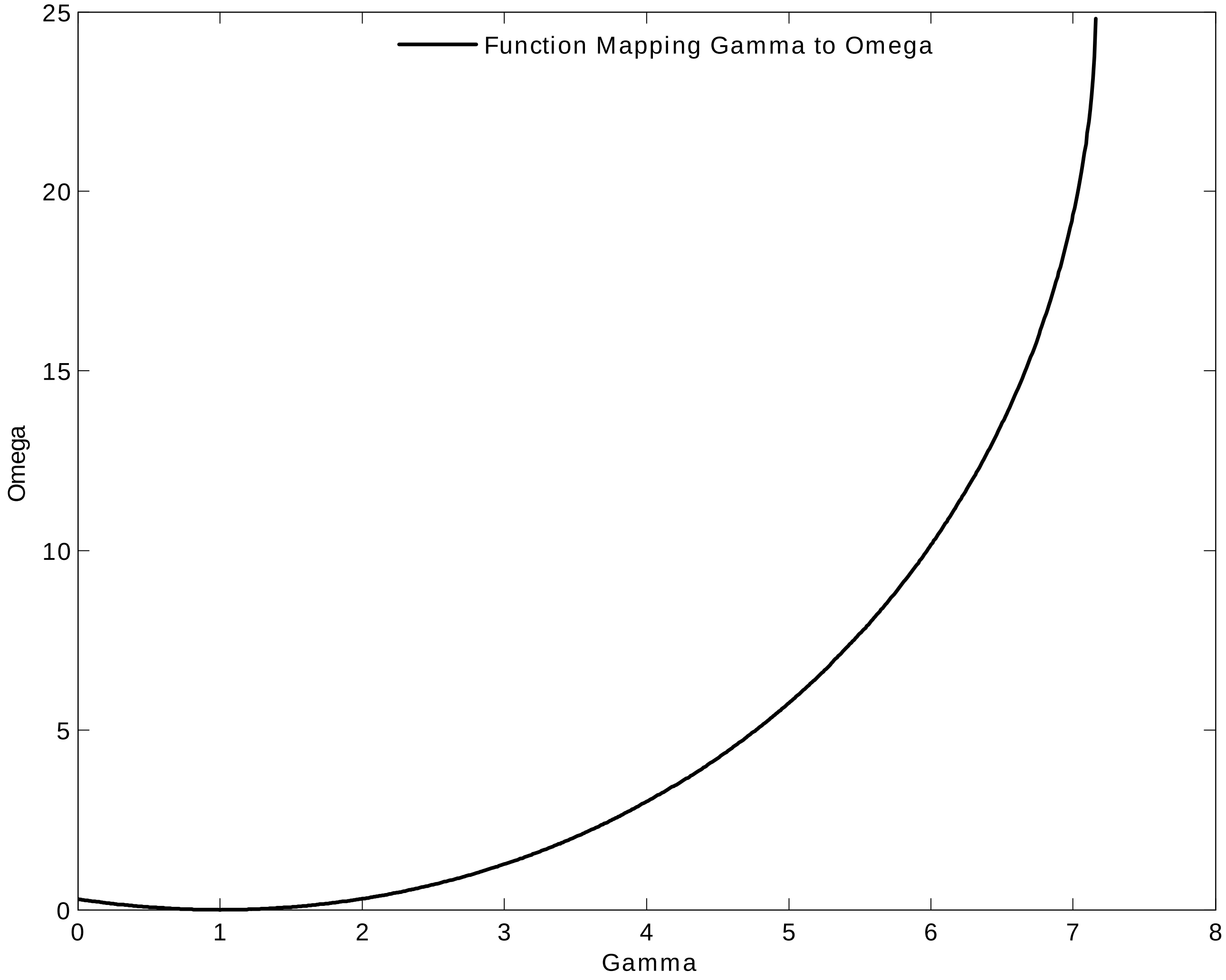


Figure 1: This figure graphs the relation between the parameter $\gamma$ and the parameter $\omega$ described by equation (22). These functions depend on the loglinearization parameter $\rho$, set to 0.95 per year and the empirically estimated VAR parameters of Table 1. $\gamma$ is the investor's risk aversion while $\omega$ is the sensitivity of news about risk, $N_{RISK}$, to news about market variance, $N_V$.

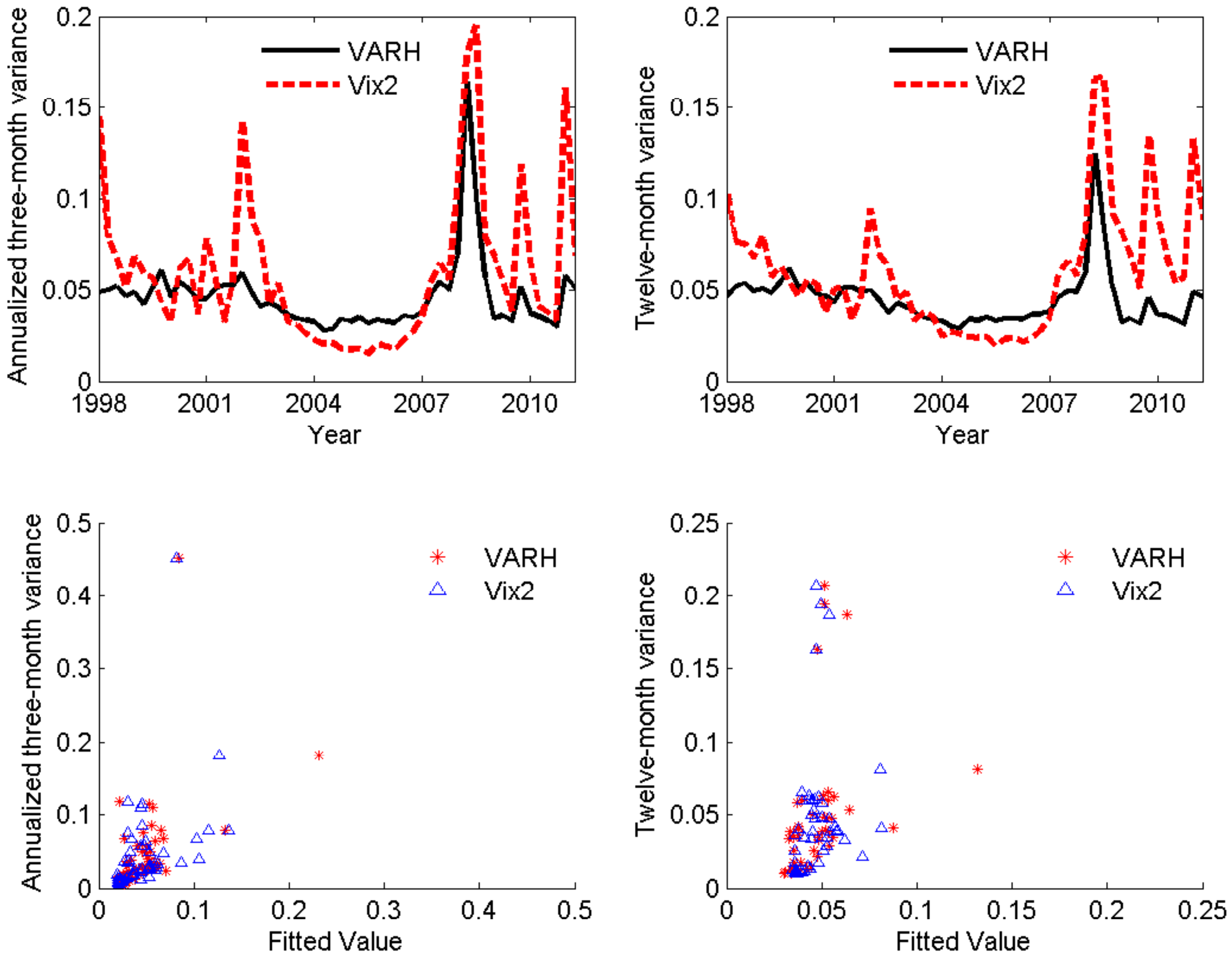


Figure 2: The top two diagrams correspond to forecasts of three-month (top left panel) and twelve-month (top right panel) variance from the VAR ($VAR_h$, solid black line) and from the option market ($VIX_h^2$, dashed red line). The bottom two diagrams correspond to scatter plots of three-month (bottom left panel) and twelve-month (bottom right panel) ralized variance against the corresponding forecast from the VAR ($VAR_h$, red asterisks) and from the option market ($VIX_h^2$, blue triangles).

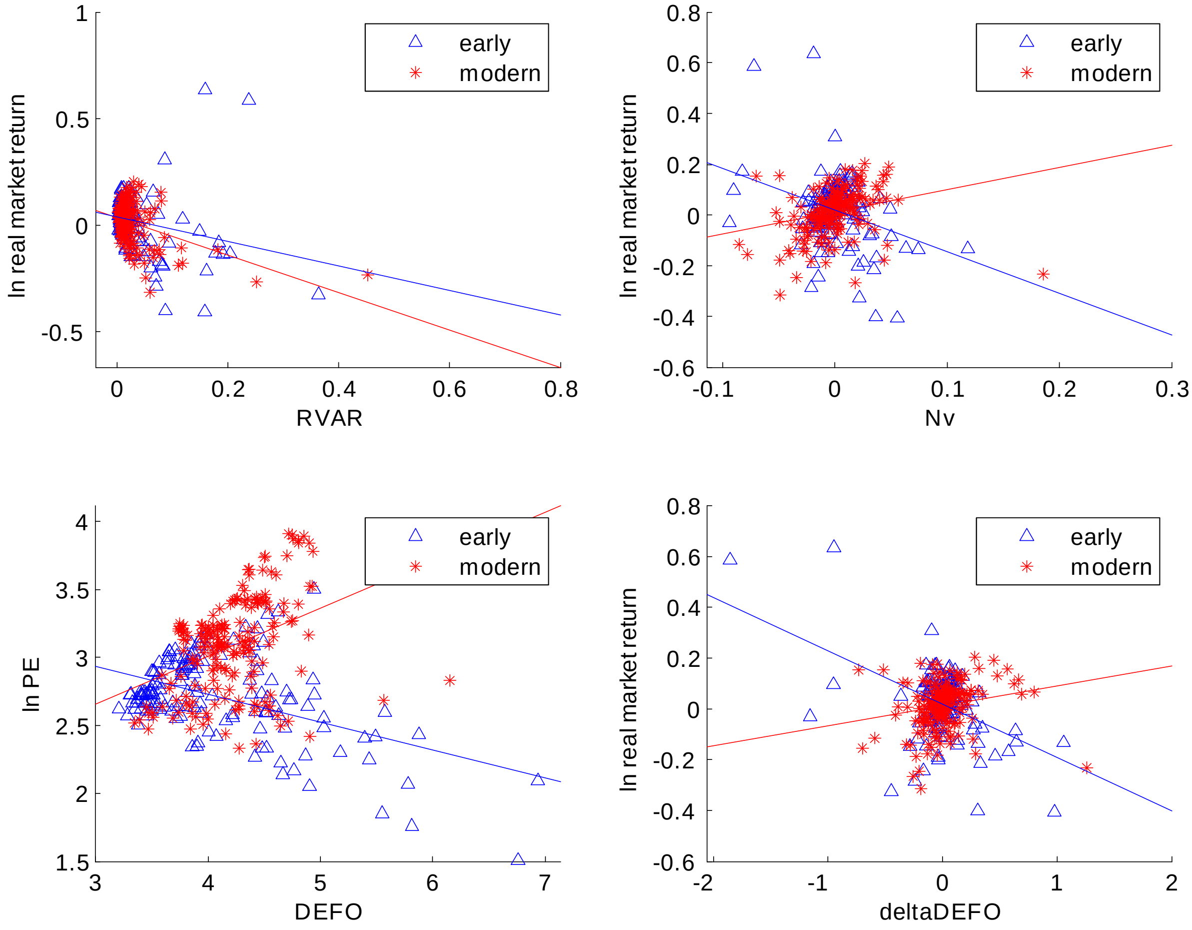


Figure 3: The top left portion of the figure plots the market return against $RVAR$. The top right portion of the figure plots the market return against volatility news, $N_V$. The bottom left of the figure plots $PE$ against $DEFO$ ($DEF$ orthogonalized to $PE$). The bottom right of the figure plots market returns against the contemporaneous change in $DEFO$, our simple proxy for news about long-horizon variance. In all four subplots, observations from the early period as denoted with blue triangles while observations from the modern period data are denoted with red asterisks.

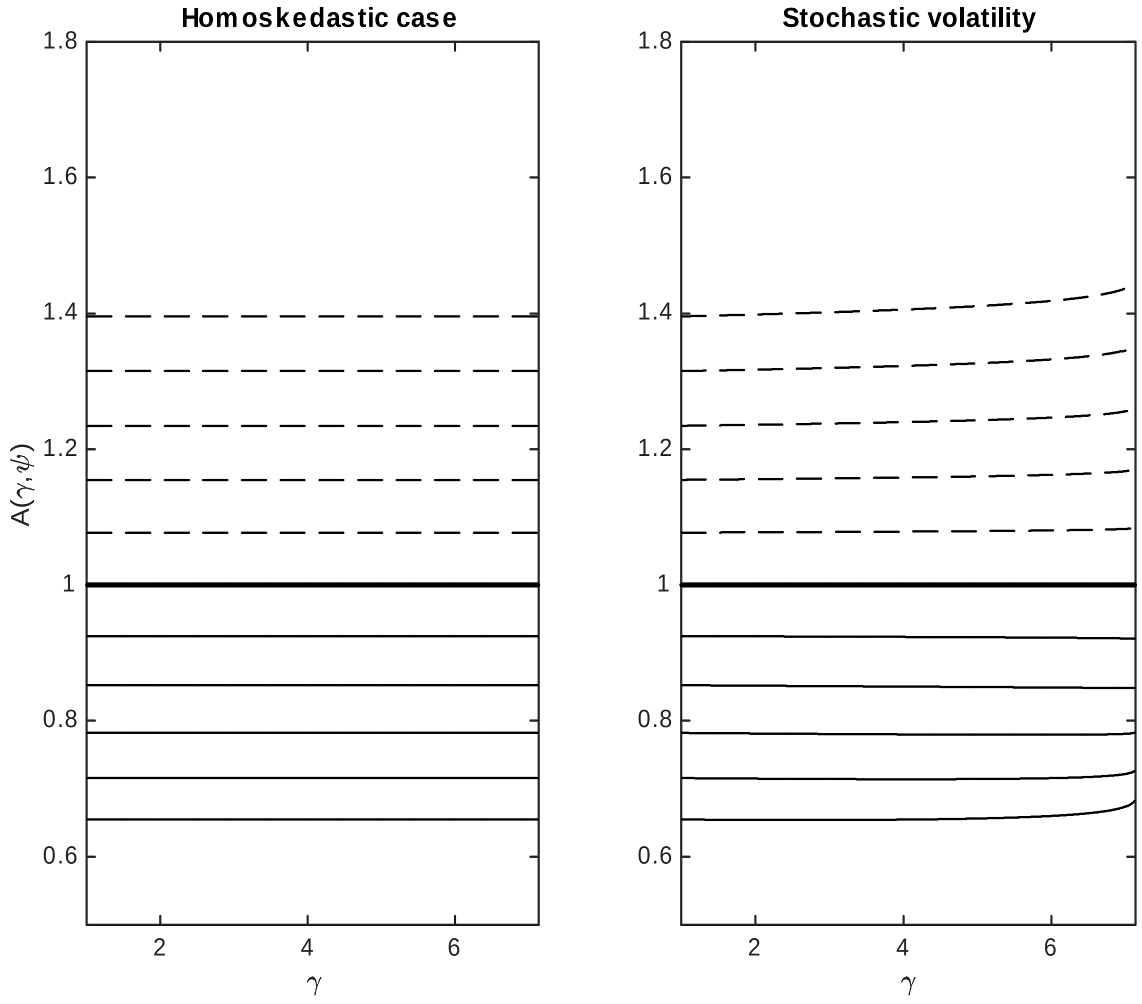


Figure 4: This figure plots plots the coefficient $A(\gamma, \psi)$ relating the conditional volatility of consumption growth to the volatility of returns for different values of $\gamma$ and $\psi$ for the homoskedastic case (left panel) and for the heteroskedastic case (right panel), where $A(\gamma, \psi)$ is a function of the variances and covariances of the *scaled* residuals $u_{t+1}$. In each panel, we plot $A(\gamma, \psi)$ as $\gamma$ varies between 1 and the maximum possible value, for different values of $\psi$. Each line corresponds to a different $\psi$, beginning with the topmost dashed line ($\psi$=0.5), incrementing $\psi$ by 0.1 until ending with the bottommost solid line ($\psi$=1.5).

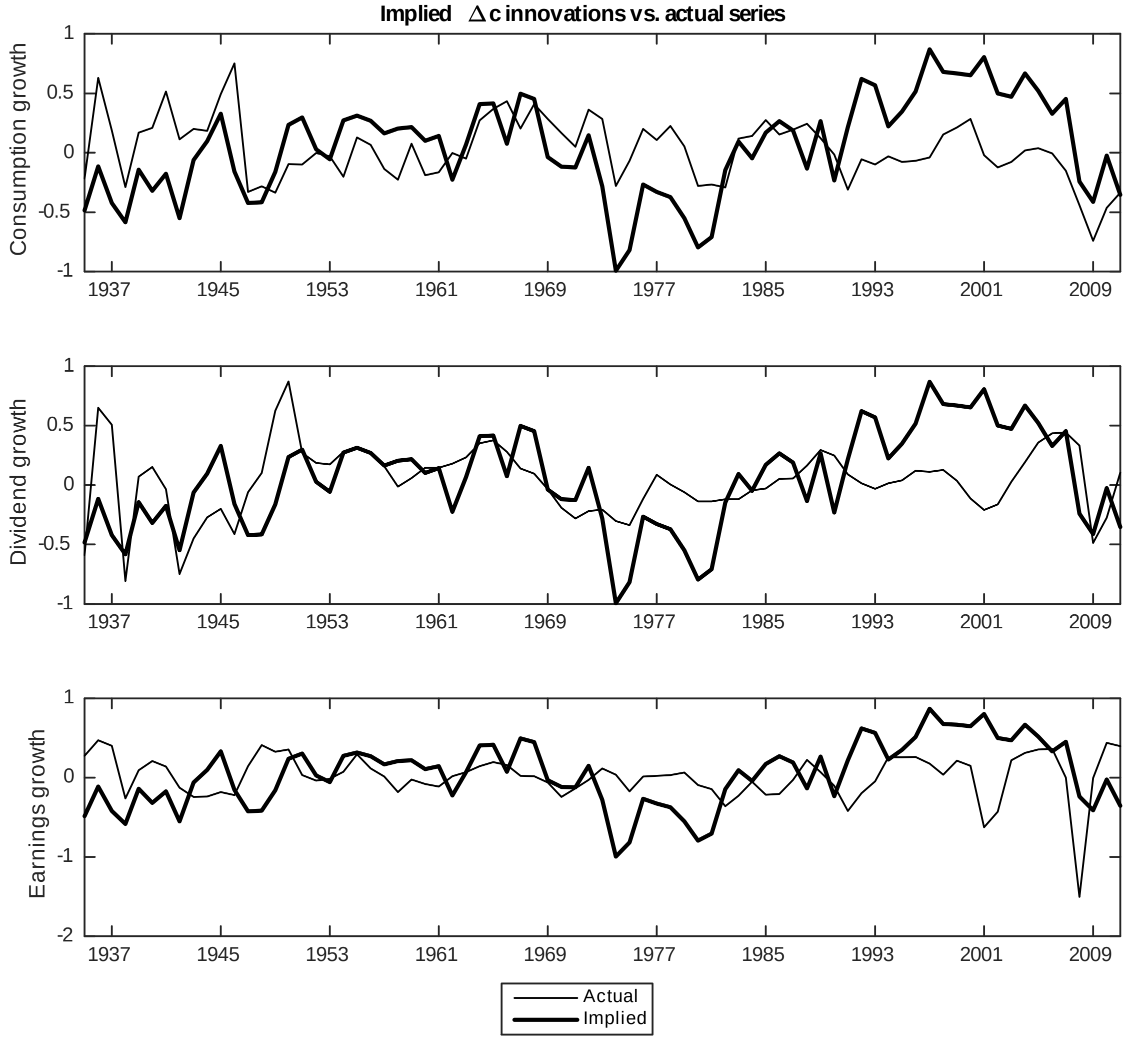


Figure 5: We plot the exponentially-weighted moving average of the series of implied consumption innovations versus actual consumption, dividends and earnings growth innovations. We standardize all series before plotting and set $\psi = 0.5$ when calibrating implied consumption innovations.

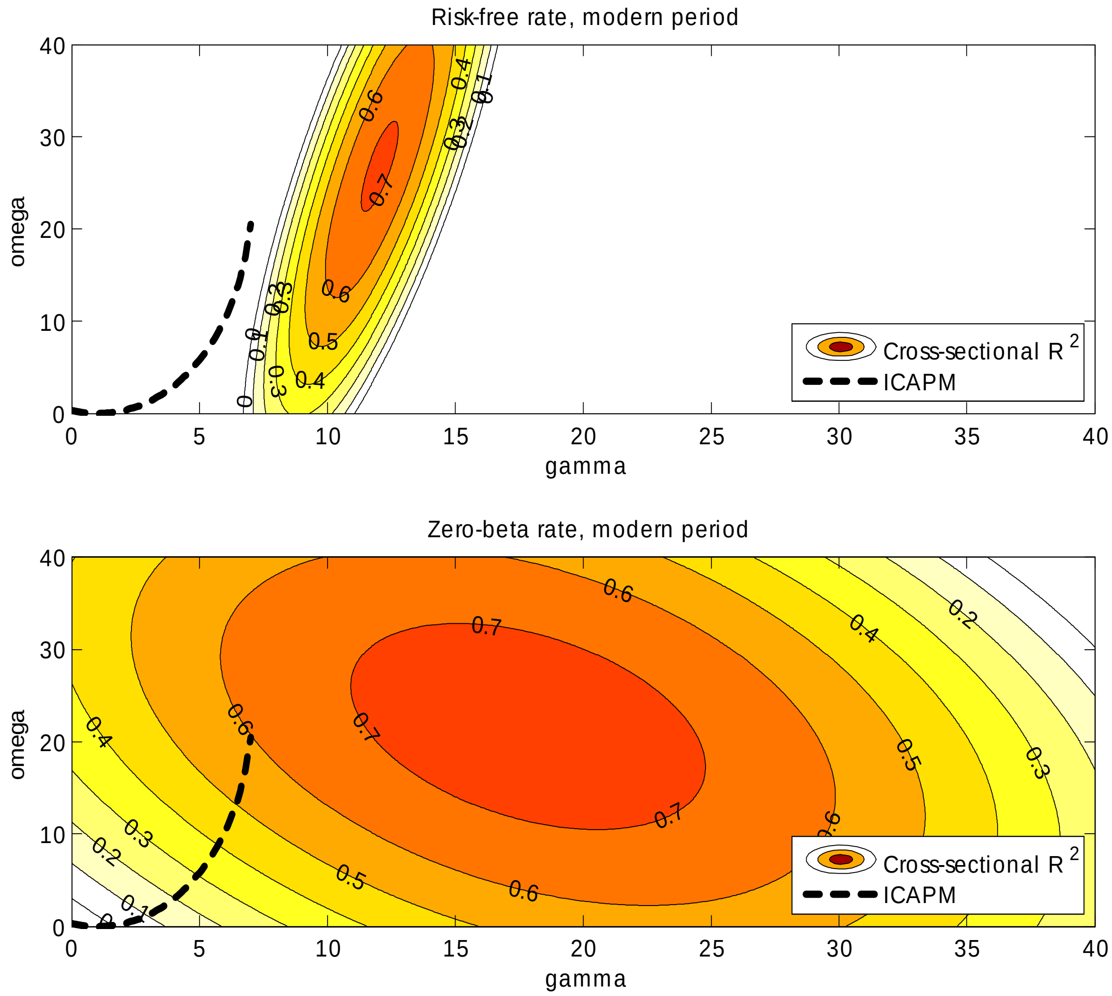


Figure 6: The two contour plots show how the $R^2$ of the cross-sectional regression explaining the average returns on the 25 size- and book-to-market portfolios varies for different values of $\gamma$ and $\omega$ for the risk-free rate (top panel) and zero-beta rate (bottom panel) three-beta ICAPM model estimated in Campbell, Giglio, Polk, and Turley (2015) Table 4 Panel B for the sample period 1963:3-2011:4. The two plots also indicate the ICAPM relation between $\gamma$ and $\omega$ described in equation (22).