Appendix to "Very Long-Run Discount Rates"

Stefano Giglio   Matteo Maggiori   Johannes Stroebel

Not for publication

A.1 Institutional Appendix - United Kingdom

In this Appendix we expand on important institutional details of leasehold ownership in England and Wales. In Appendix A.1.1 we provide historical background on the development of leasehold land law. In Appendix A.1.2 we discuss requirements for leasehold registration with the Land Registry and the Land Registration Act 2002. In Appendix A.1.3 we describe the U.K. property tax regime, and in Appendix A.1.4 we outline restrictions on leaseholders' ability to make changes to the property. In Appendix A.1.5 we describe the process of lease extensions, in Appendix A.1.6 the presence of ground rents and service charges, and in Appendix A.1.7 the impact of leasehold covenants.

A.1.1 Historical Background

The history of leasehold property ownership in England has its roots in feudalism, a system of land use and ownership that was common in Europe between the tenth and thirteenth centuries, and introduced in England following the Norman Conquest. Land was owned and controlled by a military or political sovereign ruler, who gave parts of that land to a number of lords as "tenants-in-chief," or "tenants-in-capite." The lord, in turn, could allow another person, a vassal, to use smaller portions of the land for a fixed period of time in return for pledging allegiance and military service to the lord. See Burn, Cartwright and Cheshire (2011) for a detailed review of the history of real property law.

There is no consensus as to the origins of the common lease length terms of 99, 125 and 999 years. McMichael (1921) describes the historical debate: *Matthew Bacon, author of "A Treatise on Leases and Terms for Years" published in London, England, in 1798, explains in various parts of his book that the ninety-nine year period represents three lives, but Bacon does not indicate why such a term was selected as the length of time a lease was to prevail. It is supposed by some that there was an English common law which prevented a lessor from granting a lease for 100 years and that it was therefore made for a somewhat briefer period, but no real evidence has ever been found to substantiate this theory.* 1,000 year leases were also common initially, with Jack Cade in Shakespeare's *Henry IV, Part II* exclaiming that "Now I am so hungry, that if I might have a lease of my life for a thousand years, I could stay no longer." McMichael

([1921](#)) also discusses theories of moving from 1,000 year to 999 year leases: *Lord Coke, who lived in the reign of Queen Elizabeth, in his writings on the subject of leases suggested that a lease for 1,000 years might on its face suggest fraud and it is thought that to avoid such a contingency the lessors of those early days set upon 999 years as the extreme limit for the life of a lease. Such leases, in any event, were made at that time.*

A.1.2 Leasehold Registration

For many decades, the registration of leasehold and freehold interest in the U.K. was governed by the Land Registration Act 1925. It was replaced in October 2003, when the new Land Registration Act 2002 came into effect. The two Acts regulate the registration procedures of ownership in real estate, both freeholds and leaseholds. In both cases, the law identifies cases in which registration of freehold and leasehold contracts is voluntary or mandatory, and outlines the procedure to follow to register an interest. The main objective of the Land Registration Act 2002 was to further encourage voluntary registration of contracts, and extended the cases of mandatory registration.

In particular, under the current law, registration of a contract is mandatory when a leasehold of 7 years or longer is granted or transferred, or a freehold is transferred. When no transfer occurs (or other event that triggers registration according to the law), registration is not mandatory, though encouraged by the law. This could for example be the case for lease extensions. Finally, the law establishes that failure to register an interest will make the interest lose the so-called overriding status: the owner of the interest may be vulnerable to successive transfer of the title that is registered.[1]

A.1.3 Tax Treatment of Leaseholds and Freeholds

Her Majesty Revenue and Customs (HMRC), the tax authority for England and Wales, gives equal treatment to the price paid leasehold of different maturities or for freeholds when levying Stamp Duty Land Tax (SDLT) on residential property transactions. Transactions below £125,000 are exempt from stamp duty, with rates rising progressively thereafter, to 5% for houses above £1 million and 7% for houses above £2 million.[2] The tax rate bands do not reflect marginal but total tax rates - a move into a higher tax band means that the higher rate is applied to the entire purchase. This would increase the value of the (cheaper) leasehold relative to freeholds, since they might end up in a lower tax band. Stamp Duty is also levied on the premium payable under lease extensions.

---

[1] See http://www.landregistry.gov.uk/professional/guides Practice guides 15 and 25.

[2] See complete current tax schedule at: https://www.gov.uk/stamp-duty-land-tax-rates.

A.1.4 Structural Changes to the Property

Since leasehold contracts can span several hundred years, it is important to understand the provisions that regulate the ability of leaseholders to make improvements to the property. Leaseholders are generally allowed to make minor (non-structural) improvements, unless explicitly restricted by covenants. For example, the leaseholder can renovate the kitchen or change non-structural walls. In addition, improvements to the property that are made by the leaseholder fully accrue to the leaseholder himself, since their value is excluded by law when computing the cost of a lease extension, i.e. the value of the extension is calculated as if the improvement was never made. Therefore, a leaseholder has the same incentives to improve the property as he would have had if he were a freeholder.

Structural changes to the property (like demolishing and redeveloping the building) are instead generally prohibited to the leaseholder without the consent of the freeholder. This restriction is unlikely to significantly affect our estimated discounts. First, a covenant against the making of improvements without consent is subject to the provision that consent shall not be unreasonably withheld (see Section 19(2) of the Landlord and Tenant Act 1927, and Appendix A.1.7 below). Second, all leaseholds regardless of maturity are affected by this restriction. If anything, long leaseholds are more exposed to restrictions on redevelopments, since major redevelopments of the buildings are more likely to be attractive over longer horizons of time. Third, the need for the approval by the freeholder has only a marginal effect when redeveloping a multi-unit building, since owners of individual flats who would like to redevelop the property need the consent of all (or a vast majority of) other owners, regardless of whether they are leaseholders of freeholders. The need for the consent of the freeholder imposes only a small additional burden.

A.1.5 Valuations, Leasehold Extensions, Tribunal Decisions

In this Appendix, we discuss the process of lease extensions in the U.K., focusing on the impact of lease extensions on interpreting our results. We first describe the legal rights of leaseholders to extend their lease. We then describe the role of Leasehold Valuation Tribunals (LVTs), which leaseholders can appeal to if they cannot reach an agreement with their freeholder on the premium payable for a lease extension; we discuss evidence that LVT decisions are favorable to leaseholders relative to the price discounts estimated in this paper. We next discuss the role of valuers in the lease extension process, and conclude by analyzing the net incentives to extend a lease, which trade off the possibly relatively cheap premia payable for extensions with the costs of doing an extension.

A.1.5.1 The Legal Right and Process to Extend the Lease

Over time, a number of laws have regulated the rights of leaseholders to extend the lease or purchase the freehold. The Leasehold Reform Act 1967 gave tenants of houses (not flats) with long leases the right to acquire either the freehold (a process called "enfranchisement") or an extended lease term. The Leasehold Reform, Housing and Urban Development Act 1993 conferred rights to collective enfranchisement and lease extensions by 90 years on groups of flat owners in the same building. The Commonhold and Leasehold Reform Act 2002 extended the right to lease extensions to individuals who have owned flats for at least two years.

The Acts also codify the bargaining process for a lease extension in the following way. First, the leaseholder files a proposal for extension, with an offered premium for the extra years to be acquired on the lease. The freeholder agrees, or proposes a counteroffer, and the two parties then bargain on the final price of the extension. It is common for both parties to solicit both legal representation and the advice of professional valuers. This process can be expensive and time consuming, with the administrative costs of extensions often exceeding £4,000-£5,000 and the proceedings taking over two years to complete.[3] The total cost is composed of some fixed costs, for example, administrative fees, and some variable costs such as stamp duty taxes, that increase with the premium negotiated. In addition to a fixed payment, the leaseholders' valuers generally earn a fraction of the bargaining gain, the difference between final premium and freeholder initial offer. The leaseholder is liable not only for his own costs but also for the legal, administrative, and valuation costs of the freeholder. In fact, the Acts established that the freeholder is to be compensated for "reasonable" costs incurred in connection with the lease extension; these costs are *in addition* to the premium payable for granting extra years on the lease.

A.1.5.2 Tribunal Decisions

If the two parties cannot agree on a price, the leaseholder can refer the matter to the Leasehold Valuation Tribunal (LVT) or, since July 2013, to the newly created First-Tier Tribunal (Property Chamber). Such Tribunals are part of Her Majesty's Courts and Tribunals Service. Each Tribunal usually consists of three members: a lawyer, who is often the chairman, a valuer and a lay person. There were five regionally based LVT offices (London, Northern, Midland, Eastern and Southern). There is no administrative fee payable to the U.K. government for LVT applications to determine the terms or price for enfranchisements or lease extensions. However, arguing before the LVT is generally a very last resort

---

[3]For an estimate of administrative costs of lease extensions, see http://www.telegraph.co.uk/finance/personalfinance/9060279/Home-owners-urged-to-extend-leaseholds.html.

for leaseholders because of the significant costs related to legal advice and representation, valuers' fees, and the uncertain and lengthy process involved in court decisions. Individual costs may vary, but are generally estimated to run in the "tens of thousands" of pounds.[4] Any order made by the LVT can be enforced in the same way as a county court order. An additional layer of protection is granted via the appeal process: LVT or First-Tier Tribunal decisions can be appealed by either the freeholder or the leaseholder. The appeal is then judged by the Upper Tribunal (Lands Chamber).[5] The fee for lodging an appeal is £250. Further legal and valuation advice costs are incurred for the appeal, and the court judgement can allow freeholders to recover these costs via increases in their service charges to leaseholders. A final layer of protection is to appeal the Upper Tribunal decision to the House of Lords, but this appeal can only be concerned with a general point of law and not with the specific valuations of a particular case.

The legislation provides a number of restrictions on how the properties and lease contracts are to be evaluated by the Tribunal in assessing the payable premium in case of a lease extension or the enfranchisement cost in case of outright purchase of the freehold. The Tribunal is instructed to consider the amount that the property "might be expected to realize if sold on the open market by a willing seller to a willing buyer" (Leasehold Reform Act 1967), a reference to market values, but also to disregard a number of features that would actually affect market values. For example, the valuation is not supposed to include improvements made by the leaseholders and ignore the value of the option to extend the lease under the current (shorter) lease.

In determining the valuations, the Tribunal relies both on the input of the leaseholder and the freeholder, but is actually not required to rule within that range.[6] Both the leaseholder and the freeholder submit a proposal for the premium payable as well as a rationale for the premium calculation, generally based on a valuer's report. These rationales mainly take two forms: a "comparables" analysis that considers transactions of similar properties, and a model-based derivation of the premium.

---

[4]For an example of cost estimates, see the Westminster City Council publication: https://www.westminster.gov.uk/sites/default/files/uploads/workspace/assets/publications/Homing-in-on-the-issues-Perspec-1375182291.pdf.

[5]Permission to appeal must first be requested from the First-Tier Tribunal, if this Tribunal does not grant permission to appeal, the permission can then be sought directly from the Upper Tribunal (Lands Chamber). See http://www.justice.gov.uk/tribunals/lands for more information.

[6]The Tribunals are not required to rule within the range of disagreement between the freeholder and the leaseholder, but can rule on the entire case including the validity of the lease. While in practice rulings do not often stray away from the parties inputs, large deviations are possible. For example, in a £9,000 service charge dispute, a LVT decision forced the leaseholder to forfeit the apartment (valued at £600,000) and to pay £70,000 in legal fees (see http://m.wandsworthguardian.co.uk/news/10244117.Photographer_forced_to_forfeit_home/).

The comparables analysis offered by the parties is generally based on very few properties, often one or two apartments; we discuss the potential shortcomings of the hedonic adjustments by valuers below. The second source of input is model based, often relying on a simple Gordon growth valuation. The inputs on the model are not chosen for their realism, nor because they reflect anyone's actual market valuations, but simply as a way within the assumed model to recover the desired premium. As the Tribunal reports in [LRA 55 2002]: *It would in our view prevent any significant weight being placed upon such an analysis unless there was clear evidence that the adjustments made by Ms Tolgyesi were soundly based and were not so coarsely grained as to obscure and influence (if not indeed provide) the answer which is sought to be found rather than reliably to reveal that answer.*

Since exponential discounting is very sensitive, particularly at long horizons, to the net discount rate, Tribunal decisions have focused on determining a model-based $r - g$. The most cited decision on this matter is commonly known as the "Sportelli Appeals." The Tribunal decided for a net discount rate of 5%. The estimate was reached by cumulating estimates of: 2.5% for the risk-free rate (based on inflation linked bonds), a real growth rate of GDP of 2% (based on long-term growth in Britain), and a housing risk premium of 4.5% (based on the equity premium).[7] The 2.5% average long-term risk-free rate is considerably higher than the $1 - 1.4\%$ we documented using the UK government yield curve. The 2% rate of growth of the economy is considerably higher than the real growth rate of rents (the object of interest in the formula), observed in the data at 0.5%.[8] The risk premium is a subjective estimate extrapolated from the equity market, and had previously been estimated at 2.5% by the courts (see the "Arbib" decision). While the courts made reference to a form of CAPM, at least a constant discount rate one, in setting the discount rate, they made no reference to actual market valuations. In discussing the realism of a 5% rate, the barristers Tanfield Chambers remark that: *"A lot of surveyors, particularly provincial valuers, will tell you that the prices paid in the market simply do not reflect such rates"* and conclude that a lower rate is more consistent with market valuations.[9]

Consistent with this, in ongoing work, Badarinza and Ramadorai (2014) analyze about 400 Tribunal decisions to grant lease extensions. They find Tribunals to award extensions

---

[7]See point 31 and 35 in the report "Sportelli: The Aftermath" by Tanfield Chambers: http://www.tanfieldchambers.co.uk/Asp/uploadedFiles/File/seminar%20notes/SportelliFEB.2009.pdf.

[8]Furthermore, using the growth rate of GDP or consumption of 2% for the growth rate of rents is inconsistent with the idea that total rents are a constant fraction of consumption in the long run despite the fact that people live in better and bigger houses over time. A positive quality-quantity adjustment implies that rents for a specific property must grow lower than per-capita rents.

[9]These differences between the prices for extension established by Tribunals and market price differences between freeholds and leaseholds makes it hard to infer true household preferences from data on Tribunal decisions.

at premia more favorable to leaseholders than the market valuations estimated in this paper. While Tribunal valuations are subject to a number of arbitrary criteria such as leaseholders' and freeholders' guesswork of what the property might be worth in a market transaction that is not actually observed (also see Appendix A.1.5.3 below), such results alleviate the concern that our estimated discounts are purely due to a hold-up problem. In fact, if Tribunals on average are more favorable to leaseholders, this should provide leaseholders with a more credible remedy in the bilateral negotiations with freeholders.

A.1.5.3 Leasehold Extensions and Valuers' Opinions

Valuers offer advice to freeholders and leaseholders on the premia payable in lease extensions or enfranchisements. They mainly base their valuations on personal experience in past extensions and on past Tribunal decisions. Their role is comparable to the guidance offered to clients by real estate agents in the process of buying or selling a property.

When consulting for parties during the process of lease extension, valuers are only interested in establishing the payable premium, because it is the only amount being transacted in an extension (or an enfranchisement). While the premium might be determined by also considering the percentage of the freehold value it represents, such percentage values, which are the focus of this paper, are only secondary objects of interest in extension transactions. Nonetheless, in this section of the Appendix we review a number of relative valuation curves (often called "relativity curves") produced by a variety of valuers. We focus on the summary provided by the Royal Institute of Chartered Surveyors (2009). Generally, these valuers tend to use higher discount rates than the ones we have shown to be necessary to match the data. For example, the valuations suggested by Savills are approximately consistent with a net-of-growth discount rate of 3.5%, while we have shown a rate of 1.9% to be necessary to match our price data.

While valuers' views are a potentially interesting source of information, they are not based on any systematic analysis of market data; indeed, prior to our analysis, there has been no systematic, large-scale analysis of the relative valuation of leaseholds and freeholds. Instead, as reported in the summary of 14 different relativity curves in Royal Institute of Chartered Surveyors (2009), the main inputs in the construction of those curves are: (i) "opinions," which are a key input in 6 out of 14 cases; and (ii) Tribunal decisions or past extensions (in all but 3 cases). Only 4 of the curves had any reference to actual market transactions, half of which only used transactions from before the 1990s.[10]

---

[10]For example, Savills's analysis is based on open market valuations for 240 leasehold properties with varying lease lengths in central London. For each of the properties, valuations were carried out by agents and valuers from thirteen agency and surveying practices. Each person was asked to provide valuations for a sample of 10 properties under a standard set of assumptions. See Royal Institute of Chartered Surveyors

The almost exclusive reliance of valuers' analysis on past Tribunal decisions and extensions, or alternatively personal opinions, makes it difficult to infer the true relative valuation of freeholds and leaseholds as determined by household preferences from relativity curves. This is true for two reasons:

First, as discussed above, extensions and Tribunal decisions do not necessarily reflect market values, due to distortions imposed by the various regulations mentioned above. As the Tribunal itself reports [LRA 39 2011]: *It is a false hypothesis to assume that settlements do not suffer from a number of potential distortions: there are poorly represented tenants, there are tenants who may be subject to pressures unrelated to the market place, and many settlements amount to "second guessing" what will happen at a tribunal.* The Tribunal is also aware of the limited information contained in the valuers' relativity curves [LRA 39 2011]: *We have been acutely aware of the difficulty of reaching a satisfactory conclusion on relativity in the light of the inadequacy of the available evidence, and it is clear that this is a problem that is likely to confront LVTs in all such cases. The likelihood is that decisions will be varied and inconsistent.* Similarly, the Appeals Tribunal mentions that *the "graph of graphs" – described by the LVT as "an average of averages" [of relativity curves] - are based upon limited, undefined data from all areas outside central London.* To summarize, with the words of the Tribunal [LRA 49 2008]: *In our judgment leasehold valuation tribunal decisions on relativity are not inadmissible, but the mere percentage figure adopted in a particular case is of no evidential value.*

Second, even when the "personal opinions" of valuers are based on the observation of actual transactions, the difficulty of performing the necessary hedonic analysis with small samples of very heterogeneous properties leaves large scope for influencing the outcome by arbitrarily choosing the value of the hedonic components, as shown in many documents related to the Tribunal decisions. Consider for example the following extract from [LRA 69 2006], in which the Tribunal describes the hedonic adjustment of a particular valuer: *Ms Joyce made the following adjustments to the prices paid in each case. For time, she used the Savills PCL South West Flats Index. [. . .] She deducted 12.5% to reflect the fact that flat 4, 3 Lennox Gardens, [. . .] had all been modernised and 7.5% to reflect the partial modernisation of flat D, 41 Lennox Gardens, the first floor flat at 31 Lennox Gardens and flat 8, 75-77 Cadogan Gardens. She did not make an adjustment in respect of flat 3, 27 Lennox Gardens, because she offset the value attributable to improvements against the large amount outstanding on the service charge. She considered that generally a second floor flat was worth 15% less than a first floor flat. She therefore added 17.65% to the prices paid for second floor flats in order to express them in terms of first floor values. She deducted 7.5% from the price paid for flat 3, 37 Cadogan Square to reflect its location on the more popular east side of the square and added 5% and 10% respectively*

_____

(2009) for more details.

to prices paid in Cadogan Gardens and Lennox Gardens to reflect the superior location of Cadogan Square. As a second example, the main discussion in the case [LRA 108 2008] revolved around the valuation of the *only* comparable property: a property relatively similar but with an additional "very small garden". Such garden was assigned different values by different valuers, in the range of £15,000 to £30,000; note that the total cost of the extension in that case was £18,000, not very different from the range of alternative values assigned to the garden. As a last example, consider the valuation of valuer Ms Tolgyesi as described by the Tribunal [LRA 55 2002]: *It will immediately be apparent that the making of adjustments to each comparable transaction can have a critical and indeed determinative effect upon the answer. Thus the adjustments made by Ms Tolgyesi are made in 5% increments – in some cases no adjustment is made but in others a 5% adjustment in others a 10% adjustment and in some a 15% adjustment is made. It will be seen that the size of the adjustments is greater than the difference (i.e a difference between 95% and 99%) which Ms Tolgyesi is seeking to measure.*

When discussing our findings with market participants, we found their priors on the relative valuation of leaseholds and freeholds to be very dispersed. While some valuers have argued for lower leasehold discounts than those we estimate (see above), a number of other market participants instead reported increases in the value of the lease with maturity that are much higher than those estimated by the valuers. Consistent with this, when asked to value the difference between a 138-year leasehold and a freehold [LRA 55 2002], *during cross-examination [a valuer] was asked to consider a hypothetical situation involving two identical properties – one freehold and one held on a long lease with 138 years unexpired. If acting for a purchaser of the long leasehold property, at a purchase price of £950,000, he accepted that a purchaser would be willing to pay a further £40,000 for the freehold property – although he commented that the long leasehold property would also sell for around £990,000 owing to the general demand for property.* While the Tribunal notes the obvious inconsistence in the valuer's statement that the long leasehold would at the same time be worth £990,000 and £950,000, the provided answer is further indication of the willingness of buyers to pay a substantial amount to increase the lease maturity from 138 years to infinity (particularly since, in this case, the valuer represented the leaseholder and, consequently, had an incentive to state a price difference as small as possible). In addition, websites that counsel on lease extensions often mention that the *cost* of lease extension is well below the actual increase in market value of the property. For example, the website lettingfocus.com reports: *The cost of extending leases got a bit more expensive in the last few years...But it can still make good sense [to extend] and of course will increase the value of your flat, usually by a lot more than the cost to extend the lease.*[11]

---

[11]See: http://www.lettingfocus.com/pages/myarticles_leaseextensions.html.

To conclude, we believe that prices paid for past extensions and valuers' opinions provide a potentially interesting source of information about the leasehold market. However, due to various distortions, extensions do not generally happen at market prices. At the same time, valuers' opinions are based for the most part on past extensions and Tribunal decisions, and are, at best, based on subjective hedonic analysis with little or no transaction data. In contrast, we provide an extensive formal hedonic analysis using the entire universe of transactions in England and Wales (1.8 million transactions of flats) and an extensive set of hedonic characteristics and fixed effects.

A.1.5.4 Net Incentives to Extend

As highlighted in Section 3.7, one possible concern with interpreting our results is the asymmetry in power between freeholders and leaseholders when it comes to negotiating lease extensions. The asymmetry occurs because freeholders, most often large estates or companies, generally hold a large number of properties, while leaseholders are typically individual homeowners. The laws described above were, in fact, primarily motivated as a way to mitigate the hold-up problem, granting the right to a lease extension as well as Tribunal protection if such extension cannot be achieved in bilateral negotiations with the freeholder.

The evidence on the Tribunal's decisions suggests that the laws have indeed managed to contain the average price that leaseholders pay for extensions through the Tribunal process, which in turn should mitigate the hold-up problem ex-ante. Of course, the threat of referring the matter to the Tribunal to take advantage of seemingly low premia has to be balanced with the substantial costs of such an option. A hypothetical new homebuyer that wanted to take advantage of the legislation would have to buy the property, wait for two years, and then hire a valuer and a solicitor to file for a lease extension; if a favorable extension cannot be agreed with the freeholder, the leaseholder has to file a complaint with the Tribunal, and, even after the Tribunal ruling, face the risk of an appeal to the Upper Tribunal (and in some cases to the House of Lords after that) from the freeholder. The potential length of this process runs into several years and the cost associated quickly rise to tens of thousands of pounds. However, while these prospects are likely to be daunting for most leaseholders, they do put significant bounds to the ability of the freeholder to hold up the leaseholders.

A final consideration concerns the possibility that the favorable treatment of Tribunals towards leaseholders may even over-compensate them for the hold-up problem, in fact increasing the ex-ante value of a leasehold by lowering the cost of extensions (directly in court, or indirectly outside of courts). If this was the case, it would produce a bias

against our results of large leasehold discounts, because it would imply that leasehold buyers would bid-up prices for short leaseholds in the hope of obtaining cheaper extensions in court (or possibly even out of court). We find the strict arbitrage logic between market prices and Tribunal decisions required for significant quantitative effects to run contrary to the substantial costs and waiting times (with the ensuing illiquidity) of the Tribunal process, the indivisibility of the housing asset, and the plausible segmentation of the market (see Piazzesi, Schneider and Stroebel, 2013).

Large freeholders have historically proved combative, even in the Tribunal system, and could be willing to sustain high legal costs on an individual case because of the repeated game nature of extensions on their total portfolio. This increases the risks associated with extensions. For example, the Earl of Cadogan, one of the largest freeholders in prime central London, brought cases against individual leaseholders through the three stages of court: LVT, Court of Appeal, all the way to the House of Lords. While the expenses connected with these legal battles make each of the cases unprofitable, there are large ripple effects on the rest of the freehold portfolio. Damian Greenish, the senior partner of Pemberton Greenish who acted for the Earl Cadogan in the House of Lords appeal "Earl Cadogan and others v Sportelli and others [2008 UKHL 71]," said that the favorable ruling could save big property owners like the Cadogan Estates millions of pounds and added: *We're over the moon about it. Had this gone against us it would have allowed the manipulation of claims to the detriment of landlords.*[12]

Furthermore, the legal framework we discussed in this section shows that only a limited amount of information regarding preferences or expectations can be inferred from the Land Valuation Tribunal's decisions. First, as the legal arguments happen in the narrow context of precedent (with the U.K. being a common law country), there is often little scope for parties to reveal their true valuations of properties, extensions, and discount rates: rather, all the valuations provided to the court are inputs in a predetermined framework established by the jurisprudence. Second, the fact that only a very selected sample of leaseholders and freeholders bring the discussion about extensions to court is consistent with the valuations and costs incurred by those who ultimately debate the extension in court not being representative of the valuation or costs of the average leaseholder or freeholder. For example, the fact that the leaseholder and the freeholder that we see in court sometimes dispute only small amounts of money may indicate that those parties went into trial with the expectation that the tribunal would award them more than

---

[12]See http://www.solicitorsjournal.com/news/crime/general-principles/Õhope-valueÕ-ruling-could-save-landlords-millions-collective-enfranchi.

originally contended.[13]

In addition to the costs of a lease extension, the effect of potentially cheaper lease extensions on ex-ante leasehold values would also be reduced if leasehold buyers were not fully aware of the details of the extension process at the moment they buy the property. While the tenure of the property is widely publicized on the sales listings, the technical details of the lease extension process may not be always understood by buyers until the moment when they actually decide to extend. Indeed, repeated calls by public authorities and newspapers to extend the lease when there are more than 80 years remaining suggest low general awareness of important details of the law regulating extensions.[14] In particular, the law induces a discontinuity in extension prices at 80 years remaining, which makes it significantly more expensive to extend once there are less than 80 years remaining. For leaseholds below 80 years remaining, the Tribunals determine a "marriage value," defined as the increase in value for the leasehold interest due to the longer length of the lease minus the decrease in value to the freehold interest for the same extension. The law requires that for leaseholds below 80 years, the marriage value is to be split equally between freeholder and leaseholder, while for leaseholds above 80 years, it accrues entirely to the leaseholder. This makes lease extensions below 80 years more expensive for leaseholders.[15] If buyers were fully aware of this rule, we would expect a large number of buyers to extend when more than 80 years remain on the contract, or at the very least to avoid extending just below 80 years. Instead, we find in our data that 84% of the extensions happen when the contract has less than 80 years remaining, with twice as many leaseholders extending with 75-79 years remaining as those extending with 80-84 years remaining (15% of the sample vs 7%). Since leaseholders who wait to extend until the remaining maturity falls below 80 years leave a substantial amount of money on the table, this suggests that they may not be fully aware of the costs and benefits of an extension or that other frictions are preventing them from taking advantage of the opportunity. Therefore, it would not be surprising to find other features of the lease extension process, like the relatively low average extension costs determined by the Tribunal, to have little ex-ante effect on the market prices of leaseholds.

---

[13] Alternatively, they may be parties with particularly low costs of going to trial (low personal cost of time or litigation), as in the case of [LRA 108 2008], where the parties went all the way to the Appeals court out of a maximum potential gain of £7,000, certainly an amount below the expected cost of each step of the trial. This evidence certainly does not imply that the average leaseholder would have gone all the way to the Appeals court in a dispute over £7,000, given that as mentioned the cost of legal representation can run in the tens of thousands of pounds.

[14] See for example http://www.theguardian.com/money/2011/feb/26/lease-running-out-buying-freehold

[15] The existence of marriage value, a form of failure of Modigliani and Miller theorem, is another symptom of frictions and failure of arbitrage in this market, particularly for short term leases.

Ultimately, the existence of a strong protection of leaseholders by Tribunals puts a strong bound on the potential hold-up of the leaseholder by the freeholder, and, if anything, may increase leasehold prices, biasing against us finding significant discounts. While a variety of model-based or small-sample "comparables" measures of leasehold discounts have been used in the industry, our analysis provides the first large scale systematic evidence. Finally, interesting puzzles on the optimal exercise of the extension option remain open for future research.

A.1.6 Ground Rents and Service Charges

Ground rents and service charges might also affect the value of leaseholds relative to freeholds. Below we describe the relevant institutional details. Neither is important enough to significantly affect prices paid, as discussed in Section 3 of the paper. Since ground rents and management fees are present for leaseholds of all maturities, the fact that 700+ year leaseholds trade at the same price as otherwise identical freeholds shows that they cannot contribute significantly to leasehold discounts relative to freeholds.

Ground Rents: Leaseholders sometimes have to pay annual ground rents to the freeholder, since the purchase price of the lease only covers the temporary ownership of the structure. The land still belongs to the freeholder who has the right to request that the lessee make regular payments for the use of the land.

Ground rents are set on a property by property basis and no centralized database exists. This makes it hard to control for them in a regression analysis. However, the amounts involved are usually very small (£10-£100 per year) and in many cases are either zero or a symbolic amount ("a peppercorn"). In the 2011-12 English Housing Survey, amongst those households reporting to pay ground rents, the median household reported annual rents of about £25. In addition, all leases extended under the Leasehold Reform, Housing and Urban Development Act 1993 are set at peppercorn levels. Even in cases where the ground rent is in principle positive, it is often zero in practice, because for the rent to be collected the freeholder has to make a specific written request to the leaseholder. Oftentimes such requests are not made because the amount collected would be too small to cover the administrative costs.

Service and Insurance Charges: Service charges are payments by the leaseholder for services provided by the landlord. These include maintenance and repairs, insurance of the building and, in some cases, provision of central heating, lifts, porterage, estate staff, lighting and the cleaning of common areas. In the 2011-12 English Housing Survey, 46%

of leaseholders reported paying a service charge; amongst those households the median annual payment was about £750.

While maintenance costs can be non-trivial, as long as the maintenance is carried out at fair value (the private market cost of the works) service charges do not represent a problem for our analysis, since freeholders would also have to pay for the maintenance of the property. Having the landlord conduct maintenance may even be efficient, because she will likely enjoy significant economies of scale.

A potential problem exists if freeholders might attempt to extract monopoly rents via the service charge, as suggested by some newspaper articles such as The Observer (2013). We do not, however, believe that this is a likely explanation of the leasehold discounts estimated in the data. The ability to extract rents is severely limited. First of all, under the Commonhold and Leasehold Reform Act 2002 an application can may be made to a Leasehold Valuation Tribunal to challenge the reasonableness of service charges. Secondly, the Commonhold and Leasehold Reform Act 2002 provides a right for leaseholders of flats to force the transfer of the landlord's management functions to a special company set up by them - a "right to manage" company. This does not require the landlord's consent, and significantly limits her ability to extract unreasonable service charges.

Similarly, in some cases the lease requires that the leaseholder insures the property, usually a house, through an insurer nominated or approved by the landlord. The tenant may consider that he can get cheaper insurance from different companies and may be concerned as to the cover provided. The provisions of Section 164 of the Commonhold and Leasehold Reform Act 2002 provide a right for the leaseholder to arrange his own insurance, provided he notifies the landlord.

A.1.7   Leasehold Covenants

A further concern might be that contractual covenants in leasehold deeds place restrictions on leaseholders that reduce the value of the leasehold relative to the freehold. We analyze a large number of covenants from deed titles individually downloaded from the Land Registry website to determine whether or not such contractual restrictions might explain the estimated discounts.[16] In this section, we present examples of the covenants discovered in these titles, after removing any personal identifiers. Many leaseholds do not carry restrictions; others contain only old, out-of-date restrictions, which, while technically still in place, are generally not enforced.[17] In Appendix Section A.1.7.1 we conduct

---

[16]http://www.landregistry.gov.uk/public/property-ownership allows the download of individual deeds, including any covenants on the land, for a charge of £3 per title.

[17]For example, a leasehold for a flat states that "A Conveyance dated 20 June 1864 made between (1) [Person A] and (2) [Person B] contains restrictive covenants but neither the original deed nor a certified

a systematic analysis of all leasehold covenants in a particular part of London to show that they do not vary systematically by initial lease length.

When present, we find four main types of covenants. The first and most common type of covenant involves restrictions on the **broad type of land use**, such as restricting structures to be residential rather than commercial. Usually these are consistent with council zoning regulations, which would also apply to the owners of freeholds:

> No building now or hereafter to be erected on the premises hereby transferred or any part or parts thereof shall at any time hereafter be used for any other purpose than that of private dwellinghouses buildings and appurtenances belonging thereto and no block of flats hotel factory or retail shop or other premises for the sale of goods by retail shall be erected on the premises hereby transferred and that the Purchaser and its successors in title and assigns will not at any time use exercise or carry on or permit to be carried on upon any part or parts of the premises hereby transferred or any buildings for the time being erected or to erected or standing thereon any trade manufacture or business or do any act or thing which may be or grow to be in any way offensive noxious or dangerous to the Vendor or its Superior Lessors or tenants or the owner or tenants of adjoining property forming part of the Vendor's estate or any part thereof and will not use or permit to be used the premises hereby transferred or any buildings erected thereon or any part thereof for the purpose of manufacturing storing selling supplying or distributing either by wholesale or retail ales beers wines spirits or any other intoxicating liquors nor shall any house or building erected or to be erected on the premises hereby transferred or on any part thereof be converted or used for such purposes or used as a cinema or a club or clubs at which intoxicating liquors may be stored sold or supplied.

Note that while those covenants restrict the use of land and future structures to be residential, there are no further restrictions on the construction of such new residential properties. Some leasehold covenants do place restrictions on the **construction of a new structure**. To the extent that there are such restrictions on new structures, many times they again relate to restrictions placed by the council, and would equally apply to all new structures that freeholders might want to erect (see in particular point 3 of the following leasehold covenant):

---

copy or examined abstract thereof was produced on first registration."

The land tinted pink on the filed plan is subject to the following stipulations contained in a Deed dated 10 August 1923 made between (1) [Person A], (2) [Person B] (3) [Person C].

1. No church chapel synagogue or other place of public worship or instruction manufacturing premises institution nursing home lunatic asylum sanatorium creche school public motor garage licensed premises theatre cinematograph theatre or other place of amusement shop or business premises shall be erected on the premises and no buildings now or at any time to be erected thereon shall at any time be used except as private dwellinghouses only but no objection shall be made to user of the premises at present erected on the land as a private residential hotel.

2. Any dwellinghouse when erected on the said premises shall be of the value of £900 at least in prime cost of materials and labour exclusive of any outbuildings stabling or motor garage.

3. The front wall of any dwellinghouses to be erected on the premises shall range and be set back from Poynders Road within the boundary line to be fixed by the London County Council and in accordance with the provisions of the Housing and Town Planning Scheme of the District.

Other covenants require the permission of the freeholder for certain redevelopments (i.e. placing a structure close to the property edge):

A Conveyance of the land in this title and other land dated 10 March 1914 made between (1) [Person A] (Vendor) and (2) [Person B] contains the following covenants: Covenant by Purchasers with Vendor his heirs and assigns that the Purchasers their heirs and assigns would not place or suffer to be placed any building whatsoever other than walls or fences under 6 feet in height on any part of the lands thereby conveyed nearer than 30 feet to the road called Lambton Road without the previous consent in writing of [Person A] or the person or persons claiming in succession to them.

In general, however, a covenant against the making of improvements without consent is subject to the provision that consent shall not be unreasonably withheld. This was determined by **Section 19(2) of the Landlord and Tenant Act 1927**:

In all leases whether made before or after the commencement of this Act containing a covenant condition or agreement against the making of improvements without a licence or consent, such covenant condition or agreement

shall be deemed, notwithstanding any express provision to the contrary, to be subject to a proviso that such licence or consent is not to be unreasonably withheld; but this proviso does not preclude the right to require as a condition of such licence or consent the payment of a reasonable sum in respect of any damage to or diminution in the value of the premises or any neighbouring premises belonging to the landlord, and of any legal or other expenses properly incurred in connection with such licence or consent nor, in the case of an improvement which does not add to the letting value of the holding, does it preclude the right to require as a condition of such licence or consent, where such a requirement would be reasonable, an undertaking on the part of the tenant to reinstate the premises in the condition in which they were before the improvement was executed.

As described by Burn, Cartwright and Cheshire (2011), the word "improvements" refers to improvements from the point of view of the tenant, and the statute applies even though what he proposes to do, for example, the demolition of part of the main structure of a building, will demporarily deminish the value of the premises (see *Lambert v Woolworth & Co Otd, 1938*).

The third class of covenants relate to the **joint use of infrastructure** such as access roads:

The land is subject to the following reservations contained in a Conveyance of the freehold estate in the land in this title and other land dated 10 October 1878 made between (1) [Person A] and (2) [Person B]: Reserving nevertheless to the said [Person A] his heirs and assigns owner or owners for the time being of any messuage to be erected on the adjoining land on the South of the said premises thereby granted the right of using a Drain already constructed under the South side of the back yard of the Southernmost of the said messuages thereby granted to connect the drain from the said messuage so to be erected on the adjoining land on the South of the said premises thereby granted as aforesaid with the said Drain the use whereof was thereinbefore granted by the said [Person A] with liberty for the said [Person A] his heirs and assigns tenants or occupiers and his and their servants and workmen at all reasonable times to enter the back-yard of the Southernmost of the said messuages thereby granted for he purpose of connecting and repairing such Drain such Drain in the said backyard to be kept in repair when and so soon as the same should be used by the said [Person A] his heirs or assigns tenants or occupiers at the joint expense of the said [Person A] his heirs and assigns and of the said

[Person B] his heirs and assigns and each of them would pay to the other of them on demand one moiety of the expenses incurred by the other of them in repairing such last mentioned Drain.

Unless there is specific agreement to the contrary, a tenant is free to grant his interest to a third party, either by assignment or by underlease, as described in Burn, Cartwright and Cheshire (2011). A fourth set of covenants we sometimes encountered therefore requires leaseholders to obtain the freeholder's permission to **sublet the property**, i.e. to rent it out to somebody else. These requirements usually stipulate that the freeholder cannot "unreasonably withhold" consent to a sublet, and sometimes allow the freeholder to charge a fee for registering a sublet. These terms for subletting property are regulated in **Section 19(1) of the Landlord and Tenant Act 1927**, which provides as follows:

In all leases whether made before or after the commencement of this Act containing a covenant condition or agreement against assigning, under-letting, charging or parting with possession of demised premises or any part thereof without licence or consent, such covenant condition or agreement shall, notwithstanding any express provision to the contrary, be deemed to be subject

1. to a proviso to the effect that such licence or consent is not to be unreasonably withheld, but this proviso does not preclude the right of the landlord to require payment of a reasonable sum in respect of any legal or other expenses incurred in connection with such licence or consent; and

2. (if the lease is for more than forty years, and is made in consideration wholly or partially of the erection, or the substantial improvement, addition or alteration of buildings, and the lessor is not a Government department or local or public authority, or a statutory or public utility company) to a proviso to the effect that in the case of any assignment, under-letting, charging or parting with the possession (whether by the holders of the lease or any under-tenant whether immediate or not) effected more than seven years before the end of the term no consent or licence shall be required, if notice in writing of the transaction is given to the lessor within six months after the transaction is effected.

Furthermore, the **Landlord and Tenant Act 1988** places on the landlord the burden of showing that any refusal or the imposition of any conditions was reasonable. It also gives a tenant the right to sue for damages suffered as a result of a landlord's unreasonable refusal. Subsequent common law cases have further regulated the maximum fee that

freeholders can charge for the granting of approval for a sublet. In *Holding and Management (Solitaire) Limited vs. Cherry Lilian Norton (LRX/33/2011)*, the court decided that a fee in excess of £40 + VAT was not merited.

A.1.7.1 Direct Evidence on Leasehold Covenants

In addition to our broad search for covenants across many individual leaseholds with different initial lease lengths and from different geographies, we also conduct a systematic analysis of covenants in a particular postcode of London, E16, located in the eastern part of the city (the former dockyards). This postcode was chosen because of the presence of leaseholds with significant heterogeneity in initial lease length.[18] For each of the 801 leaseholds transacting within this postcode in our sample period, we downloaded the full leasehold and freehold title (which sometimes registers the covenants on the leaseholds) from the Land Registry website. 273 transactions were of leaseholds with an initial length of 99 years, 152 with an initial length of 125 years, 147 with an initial length of 200 years and 229 with an initial length of 999 years. After manual inspection of these contracts we found 11 different classes of covenants. These are described below, together with an example from one leasehold including such covenant:

1. **Docks** - No construction or erection of docks without prior authorization. Example from Register EGL457462 (Leasehold for 120, Wards Wharf Approach, E16 2ER)
   In respect of the whole of the land assured by the above written Indenture that the Purchasers their Successors and assigns or the person or persons deriving title under them will not without the consent in writing of the London and India Dock Company formerly the Victoria London Dock Company first had and obtained make erect or construct or suffer to be made erected or constructed upon such land any dock of a similar nature or to be applied to the like purposes as the Victoria (London) Dock.

2. **Thames** - No execution of works without prior presentation of plans to the Conservators of the River Thames. Example from Register EGL457462 (Leasehold for 120, Wards Wharf Approach, E16 2ER)
   In respect of the land secondly described in the above written Indenture and thereby assured that the Purchasers their Successors and assigns or the person or persons deriving title under them will not execute or begin to execute any works upon the last mentioned land or any part thereof unless plans shewing the nature of the works proposed to be erected or executed shall have been deposited with the Conservators of the River Thames at their office

---

[18]It is hard to establish the representativeness of the covenants in this particular postcode. In principle this exercise could be repeated for other parts of the country, though the £3 charge per title acquisition makes such an analysis hard to conduct on a large scale.

and unless such plans shall have been approved on behalf of the said Conservators by their Secretary in writing before the proposed works shall be commenced Provided nevertheless that such approval shall not be unreasonably withheld.

3. **Nuisance** - Not to do anything that may be a nuisance to the Transferor. Example from Register EGL435316 (Leasehold for Lily Nichols House 6, Connaught Road, E16 2AD)
The Transferee covenants with the Transferor (...) Not to do anything which shall be or may grow to be a nuisance or annoyance to the Transferor or any other person who is registered proprietor of any part of the Estate.

4. **Roadblock** - Not to obstruct any part of the Estate roads. Example from Register EGL435316 (Leasehold for Lily Nichols House, Connaught Road, E16 2AD)
Not to park on or otherwise obstruct nor to erect any building or structure of any nature upon any part of the Estate Road.

5. **Build** - Not to erect structures or change existing ones. Example from Register EGL435316 (Leasehold for Lily Nichols House 6, Connaught Road, E16 2AD)
Not for a period of two years from the date hereof without the prior written consent of the transferor which shall not be unreasonably withheld or delayed:- (i) to construct or allow to be constructed any additional building structure or extension or lay any sewers or drains on any part of the Property; (ii) to make any alterations to any Buildings or the external appearance of any part of the Property

6. **Trees** - Not to alter/destroy any trees in the Estate. Example from Register EGL435316 (Leasehold for Lily Nichols House 6, Connaught Road, E16 2AD)
Not to fell lop or top any tree situated within the Property without the prior written consent of the Local Planning Authority nor to remove or destroy any tree or shrub planted on the Property as part of any landscaping scheme and to replace any such tree or shrub which may fail or die

7. **Parking** - Not to use the parking spaces for any other use than parking private vehicles. Example from Register EGL435316 (Leasehold for Lily Nichols House 6, Connaught Road, E16 2AD)
Not to use the parking spaces on the Property save for the purpose of parking one private motor vehicle in each space.

8. **NoisyTrade** - Not to use the Estate for manufacture, trade and business that may be dangerous and/or noisy. Example from Register EGL281787 (8, Newland Street,

E16 2DU)

"No manufacture trade business or operations of a noisome dangerous or noisy kind shall be carried on in or upon the land or any building thereon and no building thereon shall be used as an hotel public house or tavern or for the sale of beer wines and spirits."

9. **Exhibition** - No use as exhibition space in excess of 2000 square meters of area in the next 25 years. Example from Register 432232 (101 Fishguard Way, E16 2RG)
   Not within 25 years of the date hereof to construct on and/or use the Property for any single purpose-built exhibition space in excess of 2000 square metres in area.

10. **Wharf** - No use of the premises for specific trades such as wharfing or bulding materials. Example from Register EGL441147 (The Reflection 2, Flat 11, E16 2GD)
    "AND the Council hereby for themselves their successors and assigns covenant with the Grantor his heirs executors administrators and assigns that the Council .... their successors and assigns will not use or allow to be used the hereditaments and premises hereby conveyed or any part thereof directly or indirectly for the purposes of a trade of a brick lime cement pipe title slate builders material merchant Wharfinger or any similar trade without the consent in writing of the Grantor his heirs executors administrators or assigns successors in title to the hereditaments adjoining the hereditaments hereby conveyed on the East side thereof and known as Sankeys Barge House Wharf first had and obtained."

11. **Advertisement** - No use of the premises as advertising space. Example from Register EGL99625 (Dunedin House, 26, E162LA)
    No advertisement sign or display shall be erected on the land and premises hereby transferred or attached to the house erected thereon other than a name plate not exceeding eight inches by six inches in dimension giving particulars of the profession of the owner and at the appropriate times the usual signs giving notice of a proposed sale.

We inspected each of the 801 titles and determined which covenants were present for which leasehold. Appendix Figure A.2 shows the share of leaseholds of each initial lease length that have each covenant. We find no evidence in this analysis that covenants are any more restrictive on leasehold properties with shorter initial leases (99 years, 125 years) relative to leaseholds with longer initial leases (999 years).

A.2   Institutional Appendix - Singapore

Residential properties in Singapore can be classified into land titles or strata titles. Land title properties occupy land that is exclusive to the owner (like a detached house), whereas

a strata title comprises units in multi-unit housing (flats or apartments) or in condominium developments. Owners of strata properties enjoy exclusive title only to the airspace of their individual unit. The land that the development is built on is shared by all the owners of the project, based on the share of the strata title unit owned by each owner. Owners are free to sell their individual unit. In order to sell the land, they will have to go via an "en bloc" sale, which requires a minimum of 80% of the owners' consent.[19]

A large fraction of the Singaporean housing stock consists of Housing and Development Board (HDB) properties, mostly in the form of flats. In total over 80% of Singapore's population lives in HDB flats, and 90% of them are fully owned. The HDB flats are part of a state-subsidized home ownership program and leases are often granted at below market values. We exclude these properties from our analysis and focus instead on the private market, where transaction prices reflect market values of the properties.

Initial lease lengths in Singapore are almost always either 99 or 999 years. The earliest land lease was issued in 1826 with a term of 999 years. There are also other types of less common lease structures. The first are private developments with 103-year leaseholds sold on freehold land. In addition, in November 2012 a plot of land at Jalan Jurong Kechil was the first to be sold for residential development under an initial 60-year lease agreement; though houses built there do not yet appear in our data.

Property taxes are independent of the form and duration of ownership. Property taxes are levied on the *Annual Value* (AV), the tax-authority assessed 1-year rental income of the property. For rental properties, the tax rate is set at 10% of AV; for owner-occupied properties, it rises from 0% on the first $6,000 to a marginal rate of 6% for AVs exceeding $65,000.[20] The rental income, and therefore the Annual Value, of a property is unaffected by the length of the lease under which the property is owned. Property transactions are also subject to stamp duty irrespective of the form and duration of ownership.[21] As with the U.K., the progressive nature of the tax increases the relative value of short-duration leasehold properties who are cheaper and therefore taxed at the lower marginal rate.

Singapore allows homebuyers to use their pension contributions to pay off their mortgage. Recently, it was also allowed to use such contributions to pay off certain portion of down payment. However, for any leasehold property, if the number of years remaining is less than 60 years at the time of transaction, homebuyers are not allowed to use

---

[19]80% consent is necessary if the development is at least 10 years old and 90% consent is necessary if the development is less than 10 years old.

[20]Starting from January 1, 2014, property taxes were made more progressive (see: http://www.iras.gov.sg/irasHome/page04.aspx?id=2094).

[21]Stamp duties are transaction taxes, and are assessed on the purchase value of the property. The first $180,000 are assessed at 1%, the next $180,000 at 2% and each additional increase in the sales prices at 3%. See http://www.iras.gov.sg/irasHome/page04.aspx?id=8748 for details.

pension contributions to buy their properties. Consistent with this, banks have a similar restrictions for mortgage lending.

A.3 Data and Empirical Appendix

A.3.1 U.K. Leasehold Discounts - Houses

In Section 2.3 we analyze price differences between leaseholds of varying maturity and freeholds for flats in the U.K. In this section we show that the estimated price differences between leaseholds and freeholds are, if anything, larger in the transaction sample for U.K. houses. The bottom panel of Table 1 reports the composition of our sample of houses. We observe just above 6.5 million transactions between 2004 and 2013, almost all of which (95%) are transactions of freeholds. Most of the remaining leaseholds are concentrated at maturities below 125 years or above 700 years: around 300,000 transactions, or 4% of the sample, are for leaseholds with 700 or more years remaining. The number of transactions for shorter leases is much smaller, with around 0.8% below 125 years and only 0.2% between 125 and 200 years. Given the lack of data for intermediate maturities for houses, we focus on the very long maturities (700+ years remaining) and the short maturities (less than 125 years remaining) when analyzing house transactions. For houses, leaseholds are more geographically concentrated, clustering around Manchester and Newcastle. However, the overall number of transactions for houses is large, so that we have within postcode variation in lease type in most postcodes in our sample, even if one contract type comprises the majority of the sample. Overall, relative to the transaction sample for flats, for transactions of houses we have less variation across contracts, in particular for leases between 125 years and 700 years. This was the reason to focus on flats in the main body of the paper.

We next estimate the relative prices paid for leaseholds of varying remaining maturity and freeholds for houses in England and Wales. Given the lack of transactions of leaseholds with intermediate maturities, we construct the following three *MaturityGroups* for leasehold houses: 80-99 years, 100-124, and 700+ years groups. We then estimate regression (A.1) below. The unit of observation is a transaction i of a property of type g (detached, semi, terraced) in 3-digit post code h at time t. We assign each leasehold with remaining maturity $T_{i,t}$ to one of the *MaturityGroup$_j$* buckets depending on the number of years remaining on the lease at the point of sale. The excluded category are freeholds, so that the β_j coefficients capture the discounts of leaseholds with that maturity relative to otherwise similar freeholds.

$$log(Price_{i,h,t,g}) = \alpha + \sum_{j=1}^{3} \beta_j \mathbf{1}_{\{T_{i,t} \in MaturityGroup_j\}} + \gamma Controls_i + \xi_h \times \psi_t \times \phi_g + \epsilon_{i,h,t,g} \quad (A.1)$$

Appendix Table A.2 shows the results. The estimated discounts between leaseholds and freeholds are larger for houses than for flats: leaseholds of 80-99 years remaining length trade at a discount of 31% relative to freeholds, and leaseholds with 100-124 years remaining trade at a 26% discount. Very long leaseholds with more than 700 years remaining trade for a small discount of around 1%. While larger in magnitude, the results are in line with the results for flats, though they are somewhat less informative given the limited use of short leaseholds for houses as well as the geographic concentration of the leaseholds discussed above.

A.3.2 Average Real Housing Returns and Rental Growth

In this section we estimate r and g for the U.S., the U.K. and Singapore. We briefly describe our methodology and findings, and provide the details of the data and estimation procedure in section A.3.2.2 below. We employ two complementary approaches to estimating average returns to housing. The first approach, which we call the balance-sheet approach, is based on the total value of the residential housing stock and the total value of housing services consumed (the dividend from that stock). We obtain this information from countries' national accounts.[22] We control for the growth of the housing stock over time in order to back out the return series for a representative house. The second approach, which we label the price-rent approach, starts from the price-rent ratio estimated in a baseline year and constructs a time series of returns by combining a house price index and a rental price index. This approach focuses on a representative portfolio of houses and, therefore, does not need to correct for changes in the housing stock. After adjusting for inflation, both methods provide estimates of the gross real returns to housing, $E[R^G]$. To compute net returns, we subtract maintenance costs and depreciation (δ) and any tax-related decreases in return (τ). We estimate net returns as $r = E[R] = E[R^G] - \delta - \tau$.

The top panel of Table 9 presents the estimated average housing returns for the U.S., England-and-Wales, and Singapore. Our estimates for housing returns in the U.S. follow Favilukis, Ludvigson and Nieuwerburgh (2010).[23] While U.S. housing returns are not the focus of this paper, they provide a useful benchmark because they have been the subject of an extensive literature, including Gyourko and Keim (1992), Flavin and Yamashita (2002), Lustig and Van Nieuwerburgh (2005) and Piazzesi, Schneider and Tuzel (2007).

---

[22]To determine the total consumption of housing services, these measures impute the value of the owner-occupied equivalent rents, the housing services consumed by individuals from living in their own house. See Mayerhauser and Reinsdorf (2006) and McCarthy and Peach (2010) for a description of the construction of these measures.

[23]We thank Stijn van Nieuwerburgh for sharing the data and for insightful discussions on estimating housing returns.

The balance-sheet and the price-rent approaches provide similar estimates for the average annual real gross return ($E[R^G]$): 10.3% and 10.7% respectively. We calibrate the impact of maintenance and depreciation (δ) at 1.5% and the property tax impact τ at 0.67%.[24] We conclude that average real net returns in the U.S. housing market are between 8% and 8.5%. This is similar to the estimates in Flavin and Yamashita (2002), who find a real return to housing of 6.6%, and Favilukis, Ludvigson and Nieuwerburgh (2010), who find a real return of 9-10% before netting out depreciation and property taxes.

Column 3 and 4 in Table 9 report our estimates for the Singaporean housing market. The balance-sheet and price-rent approaches provide similar estimates for the average annual real gross return ($E[R^G]$): 10.3% and 10.4%, respectively. We assume the cost of maintenance and depreciation to be 1.5%, in line with the estimates for the U.S., and the property tax impact to be 0.5%.[25] A conservatively low estimate of the real net returns in the Singapore housing market is therefore between 8.3% and 8.4%.

Columns 5 and 6 of Table 9 report the estimates for the housing market in England and Wales. The balance-sheet and the price-rent approaches provide similar estimates for the average annual real gross return ($E[R^G]$): 12.5% and 10.9%, respectively. We maintain the calibration for the cost of maintenance and depreciation at 1.5%. There are no property taxes to be considered in England and Wales. Average real net returns in the U.K. housing market are approximately $9 - 11\%$.

Overall, the estimates show that real expected returns for housing are between 8% and 10% for all countries in our international panel. These estimates are in line with the existing literature, and robust to the different methodologies.[26] Our estimates for

---

[24]Malpezzi, Ozanne and Thibodeau (1987) provide an overview of the literature on depreciation. For example, Leigh (1980) estimates the annual depreciation rate of housing units in the U.S. to be between 0.36% and 1.36%. Depreciation is also a key calibration parameter for much of a recent literature in macroeconomics that considers households' portfolio and consumption decisions with housing as an additional asset. Cocco (2005) chooses a depreciation rate equal to 1% on an annual basis; Díaz and Luengo-Prado (2008) include an annual depreciation rate of 1.5%. Property taxes in the U.S. are levied at the state level and, while there is variation across states, are generally around 1% of house prices. Property taxes, however, are deductible from federal income tax. We assume that the deductibility reflects a marginal U.S. federal income tax rate of 33%. The net impact is therefore $(1 - 0.33) * 0.01 = 0.67\%$.

[25]Singapore levies a 10% annual tax on the estimated rental income of the property. A lower tax rate applies to owner-occupied properties (6%), but we use the more conservative (higher) rate for rental properties. See section 1.2 for details. The tax impact on returns is the tax rate times the average rent-price ratio, estimated at 5%. Hence, $\tau = 0.1 * 0.05 = 0.5\%$.

[26]We also note that since most movements in rent-price ratios are driven by movements in house prices and not by movements in rents, see Shiller (2007), our estimates of returns are relatively unaffected by the time period chosen. For example, since 2013 rent-price ratios in the U.S. have declined to approximately their 2000 levels (see Figure A.32), ending the sample in 2005 would have produced a slightly lower average rent-price ratio. However, focusing on that period would also exclude the house price crash from our estimates of capital gains, thus leading to higher estimated average capital gains. In the overall estimates of expected returns, the higher estimated capital gains would be offset by a lower estimated rent-price ratio.

the U.S. and the U.K. are consistent with the notion that average house price growth over extended periods of time is relatively low, as argued by Shiller (2006), and the key driver of real housing returns is the high rental yield. Our estimated average capital gains are positive but relatively small (even for Singapore where they are the highest) despite focusing on samples and countries that are often regarded as having experienced major growth in house prices.

A.3.2.1 Real Rental Growth

In order to calibrate the parameter governing rent growth (g), we estimate the average growth rate of rental income, obtained directly from rental indexes. The national accounts and the rental index provide similar growth rate estimates on the sample where both are available. The estimated real growth rate of rents is close to zero. For the U.S., our estimate (0.2%) is in line with the estimates of Campbell et al. (2009) that obtain a median growth rate of 0.4% per year. We obtain a similarly low estimate (0.2%) of average annual rental growth for Singapore, while the U.K. estimate is somewhat higher at 0.7%. As for the case of real average house price growth, our estimates of small-to-negligible real rent growth are in line with Shiller (2006). In our baseline estimates, we calibrate g to be 0.2%.

A.3.2.2 Details on Estimation Procedures

This section describes the methodology and data used to compute average real returns and rent growth for residential properties. We report the details of the calculations in an online worksheet.

The balance-sheet approach Following Favilukis, Ludvigson and Nieuwerburgh (2010), this approach uses information about the value of the stock of residential real estate to estimate the value (price) of housing and total household expenditure on housing as a measure of the value of rents in each period. Since we are only interested in the return to a representative property, we need to control for changes in the total housing stock. We proxy for the change in the stock by population growth, assuming that at least over long periods the per capita stock of housing is constant. We derive the gross return to housing in each period as:

$$R_{t+1}^{G} = \frac{V_{t+1}^{H} + CE_{t}^{H}}{V_{t}^{H}} \frac{\pi_t}{\pi_{t+1}} \frac{L_t}{L_{t+1}},$$

where V^H is the value of the housing stock, CE^H is the household expenditure on housing, π is the CPI price level index, and L is population.

- For the U.S. we follow Favilukis, Ludvigson and Nieuwerburgh (2010) and use data from the Flow of Funds (obtained from the Federal Reserve Board and the Federal Reserve Bank of St. Louis). For the value of the housing stock we sum two series: owner-occupied real estate and tenant-occupied real estate (FL155035005, FL115035023) from the Flow of Funds. From the Federal Reserve Bank of St. Louis we obtain: (i) household expenditure on housing services (DHUTRC1A027NBEA); (ii) Population estimates (POP); and (iii) the Consumer Price Index (U.S.ACPIBLS).

- For the U.K., using the same procedure, we combine the value of the total stock of housing (series CGRI) and the total expenditures on housing (series ADIZ) from the National Accounts (available from the Office of National Statistics). From the same source, we obtain the CPI (series D7BT). We adjust for the change in the stock of housing using the population growth series from ONS for England and Wales.

- We use a similar procedure for Singapore. From the National Accounts (from singstat.gov.sg), we obtain the value of the private residential stock of housing (series M013181.1.1.1 P017199) and the private consumption expenditure on housing and utilities (series M013131.1.4 P017135). We obtain the series for the population growth (that proxies for the change in the stock of housing wealth) from the World Bank (series SP.POP.GROW). Finally, we obtain the CPI series from the National Statistical Office (singstat.gov.sg).

The price-rent approach   This approach constructs a time series of returns by combining information from a house price index, a rent index, and an estimate of the price-to-rent ratio in a baseline year. Without loss of generality suppose we have the rent-to-price ration at time $t = 0$. We can derive the time series of the rent-to-price ratio as:

$$\frac{P_t}{D_{t+1}} = \frac{P_t}{P_{t-1}} \frac{D_t}{D_{t+1}} \frac{P_{t-1}}{D_t}; \qquad \frac{P_0}{D_1} \quad given.$$

where P is the price index and D the rental index. Notice that, given a baseline year $\frac{P_0}{D_1}$, only information about the growth rates in prices and rents are necessary for the calculations. We then compute real returns using the formula:

$$R_{t+1}^G = \left(\frac{D_{t+1}}{P_t} + \frac{P_{t+1}}{P_t} \right) \frac{\pi_t}{\pi_{t+1}}.$$

- For the U.S. we follow Favilukis, Ludvigson and Nieuwerburgh (2010) and use the Case-Shiller 10-city house price index (series SPCS10RSA from the Federal Reserve

Bank of St. Louis), and compute rent growth using the BLS shelter index (the component of CPI related to shelter, item CU.S.R0000SAH1 from the Federal Reserve Bank of St. Louis). However, differently from Favilukis, Ludvigson and Nieuwerburgh (2010), we choose 2012 as a baseline year for the rent-price ratio, which is estimated at 0.1, because of the availability of high quality data for that year. We obtained two independent estimates for the rent-price ratio in the base year of 2012. The first estimate is the price-rent ratio implied by the balance-sheet approach. The second estimate is a direct estimate obtained using data by the real estate portal Trulia. Figure A.31 shows the distribution of rent-price ratios across the 100 largest MSAs provided by Trulia.[27] Both independent estimates imply a rent-price ratio of 10% in 2012. Figure A.32 suggests that these rent-price ratios are close to their long-run average.

- For Singapore we obtain a time series of price and rental indices for the whole island from the Urban Redevelopment Authority (the official housing arm of the government: ura.gov.sg).

  To estimate the baseline rent-price ratio, we use data from for-sale and for-rent listings provided by iProperty.com, Asia's largest online property listing portal. We observe approximately 105,000 unique listings from the year 2012, about 46% of which are for-rent listings. To estimate the rent-price ratio we run the following regression which pools both types of listing. This follows a similar methodology as Figure A.31 in the construction of rent-price ratios for the U.S.:

$$\ln\left(ListingPrice\right)_{i,t} = \alpha + \beta_i ForRent_i + \gamma Controls_{i,t} + \epsilon_{i,t} \tag{A.2}$$

  The dependent variable, $ListingPrice$ is equal to the list-price in "for-sale" listings, and equal to the annual rent in "for-rent" listings. $ForRent_i$ is an indicator variable that is equal to one if the listing is a for-rent listing. The results are reported in Table A.5. In column (1) we control for postal code by quarter fixed effects. The estimate coefficient on β_i suggests a rent-price ratio of $e^{\beta_i} = 4.5\%$. In columns (2) - (4) we also control for other characteristics of the property, such as the property type, the number of bedrooms, bathrooms as well as the property type, size, age and the floor of the building. In columns (3) and (4) we tighten fixed effects to the month by postal

---

[27]We thank Jed Kolko and Trulia for providing these data. Trulia observes a large set of both for-sale and for-rent listings. The rent-price ratio is constructed using a MSA-level hedonic regression of log(price) on property attributes, zip code fixed effects, and a dummy for whether the unit is for sale or for rent. The rent-to-price ratio is constructed by inverting the exponent of the coefficient on this dummy variable.

code level and the month by postal code by number of bedrooms level respectively. In all specifications the estimated rent-price ratio for 2012 is 4.5%. Finally, note that if we instead used the rent-price ratio obtained from the Balance Sheet approach as a baseline estimate in 2012, we would obtain a higher total return (as the baseline in 2012 would be 6% rather than 4.5%). We choose 4.5% to be as conservative as possible.

- For England and Wales we use the house price index from the U.K. Land Registry to compute price appreciation and we use the CPI component "Actual rents for housing" (series D7CE) from the Office of National Statistics as a rental index. As a baseline we used the 6% rent-price ratio in 2012 obtained from the balance-sheet approach.

A.3.3 The Riskiness of Housing - Details

This section provides the details underlying the analysis carried out in Section 4. Section A.3.4 will provide additional evidence for the riskiness of housing. We are deeply indebted to a number of researchers, statistical agencies, and scholars that have either made their data available online, shared it with us on request, or have in general been available to discuss long-term house prices and rent behavior with us. The original sources of each series are acknowledged here and should be cited in future use of our replication dataset.

Table A.6 reports the availability of house price data and the associated financial crises and rare disasters. The second column in Table A.6 shows the time coverage of house price indices for each country. For some countries we can go back far in time; for example, we sourced data as far back as 1819 for Norway, 1890 for the U.S., and 1840 for France. The third and fourth column report the dates of any banking crises or consumption rare disasters that occur for the country in the time period provided in the first column. Banking crises dates for all countries, except Singapore, Belgium, Finland, New Zealand, South Korea, and South Africa, are from Schularick and Taylor (2012). Banking crises dates for the countries not covered by Schularick and Taylor (2012) are from Reinhart and Rogoff (2009).[28] Rare disasters dates in the last column are the year of the trough in consumption during a consumption disaster as reported by Barro and Ursua (2008). South Africa is not covered by Barro and Ursua (2008).

For each country we obtained the longest continuous and high-quality time series of house price data available. To make the data comparable across countries and time peri-

---

[28]For this second set of countries/dates, we have also consulted Bordo et al. (2001) that confirms all dates in Reinhart and Rogoff (2009) except for 1986 for South Korea and for 1989 for South Africa.

ods, we focus on real house prices at an annual frequency. Finally, to increase historical comparability across time within each time series, for each country we report for the entire time period the index for the unit of observation (for example, a city) for which the longest possible high quality time series is available. For example, since a house price index for France is only available since 1936, but a similar index is available for Paris since 1840, we focus on the Paris index for the entire history 1840-2012. We stress, however, that for each index and country we have carried out an extensive comparison analysis with other indices and in particular with indices that are available for the most recent time period in order to ensure that we are observing consistent patterns in the data. We detail here the sources for each of the 20 countries in our sample:

- **Australia**: Real annual house price indices are from Stapledon (2012). For our analysis, we use the arithmetic average of the indices (rebased such that 1880 = 100) for Melbourne and Sydney.

- **Belgium, Canada, Denmark, Finland, Germany, Japan, Italy, New Zealand, South Africa, South Korea, and Spain**: Real annual house price indices are from the Federal Reserve Bank of Dallas.[29] The sources and methodology are described in Mack and Martínez-García (2011).

- **France**: Nominal annual house price index and CPI are available from the Conseil Général de l'Environnement et du Développement Durable (CGEDD).[30] We obtain the real house price index by deflating the nominal index by CPI. For our analysis, we use the longer time series available for the Paris house price index.

- **Netherlands**: Nominal annual house price index for Amsterdam and CPI for the Netherlands are available from Eichholtz (1997) and Ambrose, Eichholtz and Lindenthal (2013).[31] We obtain the real house price index by deflating the nominal index by CPI.

- **Norway**: Nominal annual house price index and CPI are from the Norges Bank.[32] We obtain the real house price index by deflating the nominal index by CPI.

---

[29]The data are available at: http://www.dallasfed.org/institute/houseprice/, last accessed February 2014.

[30]http://www.cgedd.developpement-durable.gouv.fr/les-missions-du-cgedd-r206.html, last accessed February 2014.

[31]Part of the data are are available on Eicholtz website at: http://www.maastrichtuniversity.nl/web/Main/Sitewide/Content/EichholtzPiet.htm, last accessed February 2014.

[32]http://www.norges-bank.no/en/price-stability/historical-monetary-statistics/, last accessed February 2014.

- **Singapore**: Nominal annual house price index for the whole island is from the Urban Redevelopment Authority (http://www.ura.gov.sg). CPI is from Statistics Singapore. We obtain the real house price index by deflating the nominal index by CPI.

- **Sweden**: Nominal house price index for one-or-two-dwelling building and CPI are from Statistics Sweden. We obtain the real house price index by deflating the nominal index by CPI.

- **Switzerland**: Nominal house price index is available by Constantinescu and Francke (2013). Among the various indices the authors estimate, we focus on the local linear trend (LLT) index. The data are available for the period 1937-2007. We update the index for the period 2007-2012 by using the percentage growth of the house price index for Switzerland available from the Dallas Fed.[33] The CPI index for Switzerland is from the Office fédéral de la statistique (OFS). e obtain the real house price index by deflating the nominal index by CPI.

- **U.K.:** Annual nominal house price data are from the Nationwide House Price Index. We divide the nominal index by the U.K. Office of National Statistics "long term indicator of prices of consumer goods and services" to obtain the real house price index. The Nationwide index has a missing value for the year 2005, for that year we impute the value based on the percentage change in value of the house price index produced by the England and Wales Land Registry.

- **U.S.:** Real annual house price data are originally from Shiller (2000). Updated data are available on the author's website.[34]

For all countries the real annual consumption data are from Barro and Ursua (2008) and available on the authors' website.[35]

A.3.4 The Riskiness of Housing - Additional Evidence

In this section we provide additional evidence for the riskiness of housing to complement the analysis in Section 4.

---

[33]This source is described in the second bullet point above.

[34]Available at: http://aida.wss.yale.edu/~shiller/data.htm, last accesses February 2014.

[35]Available at: http://rbarro.com/data-sets/, last accessed February 2014.

Figure A.33 shows the time series of house prices and marks with shadowed bands years of crisis for the UK and Singapore.[36] The pattern of house price movement during crises in these two countries is similar to the average pattern described in Section 4. For example, house prices peak and then fall during major crises in the sample: the 1974-76 and 1991 banking crises in the UK, and the 1982-83 banking crisis as well as the 1997 Asian financial crisis in Singapore.[37] Similarly, both countries experience a drop in house prices during the 2007-08 global financial crisis.

Figure A.34 shows the performance of house prices during major wars, namely World War I and II (WWI and WWII). In both cases time zero is defined to be the start date of the war period, 1913 and 1939 for WWI and WWII respectively. The dotted line tracks house prices for 5 countries for the duration of WWI (1913-1918).[38] House prices fell throughout the war with a total fall in real terms close to 40%. Similarly, the solid line tracks house prices for 6 countries for the duration of WWII (1939-1945).[39] House prices fell by 20% in real terms from 1939 to 1943 and then stabilized for the last two year of the war, 1944-45. Overall we find wars to be periods of major falls in real house prices, thus contributing to the riskiness of housing as an asset.

We also provide a robustness check for the correlation between house price growth and measures of economic activity. The main body of the text focused on consumption growth in 10. By contrast, Appendix Table A.7 uses data from Mack and Martínez-García (2011) to report the correlation between annual real house price growth and personal disposable income in a panel of 20 developed and emerging countries. The average correlation is 0.37, with a minimum of 0.05 for Luxembourg and a maximum of 0.63 for Spain. Overall, this evidence further corroborates the fact that housing returns are risky.

Figure A.35 plots the growth rates of rents and personal consumption expenditures (PCE) in the U.S. since 1929. In periods of falling PCE, in particular the Great Depression, rents also fell noticeably. The bottom panel shows that there is a (weak) positive relationship between the growth rates of rents and personal consumption expenditures. This suggests that housing rents tend to increase when consumption increases and the marginal utility of consumption is low.

---

[36] All crises dates are from Reinhart and Rogoff (2009) except the periods 1997-98 and 2007-08 for Singapore. The latter dates have been added by the authors and are commonly documented to correspond to the Asian financial crisis of 1997-98 and the global financial crisis of 2007-08.

[37] The 1984 banking crisis in the UK proves the sole exception: house prices increase during this crisis.

[38] Due to data availability for house price indices during this period, the countries included are Australia, France, Netherlands, Norway, and the United States.

[39] Due to data availability for house price indices during this period, the countries included are Australia, France, Netherlands, Norway, Switzerland, and the United States.

A.4 Theoretical Appendix

A.4.1 Financing Frictions - Calibration

It is beyond the scope of this paper to provide a full general equilibrium model of housing in the presence of collateral and borrowing constraints. Instead we consider a simple deviation from the constant-discount-rate model in Section 5.1 to quantify the impact of a reduced form collateral constraint on leasehold discounts. We assume that for the last $\bar{T}$ years of lease maturity the property has lower collateral value modeled via an effective rent for the last $\bar{T}$ years that is a fraction $(1 - \alpha)$ of the true rent.[40] The value of the lease is given by:

$$
\begin{aligned}
P_t^T &= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} (1 - \alpha \mathbf{1}_{\{s > t+T-\bar{T}\}}) ds \qquad\qquad\qquad\qquad (A.3) \\
&= \int_t^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} ds - \alpha \int_{t+T-\bar{T}}^{t+T} e^{-\rho(s-t)} D_t e^{g(s-t)} ds + \\
&\quad + \mathbf{1}_{\{T<\bar{T}\}} \, \alpha \int_{t+T-\bar{T}}^{t} e^{-\rho(s-t)} D_t e^{g(s-t)} ds \\
&= \frac{D_t}{\rho - g} \left[1 - e^{-(\rho-g)T} - \alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) + \mathbf{1}_{\{T<\bar{T}\}} \, \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right) \right]
\end{aligned}
$$

Notice that the first multiplicative term in the equation above is simply the valuation of the freehold under the Gordon growth formula $\left(\frac{D_t}{\rho - g} \right)$. The first term inside the squared bracket $\left(1 - e^{-(\rho-g)T} \right)$ is the Gordon growth price adjustment for the value of a T-maturity leasehold as shown in equation (3). The second term inside the squared bracket is the loss in value for the $T > \bar{T}$ maturity leasehold due to the frictions. Notice that this term is zero whenever there are no frictions ($\alpha = 0$ and or $\bar{T} = 0$). The last term inside the squared bracket $\left(\mathbf{1}_{\{T<\bar{T}\}} \, \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right) \right)$ captures the notion that if a leasehold has already less than $\bar{T}$ years left than it would be valued at:

$$
P_t^T = \frac{D_t(1 - \alpha)}{\rho - g} (1 - e^{-(\rho-g)T}),
$$

so that the leasehold is valued as if rents were only a fraction $(1 - \alpha)$ of the original ones. Notice that the value of the freehold is unaffected by the frictions because by definition

---

[40]This loss corresponds to the per-period shadow value of liquidity (i.e., the per-period cost to the buyer of having to use own resources or a shorter duration mortgage). Alternatively, we can interpret α as the total loss in value once the leasehold reaches 60 years of remaining maturity due to the fact that new potential buyers will no longer have access to long-maturity mortgages and might need to make a larger downpayment.

it never loses its collateral value: $P_t = \lim_{T \to \infty} P_t^T = \frac{D_t}{\rho - g}$. The model-implied leasehold discount is given by:

$$Disc_t^T = e^{-(\rho-g)T} + \alpha \left(e^{-(\rho-g)(T-\bar{T})} - e^{-(\rho-g)T} \right) - \mathbf{1}_{\{T<\bar{T}\}} \, \alpha \left(e^{-(\rho-g)(T-\bar{T})} - 1 \right).$$

Let us focus on the case in which $T > \bar{T}$, i.e. if we are valuing a leasehold with maturity beyond the problematic threshold. Notice the following effects:

1. $\frac{\partial Disc_t^T}{\partial \alpha} > 0$, the discount increases the greater the per-period collateral benefit.

2. $\frac{\partial Disc_t^T}{\partial \bar{T}} > 0$, the discount increases whenever the threshold for financing increases.

3. $\frac{\partial Disc_t^T}{\partial \alpha \partial \bar{T}} < 0$ and $\lim_{T \to \infty} \frac{\partial Disc_t^T}{\partial \alpha} = 0$, the marginal effect of the loss in collateral value on the discount decreases with maturity of the lease and is zero in the limit of very long leases.

   The last property is the most relevant for our robustness exercise. It states that no matter how high the frictions are ($\uparrow \alpha$), their effect decreases with the length of the lease. This effect makes the frictions quantitatively incapable of explaining the observed discounts, especially for long term leases (125 or 200 years for example).

   To asses the quantitative implications of this friction we set $\bar{T} = 60$, we maintain the benchmark values of $r = 6.5\%$ and $g = 0.7\%$, and explore a range of values for α between 5% and 20%. A value of α of 20% implies that a 60-year leasehold is worth 20% less than it would have been in the absence of this collateral friction. Appendix Figure A.40 shows the leasehold discounts implied by such high collateral values at various horizons. Even unrealistically high assumptions on the loss of collateral value for short duration leaseholds cannot help to explain the discounts for leases of long maturities (for example 150 or 250 years). Intuitively, a lease that has 200 years left today will only incur direct losses of its collateral value 140 years from now, when the lease will have 60 years left. Any losses that occur so far into the future have little impact on present values at conventional discount rates. To illustrate this effect, we calibrate the model choosing $\alpha = 0.78$ so as to match by construction the price discount for the 80-100 year maturity bucket. Even under this calibration, there is little impact on the leasehold discounts at long maturities: 125-150 year maturity backer has a discount of 1.5% compared to the 8% estimated in the data.

A.4.2 Stochastic Discount Factor and Leasehold Discounts

Consider a claim to the risky rent at time T, denoted D_T. The present value at time t is the expected dividend $E_t[D_T]$ discounted with some discount factor $R_{t,t+T}$:

$$P_t^{D_T} = \frac{E_t[D_T]}{R_{t,t+T}}. \tag{A.4}$$

The price of a safe security that pays 1 for sure at maturity T is: $P_t^{1T} = 1/R_{t,t+T}^f$, where $R_{t,t+T}^f$ is the total return on the safe security when held to maturity. Since the rent D_T is risky, and risky cash flows are discounted at a higher rate than if they were safe, we have $R_{t,t+T} > R_{t,t+T}^F$. We can decompose $R_{t,t+T}$ into a discount factor that would be applied even if D_T were certain, and an additional discount that compensates the agents for risk, the risk premium $RP_{t,t+T}$: $R_{t,t+T} = R_{t,t+T}^f + RP_{t,t+T}$. Asset pricing theory relates the discount factors $R_{t,t+T}$ and $R_{t,t+T}^f$ to a "stochastic discount factor" (SDF) $\xi_{t,t+T}$ that reflects marginal utility in different states of the world. Assets are priced according to:

$$P_t^{D_T} = E_t[\xi_{t,t+T} D_T], \tag{A.5}$$

The values of $R_{t,t+T}$, $RP_{t,t+T}$, and $\xi_{t,t+T}$ are related via the formulas below:

$$
\begin{aligned}
R_{t,t+T}^f &= E_t[\xi_{t,t+T}]^{-1}, \\
RP_{t,t+T} &= -\frac{Cov_t[\xi_{t,t+T}, \tilde{R}_{t,t+T}]}{Var[\xi_{t,t+T}]} \frac{Var[\xi_{t,t+T}]}{E_t[\xi_{t,t+T}]} \equiv \beta_{t,t+T} \lambda_{t,t+T}.
\end{aligned}
$$

The risk-free component of the discount factor is related to the inverse of the expectation of the long-term SDF, i.e. long-term marginal utility growth. The risk premium has the opposite sign to the covariance between the stochastic discount factor and the rent, $Cov_t[\xi_{t,t+T}, D_T]$.[41] A claim that pays a higher rent in states of the world when extra resources are less valuable, i.e. when marginal utility $\xi_{t,t+T}$ is low, is less desirable and thus discounted at a higher rate. Such an asset is risky, and its risk premium is positive. The risk premium ($RP_{t,t+T}$) can be further decomposed into an asset-specific "quantity of risk" term ($\beta_{t,t+T}$), which summarizes how strongly the payoff co-varies with the stochastic discount factor, and a common "price of risk" term ($\lambda_{t,t+T}$), that only depends on $\xi_{t,t+T}$ and summarizes the compensation required for each unit of risk at that horizon.

We now provide detailed derivations. Starting with the fundamental valuation equa-

---

[41]Recall that in this case $Cov_t[\xi_{t,t+T}, \tilde{R}_{t,t+T}] = Cov_t[\xi_{t,t+T}, D_T]$ because $\tilde{R}_{t,t+T} = D_T/P_t^{D_T}$ is the stochastic return on investing in the risky asset.

tion $P_t^{D_T} = E_t[\tilde{\zeta}_{t,t+T}D_T]$ and the definition of return $\tilde{R}_{t,t+T} = \frac{D_T}{P_t^{D_T}}$, we have:

$$1 = E_t[\tilde{\zeta}_{t,t+T}\tilde{R}_{t,t+T}] = E_t[\tilde{\zeta}_{t,t+T}]E_t[\tilde{R}_{t,t+T}] + Cov_t[\tilde{\zeta}_{t,t+T}, \tilde{R}_{t,t+T}].$$

Re-arranging we obtain:

$$E_t[\tilde{R}_{t,t+T}] = E_t[\tilde{\zeta}_{t,t+T}]^{-1}\left(1 - Cov_t[\tilde{\zeta}_{t,t+T}, \tilde{R}_{t,t+T}]\right) = R_{t+t+T}^f - Cov_t[\tilde{\zeta}_{t,t+T}, \tilde{R}_{t,t+T}]E_t[\tilde{\zeta}_{t,t+T}]^{-1},$$

where the last equality follows from the definition $R_{t+t+T}^f \equiv E_t[\tilde{\zeta}_{t,t+T}]^{-1}$. Finally, we re-arrange the definition of returns, take conditional expectations, and substitute in the above derivation for expected returns to write:

$$
\begin{aligned}
P_t^{D_T} &= \frac{E_t[D_T]}{E_t[\tilde{R}_{t,t+T}]} \\
&= \frac{E_t[D_T]}{R_{t+t+T}^f - Cov_t[\tilde{\zeta}_{t,t+T}, \tilde{R}_{t,t+T}]E_t[\tilde{\zeta}_{t,t+T}]^{-1}} \\
&= \frac{E_t[D_T]}{R_{t+t+T}^f - \frac{Cov_t[\tilde{\zeta}_{t,t+T}, \tilde{R}_{t,t+T}]}{Var_t[\tilde{\zeta}_{t,t+T}]}\frac{Var_t[\tilde{\zeta}_{t,t+T}]}{E_t[\tilde{\zeta}_{t,t+T}]}},
\end{aligned}
$$

which provides the main relations by defining:

$$
\begin{aligned}
R_{t,t+T} &\equiv E_t[\tilde{R}_{t,t+T}]; \\
RP_{t,t+T} &\equiv \beta_{t,t+T}\lambda_{t,t+T}; \\
\beta_{t,t+T} &\equiv -\frac{Cov_t[\tilde{\zeta}_{t,t+T}, \tilde{R}_{t,t+T}]}{Var_t[\tilde{\zeta}_{t,t+T}]}; \\
\lambda_{t,t+T} &\equiv \frac{Var_t[\tilde{\zeta}_{t,t+T}]}{E_t[\tilde{\zeta}_{t,t+T}]}.
\end{aligned}
$$

Equation 5 can then be derived by noticing that $P_t - P_t^T$, the difference in price between the freehold and the leasehold of maturity T, is the price today of a claim to the freehold T periods from now. In brief, it is a claim today to a single payoff, P_{t+T}, T periods from now.

A.4.3 Leading Asset Pricing Models

In this appendix we consider in detail the predictions of leading asset pricing models for the evaluation of long-run risky cash flows and the implied leasehold discounts. We discuss the models briefly, focusing on the most important elements for the valuation of

long-dated claims to housing. In this section we provide an intuitive discussion of the key features of the models; in the next section, Appendix A.4.4, we report details on the derivation and calibration of the models. When calibrating the models we deviate as little as possible from the original papers' calibrations of the stochastic discount factor and cash-flows. In each model, we calibrate housing to be a risky asset with an average growth rate of rents of 0.7% and an exposure to risk that ensures an expected return of 6.5%.

In the long-run risk model of Bansal and Yaron (2004) agents have a preference for early resolution of uncertainty and are concerned about shocks that persistently affect the growth rate of consumption. Therefore, agents dislike claims to very long-term cash flows that are exposed to these long-run risks.[42] The model matches the expected return to housing only if housing is exposed to long-run risks. The model also implies that leaseholds with higher maturity are more exposed to long-run risks, and command higher risk premia.[43] This upward sloping term structure of risk premia contributes to generating smaller discounts for leaseholds relative to freeholds compared to the constant-discount-rate model. Appendix Figure A.39, which separates out risk-free yields and annualized risk premia, confirms the analysis of Hansen, Heaton and Li (2008) who show that the result is driven by risk premia that increase with maturity.[44]

In the external habit model of Campbell and Cochrane (1999) agents care about their surplus consumption relative to a habit level, which itself depends on the history of aggregate consumption. Negative shocks to consumption, with which rents are correlated, induce increases in risk premia because they bring current consumption closer to the habit level. Long-term claims, due to their high duration, are particularly exposed to these shocks and are therefore particularly risky. The model implies an upward sloping term structure of risk premia that contributes to generate low discounts for leaseholds compared to freeholds. Appendix Figure A.39 confirms this intuition by showing that while

---

[42]Dew-Becker and Giglio (2013) show that half the total price of risk in the long-run risk model comes from fluctuations in consumption with cycles longer than 230 years and three quarters of the risk prices come from fluctuations longer than 75 years. These horizons correspond closely to the maturities of the leaseholds we consider in this paper.

[43]A similar result obtains when agents have power utility but are ambiguity averse, as in Hansen and Sargent (2001). When the agent has a preference for robustness, she can be viewed as having a reference distribution for the relevant shocks (the true distribution) and a worst-case distribution, which is what she uses to price assets. Under the worst-case distribution she places relatively more weight on bad states of the world, which correspond to states with persistently low consumption growth. Therefore, the model has similar asset pricing implications to the long-run risk model.

[44]We calibrate the yield $r^f_{t,T}$ and the total per-period discount rate for a T-maturity asset r_t using the model, and report $rp_{t,T}$ as the difference between the two. As mentioned in the previous section, this per-period $rp_{t,T}$ does not have the interpretation of an expected return, which would correspond to the expectation of a holding period return for one period.

risk-free yields are constant across maturities, risk premia are increasing in maturity.

In the rare disasters model of Barro (2006) and Gabaix (2012) consumption growth is subject to rare but large negative shocks, the disasters. Agents dislike assets that are exposed to these disasters. While the presence of rare disasters increases risk premia, it does so uniformly across maturities because claims to cash flows at all horizons are equally exposed to the disaster risk. Therefore, discount rates will be the same at all horizons and equal to the average return. These high constant-discount rates produce leasehold price discounts compared to freeholds similar to those of the constant-discount-rate model.

Appendix Figure A.38 shows the discounts for long-dated leaseholds relative to freeholds implied by these three leading models together with those observed in the data. In all cases, the models produce discounts similar, or even smaller, to the constant-discount-rate model discussed in Section 5.1.

A.4.4 Model appendix - Algebra

This section reports the main equations used to compute the price of dividend strips in the main asset pricing models. In each case, the discount for a group of years remaining (say, 80 to 100) is obtained by first computing the prices of dividend strips in the model P_t^i, claims to dividends at maturity i, then computing the price of leaseholds with T years remaining as the sum of the prices of all dividend strips up to maturity T, and finally averaging the discount with respect to the infinite claim across the group of maturities (for example, 80 to 100 years).

A.4.4.1 Long-Run Risk

Setup of the model

Following Bansal and Yaron (2004) and Beeler and Campbell (2012), the setup of the model is:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_t \eta_{t+1}$$

$$x_{t+1} = \rho x_t + \phi_e \sigma_t e_{t+1}$$

$$\sigma_{t+1}^2 = \bar{\sigma}^2 + v(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1}$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \phi_d \sigma_t u_{t+1}$$

All shocks are standard normal iid shocks, all uncorrelated. The representative agent has

Epstein-Zin utility, so its SDF is determined as:

$$m_{t+1} = \theta ln\delta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{a,t+1}$$

Applying the Campbell-Shiller decomposition to the total return of wealth (see Beeler and Campbell (2012)) we obtain:

$$r_{a,t+1} = k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1}$$

where z_t is the log wealth-consumption ratio, and k_0 and k_1 are loglinearization parameters. Defining z_t^m as the log price-dividend ratio of the risky asset (here: housing), we similarly have:

$$r_{m,t+1} = k_0^m + k_1^m z_{t+1}^m - z_t^m + \Delta d_{t+1}$$

Calibration

We follow the same (monthly) calibration as in Bansal and Yaron (2004), with $\phi_e = 0.044$, $\phi_d = 4.5$, $\bar{\sigma} = 0.0078$, $\mu_c = 0.0015$, $v = 0.987$, $\delta = 0.998$, $\rho = 0.979$, $\sigma_w = 0.23 \cdot 10^{-5}$, $\gamma = 10$, $\psi = 1.5$. However, we choose $\mu_d = 0.007/12$, $\phi_d = 3$ and $\phi = 2.4$ to match the dividend growth and average return of the housing asset.

Solution of the model

Bansal and Yaron (2004) show that we can write:

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2$$

$$z_t^m = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2$$

for parameters $A_0, A_1, A_2, A_{0,m}, A_{1,m}, A_{2,m}$. As reported in Beeler and Campbell (2012), these parameters satisfy:

$$A_0 = \frac{ln\delta + \mu_c(1 - \frac{1}{\psi}) + k_0 + k_1 A_2\bar{\sigma}^2(1 - v) + \frac{1}{2}\theta k_1^2 A_2^2\sigma_w^2}{(1 - k_1)}$$

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - k_1\rho}$$

$$A_2 = \frac{\frac{1}{2}\left[(\theta - \frac{\theta}{\psi})^2 + (\theta k_1 A_1\phi_e)^2\right]}{\theta(1 - vk_1)}$$

A.40

while:

$$A_{0,m} = \frac{1}{1 - k_{1,m}} [\theta \ln \delta + \mu_c(\theta - \frac{\theta}{\psi} - 1) - (1 - \theta)A_2 k_1 \bar{\sigma}^2 (1 - v) + (\theta - 1)(k_0 + A_0(k_1 - 1))$$

$$+ k_{0,m} + k_{1,m} A_{2,m} \bar{\sigma}^2 (1 - v) + \mu_d + \frac{1}{2}(k_{1,m}A_{2,m} - (1 - \theta)A_2 k_1)^2 \sigma_w^2]$$

$$A_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - k_{1,m}\rho}$$

$$A_{2,m} = \frac{(1 - \theta)A_2(1 - k_1 v) + \frac{1}{2}\left[\gamma^2 + (k_{1,m}A_{1,m}\phi_e - (1 - \theta)k_1 A_1 \phi_e)^2 + \phi_d^2\right]}{(1 - k_{1,m}v)}$$

When solving for these parameters, it is important to remember that the parameters A_0, A_1, A_2 depend on k_0 and k_1 parameters, and viceversa. So, the five parameters A_0, A_1, A_2, k_0 and k_1 need to be solved for jointly: k_0 and k_1 need to satisfy the loglinearization equations:

$$k_1 = \frac{exp(\bar{z})}{1 + exp(\bar{z})}$$

$$k_0 = ln(1 + exp(\bar{z})) - k_1\bar{z}$$

where

$$\bar{z} = A_0 + A_2\bar{\sigma}^2$$

Therefore, we have 5 equations in 5 unknowns. The same applies to finding $A_{0,m}, A_{1,m}$ and $A_{2,m}$, which depend on $k_{0,m}$ and $k_{1,m}$. The additional equations in that case are:

$$k_{1,m} = \frac{exp(\bar{z}_m)}{1 + exp(\bar{z}_m)}$$

$$k_{0,m} = ln(1 + exp(\bar{z}_m)) - k_{1,m}\bar{z}_m$$

where

$$\bar{z}_m = A_{0,m} + A_{2,m}\bar{\sigma}^2$$

These parameters allow us to derive a simple expression for the innovation in the SDF, see Bansal and Yaron (2004), $m_{t+1} - E_t m_{t+1}$:

$$m_{t+1} - E_t m_{t+1} = -\gamma \sigma_t \eta_{t+1} - (1 - \theta)k_1 A_1 \phi_e \sigma_t e_{t+1} - (1 - \theta)A_2 k_1 \sigma_w w_{t+1}$$

A.41

as well as the mean and variance of the SDF: $E_t m_{t+1}$ and $V_t m_{t+1}$:

$$E_t m_{t+1} = \theta ln\delta - \frac{\theta}{\psi} E_t \Delta c_{t+1} + (\theta - 1) E_t r_{a,t+1}$$

$$= \theta ln\delta - \frac{\theta}{\psi}(\mu_c + x_t) + (\theta - 1)(k_0 + k_1 E_t z_{t+1} - z_t + E_t \Delta c_{t+1})$$

$$= \theta ln\delta - \frac{\theta}{\psi}(\mu_c + x_t) + (\theta - 1)(k_0 + k_1(A_0 + A_1 \rho x_t + A_2(\bar{\sigma}^2 + v(\sigma_t^2 - \bar{\sigma}^2))))$$

$$-(A_0 + A_1 x_t + A_2 \sigma_t^2) + \mu_c + x_t)$$

$$= \theta ln\delta - \frac{\theta}{\psi}\mu_c - \frac{\theta}{\psi}x_t + (\theta - 1)k_0 + (\theta - 1)k_1 A_0 + (\theta - 1)k_1 A_1 \rho x_t + (\theta - 1)k_1(1 - v)A_2\bar{\sigma}^2$$

$$+(\theta - 1)k_1 v A_2 \sigma_t^2 - (\theta - 1)A_0 - (\theta - 1)A_1 x_t - (\theta - 1)A_2 \sigma_t^2 + (\theta - 1)\mu_c + (\theta - 1)x_t$$

$$= \theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta - 1)k_0 + (\theta - 1)k_1 A_0 + (\theta - 1)k_1(1 - v)A_2\bar{\sigma}^2 - (\theta - 1)A_0 + (\theta - 1)\mu_c$$

$$-\frac{\theta}{\psi}x_t + (\theta - 1)k_1 A_1 \rho x_t - (\theta - 1)A_1 x_t + (\theta - 1)x_t$$

$$+(\theta - 1)k_1 v A_2 \sigma_t^2 - (\theta - 1)A_2 \sigma_t^2$$

So:

$$E_t m_{t+1} = a_0^m + a_1^m x_t + a_2^m \sigma_t^2$$

with:

$$a_0^m = \theta ln\delta - \frac{\theta}{\psi}\mu_c + (\theta - 1)k_0 + (\theta - 1)k_1 A_0 + (\theta - 1)k_1(1 - v)A_2\bar{\sigma}^2 - (\theta - 1)A_0 + (\theta - 1)\mu_c$$

$$a_1^m = -\frac{\theta}{\psi} + (\theta - 1)k_1 A_1 \rho - (\theta - 1)A_1 + (\theta - 1)$$

$$a_2^m = (\theta - 1)k_1 v A_2 - (\theta - 1)A_2$$

Similarly,

$$V_t m_{t+1} = V_t[-\gamma\sigma_t \eta_{t+1} - (1 - \theta)k_1 A_1 \phi_e \sigma_t e_{t+1} - (1 - \theta)A_2 k_1 \sigma_w w_{t+1}]$$

$$= \gamma^2 \sigma_t^2 + (1 - \theta)^2 k_1^2 A_1^2 \phi_e^2 \sigma_t^2 + (1 - \theta)^2 k_1^2 A_2^2 \sigma_w^2$$

$$= b_0^m + b_2^m \sigma_t^2$$

with:

$$b_0^m = (1 - \theta)^2 k_1^2 A_2^2 \sigma_w^2$$

$$b_2^m = \gamma^2 + (1 - \theta)^2 k_1^2 A_1^2 \phi_e^2$$

Pricing dividend strips

The prices of dividend strips can be found recursively and are also exponentially affine:

$$pd_t^n = A_0^{d(n)} + A_1^{d(n)} x_t + A_2^{d(n)} \sigma_t^2$$

where pd_t^n is the log price dividend ration of the n-maturity strip. The one-month dividend strip satisfies:

$$PD_t^1 = E_t(exp(m_{t+1} + \Delta d_{t+1})) = exp\{E_t m_{t+1} + \frac{1}{2} V_t m_{t+1} + E_t \Delta d_{t+1} + \frac{1}{2} V_t \Delta d_{t+1} + Cov_t(\Delta d_{t+1}, m_{t+1})\}$$

Now,

$$E_t \Delta d_{t+1} = \mu_d + \phi x_t$$

$$V_t \Delta d_{t+1} = \phi_d^2 \sigma_t^2$$

$$Cov_t(\Delta d_{t+1}, m_{t+1}) = 0$$

Therefore:

$$PD_t^1 = exp\{a_0^m + a_1^m x_t + a_2^m \sigma_t^2 + \frac{1}{2}(b_0^m + b_2^m \sigma_t^2) + \mu_d + \phi x_t + \frac{1}{2} \phi_d^2 \sigma_t^2\}$$

So:

$$A_{0,d}(1) = a_0^m + \frac{1}{2} b_0^m + \mu_d$$

$$A_{1,d}(1) = a_1^m + \phi$$

$$A_{2,d}(1) = a_2^m + \frac{1}{2} b_2^m + \frac{1}{2} \phi_d^2$$

We can find all other maturities recursively:

$$PD_t^n = exp\{A_{0,d}(n) + A_{1,d}(n) x_t + A_{2,d}(n) \sigma_t^2\}$$

and

$$PD_t^{n+1} = E_t(exp(m_{t+1}) PD_{t+1}^n \frac{D_{t+1}}{D_t}))$$

Therefore:

$$PD_t^{n+1} = exp\{E_t m_{t+1} + \frac{1}{2}V_t m_{t+1} + E_t\left\{\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2\right\}$$

$$+\frac{1}{2}V_t\left\{\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2\right\}$$

$$+Cov_t(\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2, m_{t+1})\}$$

Now,

$$\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2 =$$

$$\mu_d + \phi x_t + \phi_d \sigma_t u_{t+1} + A_{0,d}(n) + A_{1,d}(n)(\rho x_t + \phi_e \sigma_t e_{t+1}) + A_{2,d}(n)(\bar{\sigma}^2 + v(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1})$$

so:

$$E_t\left\{\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2\right\} =$$

$$= \mu_d + \phi x_t + A_{0,d}(n) + A_{1,d}(n)\rho x_t + A_{2,d}(n)(v\sigma_t^2 + (1-v)\bar{\sigma}^2)$$

and

$$V_t\left\{\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2\right\} =$$

$$= \phi_d^2 \sigma_t^2 + A_{1,d}(n)^2 \phi_e^2 \sigma_t^2 + A_{2,d}(n)^2 \sigma_w^2$$

and

$$Cov_t(\Delta d_{t+1} + A_{0,d}(n) + A_{1,d}(n)x_{t+1} + A_{2,d}(n)\sigma_{t+1}^2, m_{t+1}) =$$

$$Cov_t(\phi_d \sigma_t u_{t+1} + A_{1,d}(n)\phi_e \sigma_t e_{t+1} + A_{2,d}(n)\sigma_w w_{t+1},$$

$$-\gamma \sigma_t \eta_{t+1} - (1-\theta)k_1 A_1 \phi_e \sigma_t e_{t+1} - (1-\theta)A_2 k_1 \sigma_w w_{t+1})$$

$$= -(1-\theta)k_1 A_1 \phi_e A_{1,d}(n)\phi_e \sigma_t^2 - A_{2,d}(n)(1-\theta)A_2 k_1 \sigma_w^2$$

Putting all together:

$$pd_t^{n+1} = log(PD_t^{n+1}) = a_0^m + a_1^m x_t + a_2^m \sigma_t^2 + \frac{1}{2}(b_0^m + b_2^m \sigma_t^2)$$

$$+\mu_d + \phi x_t + A_{0,d}(n) + A_{1,d}(n)\rho x_t + A_{2,d}(n)(v\sigma_t^2 + (1-v)\bar{\sigma}^2)$$

$$+\frac{1}{2}\phi_d^2 \sigma_t^2 + \frac{1}{2}A_{1,d}(n)^2 \phi_e^2 \sigma_t^2 + \frac{1}{2}A_{2,d}(n)^2 \sigma_w^2$$

$$-(1-\theta)k_1 A_1 \phi_e A_{1,d}(n)\phi_e \sigma_t^2 - A_{2,d}(n)(1-\theta)A_2 k_1 \sigma_w^2$$

Or:

$$A_{0,d}(n+1) = a_0^m + \frac{1}{2}b_0^m + \mu_d + A_{0,d}(n) + A_{2,d}(n)(1-v)\bar{\sigma}^2 + \frac{1}{2}A_{2,d}(n)^2\sigma_w^2 - A_{2,d}(n)(1-\theta)A_2 k_1 \sigma_w^2$$

$$A_{1,d}(n+1) = a_1^m + \phi + A_{1,d}(n)\rho$$

$$A_{2,d}(n+1) = a_2^m + \frac{1}{2}b_2^m + A_{2,d}(n)v + \frac{1}{2}\phi_d^2 + \frac{1}{2}A_{1,d}(n)^2\phi_e^2 - (1-\theta)k_1 A_1 \phi_e^2 A_{1,d}(n)$$

Dividend strip prices at the steady state can be computed by setting $x_t = 0$ and $\sigma_t^2 = \bar{\sigma}^2$.

Term structure of real interest rates

The risk-free rate in this economy is, see [Bansal and Yaron (2004)](#):

$$r_{f,t+1} = -\ln\delta + \frac{1}{\psi}E_t\Delta c_{t+1} + \frac{(1-\theta)}{\theta}E_t[r_{a,t+1} - r_{f,t}] - \frac{1}{2\theta}V_t(m_{t+1})$$

Now,

$$E_t\Delta c_{t+1} = \mu_c + x_t$$

$$E_t[r_{a,t+1} - r_{f,t}] = -\frac{1}{2}V_t(r_{a,t+1}) + \gamma\sigma_t^2 + (1-\theta)k_1^2 A_1^2 \phi_e^2 \sigma_t^2 + (1-\theta)k_1^2 A_2^2 \sigma_w^2$$

$$V_t(m_{t+1}) = b_0^m + b_2^m \sigma_t^2$$

We can find $V_t(r_{a,t+1})$ as:

$$V_t(r_{a,t+1}) = V_t(k_0 + k_1 z_{t+1} - z_t + \Delta c_{t+1}) = V_t(k_1 z_{t+1} + \Delta c_{t+1})$$

$$= V_t(k_1 A_0 + k_1 A_1 x_{t+1} + k_1 A_2 \sigma_{t+1}^2 + \Delta c_{t+1})$$

$$= V_t(k_1 A_1 \phi_e \sigma_t e_{t+1} + k_1 A_2 \sigma_w w_{t+1} + \sigma_t \eta_{t+1})$$

$$= (k_1^2 A_1^2 \phi_e^2 + 1)\sigma_t^2 + k_1^2 A_2^2 \sigma_w^2$$

So we can write:

$$E_t[r_{a,t+1} - r_{f,t}] = -\frac{1}{2}(k_1^2 A_1^2 \phi_e^2 + 1)\sigma_t^2 - \frac{1}{2}k_1^2 A_2^2 \sigma_w^2 + \gamma\sigma_t^2 + (1-\theta)k_1^2 A_1^2 \phi_e^2 \sigma_t^2 + (1-\theta)k_1^2 A_2^2 \sigma_w^2$$

$$= -\frac{1}{2}(k_1^2 A_1^2 \phi_e^2 + 1)\sigma_t^2 + \gamma\sigma_t^2 + (1-\theta)k_1^2 A_1^2 \phi_e^2 \sigma_t^2 + (\frac{1}{2} - \theta)k_1^2 A_2^2 \sigma_w^2$$

Putting everything together:

$$r_{f,t+1} = -\ln\delta + \frac{1}{\psi}\mu_c + \frac{1}{\psi}x_t$$

$$+\frac{(1-\theta)}{\theta}[-\frac{1}{2}(k_1^2 A_1^2 \phi_e^2 + 1)\sigma_t^2 + \gamma\sigma_t^2 + (1-\theta)k_1^2 A_1^2 \phi_e^2 \sigma_t^2 + (\frac{1}{2} - \theta)k_1^2 A_2^2 \sigma_w^2]$$

$$-\frac{1}{2\theta}Var_t(m_{t+1})$$

or:

$$r_{f,t+1} = -\ln\delta + \frac{1}{\psi}\mu_c + \frac{1}{\psi}x_t$$

$$-\frac{1}{2}\frac{(1-\theta)}{\theta}(k_1^2 A_1^2 \phi_e^2 + 1)\sigma_t^2 + \frac{(1-\theta)}{\theta}\gamma\sigma_t^2 + \frac{(1-\theta)^2}{\theta}k_1^2 A_1^2 \phi_e^2 \sigma_t^2 + \frac{(1-\theta)}{\theta}(\frac{1}{2} - \theta)k_1^2 A_2^2 \sigma_w^2$$

$$-\frac{1}{2\theta}b_0^m - \frac{1}{2\theta}b_2^m \sigma_t^2$$

$$= -C_0(1) - C_1(1)x_t - C_2(1)\sigma_t^2$$

with:

$$C_0(1) = \ln\delta - \frac{1}{\psi}\mu_c - \frac{(1-\theta)}{\theta}(\frac{1}{2} - \theta)k_1^2 A_2^2 \sigma_w^2 + \frac{1}{2\theta}b_0^m$$

$$C_1(1) = -\frac{1}{\psi}$$

$$C_2(1) = \frac{1}{2}\frac{(1-\theta)}{\theta}(k_1^2 A_1^2 \phi_e^2 + 1) - \frac{(1-\theta)}{\theta}\gamma - \frac{(1-\theta)^2}{\theta}k_1^2 A_1^2 \phi_e^2 + \frac{1}{2\theta}b_2^m$$

The log price of a 1-period zero coupon bond is:

$$p_{1,t} = -r_{f,t+1} = C_0(1) + C_1(1)x_t + C_2(1)\sigma_t^2$$

Recursively,

$$p_{n+1,t} = E_t(m_{t+1} + p_{n,t+1}) + \frac{1}{2}V_t(m_{t+1} + p_{n,t+1})$$

$$= E_t m_{t+1} + \frac{1}{2}V_t m_{t+1} + E_t p_{n,t+1} + \frac{1}{2}V_t p_{n,t+1} + Cov_t(p_{n,t+1}, m_{t+1})$$

Now,

$$p_{n,t+1} = C_0(n) + C_1(n)x_{t+1} + C_2(n)\sigma_{t+1}^2 =$$

$$C_0(n) + C_1(n)(\rho x_t + \phi_e \sigma_t e_{t+1}) + C_2(n)(\bar{\sigma}^2 + v(\sigma_t^2 - \bar{\sigma}^2) + \sigma_w w_{t+1})$$

We have:

$$E_t \left\{ C_0(n) + C_1(n)x_{t+1} + C_2(n)\sigma_{t+1}^2 \right\} = C_0(n) + C_1(n)\rho x_t + C_2(n)(v\sigma_t^2 + (1-v)\bar{\sigma}^2)$$

$$V_t \left\{ C_0(n) + C_1(n)x_{t+1} + C_2(n)\sigma_{t+1}^2 \right\} = C_1(n)^2 \phi_e^2 \sigma_t^2 + C_2(n)^2 \sigma_w^2$$

$$Cov_t(C_0(n) + C_1(n)x_{t+1} + C_2(n)\sigma_{t+1}^2, m_{t+1}) =$$

$$Cov_t(C_1(n)\phi_e \sigma_t e_{t+1} + C_2(n)\sigma_w w_{t+1},$$

$$-\gamma \sigma_t \eta_{t+1} - (1-\theta)k_1 A_1 \phi_e \sigma_t e_{t+1} - (1-\theta)A_2 k_1 \sigma_w w_{t+1})$$

$$= -(1-\theta)k_1 A_1 \phi_e^2 C_1(n)\sigma_t^2 - C_2(n)(1-\theta)A_2 k_1 \sigma_w^2$$

Therefore:

$$p_{n+1,t} = a_0^m + a_1^m x_t + a_2^m \sigma_t^2 + \frac{1}{2}b_0^m + \frac{1}{2}b_2^m \sigma_t^2$$

$$+ C_0(n) + C_1(n)\rho x_t + C_2(n)(v\sigma_t^2 + (1-v)\bar{\sigma}^2)$$

$$+ \frac{1}{2}C_1(n)^2 \phi_e^2 \sigma_t^2 + \frac{1}{2}C_2(n)^2 \sigma_w^2$$

$$- (1-\theta)k_1 A_1 \phi_e^2 C_1(n)\sigma_t^2 - C_2(n)(1-\theta)A_2 k_1 \sigma_w^2$$

So:

$$C_0(n+1) = a_0^m + \frac{1}{2}b_0^m + C_0(n) + C_2(n)(1-v)\bar{\sigma}^2 + \frac{1}{2}C_2(n)^2 \sigma_w^2 - C_2(n)(1-\theta)A_2 k_1 \sigma_w^2$$

$$C_1(n+1) = a_1^m + C_1(n)\rho$$

$$C_2(n+1) = a_2^m + \frac{1}{2}b_2^m + C_2(n)v + \frac{1}{2}C_1(n)^2 \phi_e^2 - (1-\theta)k_1 A_1 \phi_e^2 C_1(n)$$

Per-period discount rate

To compute the per-period discount, we need to compute the expectation of dividend growth, since r^n is that discount rate such that:

$$PD_t^n = \frac{E_t[\frac{D_{t+n}}{D_t}]}{(1+r_n)^n}$$

So

$$(1+r_n)^n = \frac{E_t[\frac{D_{t+n}}{D_t}]}{PD_t^n}$$

A.47

We obtain the discount rates r for each maturity by simulating the model and obtaining average dividend growth and dividend strip price for each horizon.

A.4.4.2 Lettau and Wachter (2007) Model

Setup

Call ϵ_{t+1} a 3x1 vector of independent standard normal shocks. Call D_t aggregate dividends (rents) in the economy, and call $d_t = log D_t$. Assume that:

$$\Delta d_{t+1} = g + z_t + \sigma_d \epsilon_{t+1}$$

with

$$z_{t+1} = \phi_z z_t + \sigma_z \epsilon_{t+1}$$

The stochastic discount factor (SDF) in the economy is:

$$M_{t+1} = exp\{-r^f - \frac{1}{2}x_t^2 - x_t \epsilon_{d,t+1}\}$$

with

$$\epsilon_{d,t+1} = \frac{\sigma_d}{||\sigma_d||}\epsilon_{t+1}$$

and x_t follows:

$$x_{t+1} = (1 - \phi_x)\bar{x} + \phi_x x_t + \sigma_x \epsilon_{t+1}$$

Here, $\sigma_d, \sigma_z, \sigma_x$ are row vectors.

Calibration

Following Lettau and Wachter (2007), we choose the parameters as follows (at the quarterly level): $g = 0.0228/4$, $r_f = 0.0193/4$, $\bar{x} = 0.625$, $\sigma_z = [-0.00136, 0.0009, 0]$, $\sigma_x = [0, 0, 0.12]$, $\phi_x = 0.87^{(1/4)}$. To calibrate the housing market, we set $\phi_z = 0.865^{(1/4)}$, $\sigma_d = [0.04, 0, 0]$, $g = 0.0032/4$. This calibration implies an average growth rate of dividends of around 0.7% per year, an expected return of the freehold 4%, and the expected return of a one-year dividend strip of 12%.

Solution and prices of dividend strips

As shown in Lettau and Wachter (2007), the price-dividend ratio of dividend strips of maturity n follows:

$$PD_t^n = \frac{P_t^n}{D_t} = exp\{A(n) + B_x(n)x_t + B_z(n)z_t\}$$

with:

$$A(0) = B_x(0) = B_z(0) = 0$$

and

$$A(n) = A(n-1) - r^f + g + B_x(n-1)(1-\phi_x)\bar{x} + 0.5V_{n-1}V'_{n-1}$$

$$B_x(n) = B_x(n-1)(\phi_x - \sigma_x\frac{\sigma'_d}{||\sigma_d||}) - (\sigma_d + B_z(n-1)\sigma_z)\frac{\sigma'_d}{||\sigma_d||}$$

$$B_z(n) = \frac{1-\phi_z^n}{1-\phi_z}$$

$$V_{n-1} = \sigma_d + B_z(n-1)\sigma_z + B_x(n-1)\sigma_x$$

Note that $\frac{\sigma'_d}{||\sigma_d||}$ is just [1,0,0] given the baseline calibration.
The risk premium of each dividend strip can be found as:

$$E_t(R_{n,t+1} - R^f) \simeq E_t[r_{n,t+1} - r^f] + \frac{1}{2}V_t(r_{n,t+1}) = (\sigma_d + B_x(n-1)\sigma_x + B_z(n-1)\sigma_z)\frac{\sigma'_d}{||\sigma_d||}x_t$$

which also directly gives us the expected return on each strip. The expected return of the freehold can be found as a weighted average of the expected returns of the strips:

$$E_t(R_{t+1}^{fh}) = \sum_{n=1}^{\infty}\left[\frac{PD_t^n}{PD_t^{fh}}E_t(R_{t+1}^n)\right]$$

Per-period discount rates

To compute the per-period discount, we need to compute the expectation of dividend growth, since r^n is that discount rate such that:

$$PD_t^n = \frac{E_t[\frac{D_{t+n}}{D_t}]}{(1+r_n)^n}$$

A.49

Now,

$$E_t[\frac{D_{t+n}}{D_t}] = E_t[exp\{\sum_{j=1}^{n} \Delta d_{t+j}\}]$$

$$= E_t[exp\{(g + z_t + \sigma_d \epsilon_{t+1})$$

$$+ (g + z_{t+1} + \sigma_d \epsilon_{t+2})$$

$$+ (g + z_{t+2} + \sigma_d \epsilon_{t+3})$$

$$+ (g + z_{t+3} + \sigma_d \epsilon_{t+4})$$

$$...$$

$$+ (g + z_{t+n-1} + \sigma_d \epsilon_{t+n})\}]$$

$$= E_t[exp\{(g + z_t + \sigma_d \epsilon_{t+1})$$

$$+ (g + (\phi_z z_t + \sigma_z \epsilon_{t+1}) + \sigma_d \epsilon_{t+2})$$

$$+ (g + (\phi_z z_{t+1} + \sigma_z \epsilon_{t+2}) + \sigma_d \epsilon_{t+3})$$

$$+ (g + (\phi_z z_{t+2} + \sigma_z \epsilon_{t+3}) + \sigma_d \epsilon_{t+4})$$

$$+ ...$$

$$= E_t[exp\{(g + z_t + \sigma_d \epsilon_{t+1})$$

$$+ (g + (\phi_z z_t + \sigma_z \epsilon_{t+1}) + \sigma_d \epsilon_{t+2})$$

$$+ (g + (\phi_z(\phi_z z_t + \sigma_z \epsilon_{t+1}) + \sigma_z \epsilon_{t+2}) + \sigma_d \epsilon_{t+3})$$

$$+ (g + (\phi_z(\phi_z z_{t+1} + \sigma_z \epsilon_{t+2}) + \sigma_z \epsilon_{t+3}) + \sigma_d \epsilon_{t+4})$$

$$+ ...$$

$$= E_t[exp\{(g + z_t + \sigma_d \epsilon_{t+1})$$

$$+ (g + (\phi_z z_t + \sigma_z \epsilon_{t+1}) + \sigma_d \epsilon_{t+2})$$

$$+ (g + (\phi_z(\phi_z z_t + \sigma_z \epsilon_{t+1}) + \sigma_z \epsilon_{t+2}) + \sigma_d \epsilon_{t+3})$$

$$+ (g + (\phi_z(\phi_z(\phi_z z_t + \sigma_z \epsilon_{t+1}) + \sigma_z \epsilon_{t+2}) + \sigma_z \epsilon_{t+3}) + \sigma_d \epsilon_{t+4})$$

$$+ ...$$

$$= exp\{gn\}E_t[exp\{(\sigma_d + \sigma_z + \phi_z \sigma_z + ... + \phi_z^{n-2}\sigma_z)\epsilon_{t+1}$$

A.50

$$+(\sigma_d + \sigma_z + \phi_z\sigma_z + ... + \phi_z^{n-3}\sigma_z)\epsilon_{t+2}$$

$$+...$$

$$+(\sigma_d + \sigma_z)\epsilon_{t+n-1}$$

$$+(\sigma_d)\epsilon_{t+n}$$

$$+(1 + \phi_z + \phi_z^2 + ... + \phi_z^{n-1})z_t\}]$$

Since this is a geometric sum, we can rewrite:

$$(\sigma_d + \sigma_z + \phi_z\sigma_z + ... + \phi_z^{n-2}\sigma_z) = (\sigma_d + \sigma_z(1 + \phi_z + ... + \phi_z^{n-2})) = (\sigma_d + \sigma_z\frac{(1 - \phi_z^{n-1})}{1 - \phi_z})$$

$$(\sigma_d + \sigma_z + \phi_z\sigma_z + ... + \phi_z^{n-3}\sigma_z) = (\sigma_d + \sigma_z\frac{(1 - \phi_z^{n-2})}{1 - \phi_z})$$

and so on, so we get:

$$= exp\{gn\}E_t[exp\{(\sigma_d + \sigma_z\frac{(1 - \phi_z^{n-1})}{1 - \phi_z})\epsilon_{t+1}$$

$$+(\sigma_d + \sigma_z\frac{(1 - \phi_z^{n-2})}{1 - \phi_z})\epsilon_{t+2}$$

$$+...$$

$$+(\sigma_d + \sigma_z)\epsilon_{t+n-1}$$

$$+(\sigma_d)\epsilon_{t+n}$$

$$+\frac{(1 - \phi_z^n)}{1 - \phi_z}z_t\}]$$

The latter term can be brought out and set at its long-run level (0):

$$= exp\{gn\}E_t[exp\{(\sigma_d + \sigma_z\frac{(1 - \phi_z^{n-1})}{1 - \phi_z})\epsilon_{t+1}$$

$$+(\sigma_d + \sigma_z\frac{(1 - \phi_z^{n-2})}{1 - \phi_z})\epsilon_{t+2}$$

$$+...$$

$$+(\sigma_d + \sigma_z)\epsilon_{t+n-1}$$

$$+(\sigma_d)\epsilon_{t+n}\}]$$

A.51

We then compute the variance term:

$$V\left[(\sigma_d + \sigma_z \frac{(1 - \phi_z^{n-1})}{1 - \phi_z})\epsilon_{t+1} + (\sigma_d + \sigma_z \frac{(1 - \phi_z^{n-2})}{1 - \phi_z})\epsilon_{t+2} + (\sigma_d + \sigma_z)\epsilon_{t+n-1} + (\sigma_d)\epsilon_{t+n}\right]$$

But the ϵ are all independent, so the variance reduces to:

$$V_n = \sum_{i=1}^{n} \tilde{V}_i$$

where:

$$\tilde{V}_1 = \sigma_d \sigma_d'$$

$$\tilde{V}_2 = (\sigma_d + \sigma_z)(\sigma_d + \sigma_z)'$$

$$\tilde{V}_3 = (\sigma_d + \sigma_z \frac{(1 - \phi_z^2)}{1 - \phi_z})(\sigma_d + \sigma_z \frac{(1 - \phi_z^2)}{1 - \phi_z})'$$

$$\tilde{V}_4 = (\sigma_d + \sigma_z \frac{(1 - \phi_z^3)}{1 - \phi_z})(\sigma_d + \sigma_z \frac{(1 - \phi_z^3)}{1 - \phi_z})'$$

$$\tilde{V}_n = (\sigma_d + \sigma_z \frac{(1 - \phi_z^{n-1})}{1 - \phi_z})(\sigma_d + \sigma_z \frac{(1 - \phi_z^{n-1})}{1 - \phi_z})'$$

Therefore:

$$E_t[\frac{D_{t+n}}{D_t}] = E_t[exp\{\sum_{j=1}^{n} \Delta d_{t+j}\}]$$

$$= exp\{gn + 0.5V_n\}$$

and

$$PD_t^n = \frac{exp\{gn + 0.5V_n\}}{(1 + r_n)^n}$$

or:

$$r_n = \frac{exp\{g + 0.5V_n/n\}}{\sqrt[n]{PD_t^n}} - 1$$

A.4.4.3 External Habit Formation

Setup

The model is calibrated following the procedure outlined in Wachter (2006). Calling C_t the representative agent's consumption, S_t the consumption surplus, and γ the curvature parameter, the stochastic discount factor (SDF) in the Campbell and Cochrane (1999)

model can be written as:

$$M_{t+1} = \delta \left(\frac{S_{t+1}}{S_t} \right)^{-\gamma} \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

Denote $s_t = log(S_t)$, $c_t = log(C_t)$. The two variables follow the process:

$$\Delta c_{t+1} = g + v_{t+1}$$

$$s_{t+1} = (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)(c_{t+1} - c_t - g)$$

$$= (1 - \phi)\bar{s} + \phi s_t + \lambda(s_t)v_{t+1}$$

$$v_{t+1} \sim N(0, \sigma_v^2)$$

The function $\lambda(s_t)$ is chosen to ensure that the risk-free rate is constant; in particular:

$$\lambda(s_t) = (1/\bar{S})\sqrt{1 - 2(s_t - \bar{s})} - 1$$

$$\lambda(s_t) = 0 \quad s_t > s_{max}$$

where

$$\bar{S} = \sigma_v \sqrt{\frac{\gamma}{1 - \phi}}$$

$$\bar{s} = log(\bar{S})$$

and

$$s_{max} = \bar{s} + \frac{1}{2}(1 - \bar{S}^2)$$

Substituting in the expression for the SDF, we obtain:

$$M_{t+1} = \delta exp\{-\gamma g - \gamma(s_{t+1} - s_t + v_{t+1})\}$$

Noting that:

$$s_{t+1} - s_t = (1 - \phi)(\bar{s} - s_t) + \lambda(s_t)v_{t+1}$$

we can rewrite:

$$M_{t+1} = \delta exp\{-\gamma g - \gamma(1 - \phi)(\bar{s} - s_t)\}exp\{-\gamma \lambda(s_t)v_{t+1} - \gamma v_{t+1}\}$$

Finally, log rent d_t is assumed to follow the process:

$$\Delta d_{t+1} = g_d + w_{t+1}$$

$$w_{t+1} = N(0, \sigma_w^2)$$

A.53

$$Corr_t(w_{t+1}, v_{t+1}) = \rho$$

Parameters

Following Campbell and Cochrane (1999), the parameters of the model (calibrated at the yearly frequency) are: $g = 0.0189$; $\sigma_v = 0.015$; $\sigma_w = 0.112$; $\phi = 0.87$; $\gamma = 2$; $\delta = 0.9$. When calibrating the housing asset, we will set $g_d = 0.007$ and $\rho = 0.27$.

Solution

We solve the model on a grid for S_t, obtained by dividing the interval 0 to S_{max} (included) in 101 equally-spaced points, and adding to this a lower segment. The lower segment consists of 100 points logarithmically spaced between the lowest point in the upper segment and -300.

The price of the claim to a one-period dividend is solved first as:

$$P_t^1 = E_t[M_{t+1}D_{t+1}]$$

Equivalently, the price-dividend ratio $PD_t^1 \equiv \frac{P_t^1}{D_t}$ can be solved as:

$$PD_t^1 = E_t[M_{t+1}\frac{D_{t+1}}{D_t}]$$

$$= E_t[\delta exp\{-\gamma g - \gamma(1-\phi)(\bar{s} - s_t)\}exp\{-\gamma\lambda(s_t)v_{t+1} - \gamma v_{t+1})\}exp\{g_d + w_{t+1}\}]$$

This step solves pd_t^1 as a function of the state variable s_t along the grid.

The expectation is solved numerically using multivariate Gaussian quadrature over a support of $20\sigma_v$ and $20\sigma_w$.

The price-dividend ratio for all other dividend strips can be obtained recursively as:

$$PD_t^n = E_t[M_{t+1}PD_{t+1}^{n-1}\frac{D_{t+1}}{D_t}]$$

$$= \delta exp\{(1-\gamma)g - \gamma(1-\phi)(\bar{s} - s_t)\}E_t[exp\{-\gamma\lambda(s_t)v_{t+1} - \gamma v_{t+1} + w_{t+1}\}PD_{t+1}^{n-1}]$$

again solved by multivariate Gaussian quadrature. We plot price-dividend ratios for s_t at its mean, $\bar{s}$.

The per-period discount rate for maturity n is given by the r_n such that:

$$P_t^n = \frac{E_t[D_{t+n}]}{(1+r_n)^n}$$

A.54

or equivalently:

$$PD_t^n = \frac{E_t\left[\frac{D_{t+n}}{D_t}\right]}{(1+r_n)^n} = \frac{E_t[exp\{\sum_{i=1}^n \Delta d_{t+i}\}]}{(1+r_n)^n} = \frac{exp\{\sum_{i=1}^n g_d + \frac{\sigma_d^2}{2}\}}{(1+r_n)^n} = \frac{exp\{n(g_d + \frac{\sigma_d^2}{2})\}}{(1+r_n)^n}$$

Therefore,

$$r_n = \frac{exp\{(g_d + \frac{\sigma_d^2}{2})\}}{\sqrt[n]{PD_t^n}} - 1$$

The expected return on the infinitely-maturity asset is computed by simulating the model 50,000 times using the calibration above.

Finally, the $\lambda(s_t)$ is chosen such that the risk-free rate is constant and given by:

$$r^f = -ln\delta + \gamma g - \frac{1}{2}(1-\phi)\gamma$$

The yield curve is flat and all yields are equal to r^f.

A.4.4.4   Rare disasters

Setup

The setup is a standard power utility model with risk aversion γ and time discount rate ρ. Consumption growth follows:

$$\frac{C_{t+1}}{C_t} = e^{g_c} \times \begin{cases} 1 & no\ disaster \\ B_{t+1} & disaster \end{cases}$$

where B_{t+1} captures the severity of the disaster.

The pricing kernel is:

$$\frac{M_{t+1}}{M_t} = e^{-\delta} \times \begin{cases} 1 & no\ disaster \\ B_{t+1}^{-\gamma} & disaster \end{cases}$$

with $\delta = \rho + \gamma g_c$.

Dividends (rents) follow:

$$\frac{D_{i,t+1}}{D_{it}} = e^{g_{iD}}(1 + e_{i,t+1}^D) \times \begin{cases} 1 & no\ disaster \\ F_{i,t+1} & disaster \end{cases}$$

where e_{it}^D is a mean-zero shock independent of the disaster and with variance σ_D^2. F_{it} is

the recovery rate in case of disaster.

Many of the main moments can be expressed in terms of "resiliens" H_{it} of an asset i, defined as:

$$H_{it} = p_t E_t^D [B_{t+1}^{-\gamma} F_{i,t+1} - 1]$$

where p_t is the disaster probability and E^D is the expectation conditional on a disaster happening.

The model directly specifies the process for resilience, a "twisted AR" process:

$$\hat{H}_{it+1} = \frac{1 + H_{i*}}{1 + H_{it}} e^{-\phi_H} \hat{H}_{it} + \epsilon_{it+1}^H$$

with ϵ^H a zero-mean shock uncorrelated with ϵ^D and the disaster event, and with variance σ_H^2. Here,

$$\hat{H}_{it} \equiv H_{it} - H_{i*}$$

where H_{i*} is the mean of H_{it}.

The price-dividend ratio of a n-period strip is, see Gabaix (2012), Online Appendix:

$$PD_t^n = e^{-\delta_i n}(1 + \frac{1 - e^{-\phi_H n}}{\phi_H} \hat{H}_{it})$$

while the stock (freehold) price-dividend ratio is:

$$PD_t^{fh} = \frac{1}{\delta_i}(1 + \frac{\hat{H}_{it}}{\delta_i + \phi_H})$$

with

$$\delta^i = \delta - g_{iD} - h_{i*}$$

$$h_{i*} = ln(1 + H_{i*})$$

At the long-run mean, dividend strips and freehold prices are:

$$PD_t^n = e^{-\delta_i n}$$

$$PD_t^{fh} = \frac{1}{\delta_i}$$

Expected stock (freehold) returns are:

$$r_{it}^e = (1 - p_t)(\delta - H_{it}) - p_t E_t^D [1 - F_{i,t+1}]$$

A.56

At the long-run (typical) values, we get:

$$r_i^e = (1-p)(\delta - H_{i*}) - p(1 - F_{i*})$$

while conditional on no disaster is:

$$r_i^e = \delta - H_{i*}$$

The risk-free rate is set as:

$$r_f = \delta - p_t E_t^D[B_{t+1}^{-\gamma}] + p_t$$

and is modeled to be constant, so that the yield curve is flat.

Calibration

Our calibration is close to the one in Gabaix (2012): $p = 3.63\%$, $E[B^{-\gamma}] = 5.29$, $F_{i*} = 0.66$, $\delta = 16.6\%$, $H_{i*} = 9\%$. To match the observed growth rate of rents, we set $g = 0.7\%$.

Dividend strips and implied discount rates

The price dividend ratio of a strip at the long-run mean is:

$$PD^n = e^{-\delta_i n}$$

The long-run growth rate of dividends is:

$$E[\frac{D_{t+1}}{D_t}] = e^{g_{iD}}[pF_{i*} + (1-p)] = exp\{g_{iD} + log(pF_{i*} + (1-p))\}$$

So:

$$PD^n = e^{-\delta_i n} = exp\{n(g_{iD} + log(pF_{i*} + (1-p)) - r_n\}$$

so:

$$e^{r_n} = exp\{n(g_{iD} + log(pF_{i*} + (1-p)))\}/exp\{\delta_i n\}$$

$$= exp\{n(g_{iD} + log(pF_{i*} + (1-p)) + \delta_i)\}$$

$$\frac{r_n}{n} = \delta_i + g_{iD} + log(pF_{i*} + (1-p)) = \delta - h_{i*} + log(pF_{i*} + (1-p))$$

$$= \delta - log(1 + H_{i*}) + log(1 - p(1 - F_{i*})) \simeq r_i^e$$

to a first-order approximation, for small p.

A.4.5 Details on Hyperbolic-Exponential Discounting

We include here details for the derivations in Section 6.1 of the paper. First, let us focus on a model of pure hyperbolic discounting. In continuous time, the hyperbolic discount function is simply $\frac{1}{1+\kappa s}$ where $\kappa > 0$ is the subjective hyperbolic parameter. To gather intuition, assume that rents were constant at D. Let us value the T lease contract. For simplicity consider the $t = 0$ starting condition.

$$P_t^T = \int_0^T \frac{1}{1+\kappa s} Dds = D\frac{\ln(1+\kappa\,T)}{\kappa}.$$

The obvious problem with this type of discounting when applied to longer term assets is that the valuation of claims diverges (even without dividend growth) as the horizon T increases ($T \to \infty$).

In the paper, therefore, we augmented the hyperbolic discount function to include an exponential term: $\frac{e^{-\rho s}}{1+\kappa s}$, where $\rho > 0$ is the subjective discount rate associated with exponential discounting. This form of discounting tends to behave like hyperbolic discounting in the short run and like exponential discounting in the long run. Since the long-run discount rate approaches ρ, finite prices for long-run securities in the presence of cash-flow growth g are guaranteed by $\rho > g$. The T-maturity leasehold is valued at:

$$P_0^T = \int_0^T \frac{e^{-(\rho-g)s}}{1+\kappa s} D_0 ds = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \left(Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)\right)}{\kappa},$$

where $Ei(x)$ is the Exponential Integral function defined as:

$$Ei(x) \equiv -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt.$$

The freehold is correspondingly valued at:

$$P_0 = D_0 \frac{e^{\frac{\rho-g}{\kappa}} \Gamma\left(0, \frac{\rho-g}{\kappa}\right)}{\kappa},$$

where $\Gamma(x)$ is the Upper Incomplete Gamma Function defined as:[45]

$$\Gamma(0, x) \equiv \int_x^{\infty} \frac{e^{-t}}{t} dt.$$

---

[45]Notice $\Gamma(0, x) = -Ei(-x)$.

A.58

The discount is now:

$$Disc_0^T = \frac{Ei\left(\frac{(T\kappa+1)(g-\rho)}{\kappa}\right) - Ei\left(\frac{g-\rho}{\kappa}\right)}{\Gamma\left(0, \frac{\rho-g}{\kappa}\right)} - 1.$$

The per-period equivalent constant discount rate r_T for horizon T solves $e^{-r_T T} = R_{0,T} = \frac{e^{-\rho T}}{1+\kappa T}$, and is hence obtained via the formula:

$$r_T = \rho + \frac{\ln(1+\kappa T)}{T}.$$

This is the formula reported in the main text. Notice that we also have $\lim_{T\downarrow 0} r_T = \rho + \kappa$ and $\lim_{T\to\infty} r_T = \rho$.[46] So that total discount rates start at $\rho + \kappa$ and then decay over the horizon to ρ.

Similarly, marginal discount rates $r(s)$ can be derived by defining the discount function as $R_{0,T} = exp\left(-\int_0^T r(s)ds\right)$. Then an application of Leibniz's rule for differentiation under the integral sign yields: $\dot{R}_{0,T} = -r(T)R_{0,T}$, where $\dot{R}_{0,T}$ is the time derivative of function $R_{0,T}$. Hence, we have the result hat $r(T) = -\frac{\dot{R}_{0,T}}{R_{0,T}}$. Finally, applying this formula to the exponential-hyperbolic discount function, $R_{0,T} = \frac{e^{-\rho T}}{1+\kappa T}$, one obtains the result:

$$r(T) = -\frac{\dot{R}_{0,T}}{R_{0,T}} = \rho + \frac{\kappa}{1+\kappa t}.$$

Marginal discount rates are therefore monotonically decreasing from $\rho + \kappa$ to ρ. We conclude, therefore, that r_T are also monotonically decreasing.

We next derive the expected, instantaneous returns to the freehold. Before deriving the expression for the current hyperbolic-exponential model, we report the derivation for the simple Gordon growth exponential model. In the main draft we have argued that the constant discount rate r of the Gordon growth model should be calibrated to the average return of the freehold. We confirm here that this logic is correct. The return on the freehold is given by:

$$\frac{dP_t + D_t dt}{P_t}.$$

In the Gordon growth environment capital gains are $\frac{dP_t}{P_t} = gdt$. This can be derived recalling that $P_t = \frac{D_t}{r-g} = \frac{D_0 e^{gt}}{r-g}$ and taking the time derivative. The rental yield is $\frac{D_t}{P_t} =$

---

[46]The first limit follows from an application of l'Hopital's rule.

$r - g$. We conclude that total returns on the freehold in the Gordon growth model are:

$$\frac{dP_t + D_t dt}{P_t} = gdt + (r - g)dt = rdt.$$

We now derive the formula for expected returns to the freehold in our hyperbolic-exponential model by analogy with the Gordon growth model derivation above. The capital gains in our hyperbolic-exponential model are $\frac{dP_t}{P_t} = gdt$. This can be derived by recalling that $P_t = D_t \frac{e^{\frac{\rho - g}{\kappa}} \Gamma\left(0, \frac{\rho - g}{\kappa}\right)}{\kappa} = D_0 e^{gt} \frac{e^{\frac{\rho - g}{\kappa}} \Gamma\left(0, \frac{\rho - g}{\kappa}\right)}{\kappa}$, and taking the time derivative. The rental yield is $\frac{D_t}{P_t} = \frac{\kappa}{e^{\frac{\rho - g}{\kappa}} \Gamma\left(0, \frac{\rho - g}{\kappa}\right)}$. We conclude that total returns on the freehold in the hyperbolic-exponential model are:

$$\frac{dP_t + D_t dt}{P_t} = gdt + \frac{\kappa}{e^{\frac{\rho - g}{\kappa}} \Gamma\left(0, \frac{\rho - g}{\kappa}\right)} dt.$$

If $\kappa = 0$ then the return to the freehold is simply ρ, and we are back to the exponential discounting model. An increase in κ for a given ρ has the following comparative statics: the returns to the freehold increase, short term discount rates increase, long-term discount rates are unchanged, and leasehold discounts (*Disc*) increase in absolute value. These dynamics are precisely what allow the reduced-form hyperbolic-exponential model to reconcile the long-run valuation pattern.

Appendix References

Ambrose, Brent W, Piet Eichholtz, and Thies Lindenthal. 2013. "House prices and fundamentals: 355 years of evidence." *Journal of Money, Credit and Banking*, 45(2-3): 477–491.

Badarinza, Cristian, and Tarun Ramadorai. 2014. "Long-Run Discounting: Evidence from the UK Leasehold Valuation Tribunal." *Social Science Research Network Working Paper Series*.

Bansal, Ravi, and Amir Yaron. 2004. "Risks for the long run: A potential resolution of asset pricing puzzles." *Journal of Finance*, 59(4): 1481–1509.

Barro, Robert J. 2006. "Rare disasters and asset markets in the twentieth century." *Quarterly Journal of Economics*, 121(3): 823–866.

Barro, Robert J., and Jose F. Ursua. 2008. "Macroeconomic Crises since 1870." *Brookings Papers on Economic Activity*, 39(1 (Spring): 255–350.

Beeler, Jason, and John Y Campbell. 2012. "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment." *Critical Finance Review*, 1(1): 141–182.

Bordo, Michael, Barry Eichengreen, Daniela Klingebiel, and Maria Soledad Martinez-Peria. 2001. "Is the crisis problem growing more severe?" *Economic policy*, 16(32): 51–82.

Burn, Edward Hector, John Cartwright, and Geoffrey C. Cheshire. 2011. *Cheshire and Burn's Modern Law of Real Property.* Oxford University Press.

Campbell, John Y., and John H. Cochrane. 1999. "By Force of Habit: A Consumption Based Explanation of Aggregate Stock Market Behavior." *Journal of Political Economy*, 107(2): 205–251.

Campbell, Sean D, Morris A Davis, Joshua Gallin, and Robert F Martin. 2009. "What moves housing markets: A variance decomposition of the rent–price ratio." *Journal of Urban Economics*, 66(2): 90–102.

Cocco, Joao F. 2005. "Portfolio choice in the presence of housing." *Review of Financial studies*, 18(2): 535–567.

Constantinescu, Mihnea, and Marc Francke. 2013. "The historical development of the Swiss rental market - A new price index." *Journal of Housing Economics*, 22(2): 135 – 145.

Dew-Becker, Ian, and Stefano Giglio. 2013. "Asset pricing in the frequency domain: theory and empirics." *Unpublished manuscript, University of Chicago*.

Díaz, Antonia, and María José Luengo-Prado. 2008. "On the user cost and homeownership." *Review of Economic Dynamics*, 11(3): 584–613.

Eichholtz, Piet. 1997. "A long run house price index: The Herengracht index, 1628–1973." *Real Estate Economics*, 25(2): 175–192.

Favilukis, Jack, Sydney C. Ludvigson, and Stijn Van Nieuwerburgh. 2010. "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium." National Bureau of Economic Research Working Paper 15988.

Flavin, Marjorie, and Takashi Yamashita. 2002. "Owner-occupied housing and the composition of the household portfolio." *American Economic Review*, 92(1): 345–362.

Gabaix, Xavier. 2012. "Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance." *Quarterly Journal of Economics*, 127(2): 645–700.

Gyourko, Joseph, and Donald B. Keim. 1992. "What does the stock market tell us about real estate returns?" *Real Estate Economics*, 20(3): 457–485.

Hansen, Lars P., and Thomas J. Sargent. 2001. "Robust control and model uncertainty." *American Economic Review*, 91: 60–66.

Hansen, Lars Peter, John C Heaton, and Nan Li. 2008. "Consumption strikes back? Measuring long-run risk." *Journal of Political Economy*, 116(2): 260–302.

Lettau, Martin, and Jessica A. Wachter. 2007. "Why Is Long-Horizon Equity Less Risky? A Duration-Based Explanation of the Value Premium." *Journal of Finance*, 62(1): 55–92.

Lustig, Hanno N., and Stijn Van Nieuwerburgh. 2005. "Housing collateral, consumption

insurance, and risk premia: An empirical perspective." *Journal of Finance*, 60(3): 1167–1219.

Mack, Adrienne, and Enrique Martínez-García. 2011. "A cross-country quarterly database of real house prices: a methodological note." *Federal Reserve Bank of Dallas Globalization and Monetary Policy Institute Working Paper*, , (99).

Malpezzi, Stephen, Larry Ozanne, and Thomas G Thibodeau. 1987. "Microeconomic estimates of housing depreciation." *Land Economics*, 63(4): 372–385.

Mayerhauser, Nicole, and Marshall Reinsdorf. 2006. "Housing Services in the National Economic Accounts." *US Bureau of Economic Analysis*.

McCarthy, Jonathan, and Richard Peach. 2010. "The measurement of rent inflation." *FRB of New York Staff Report*, , (425).

McMichael, Stanley L. 1921. *Long Term Land Leaseholds: Including Ninety-nine Year Leases.* SL McMichael.

Piazzesi, Monika, Martin Schneider, and Selale Tuzel. 2007. "Housing, consumption and asset pricing." *Journal of Financial Economics*, 83(3): 531–569.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2009. *This time is different: Eight centuries of financial folly.* Princeton University Press.

Royal Institute of Chartered Surveyors. 2009. "Leasehold Reform: Graphs of Relativity." *Research Report*.

Schularick, Moritz, and Alan M. Taylor. 2012. "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 18702008." *American Economic Review*, 102(2): 1029–1061.

Shiller, Robert. 2000. *Irrational Exuberance.* Princeton University Press, Princeton.

Shiller, Robert J. 2006. "Long-Term Perspectives on the Current Boom in Home Prices." *The Economists' Voice*, 3(4): 1–11.

Shiller, Robert J. 2007. "Understanding recent trends in house prices and home ownership." National Bureau of Economic Research.

Stapledon, Nigel. 2012. "Trends and Cycles in Sydney and Melbourne House Prices from 1880 to 2011." *Australian Economic History Review*, 52(3): 293–317.

The Observer. 2013. "Beware the 'cheaper' leasehold option that could cost more in the long run."

Appendix Figures

Figure A.1: Distribution of years remaining at lease extension
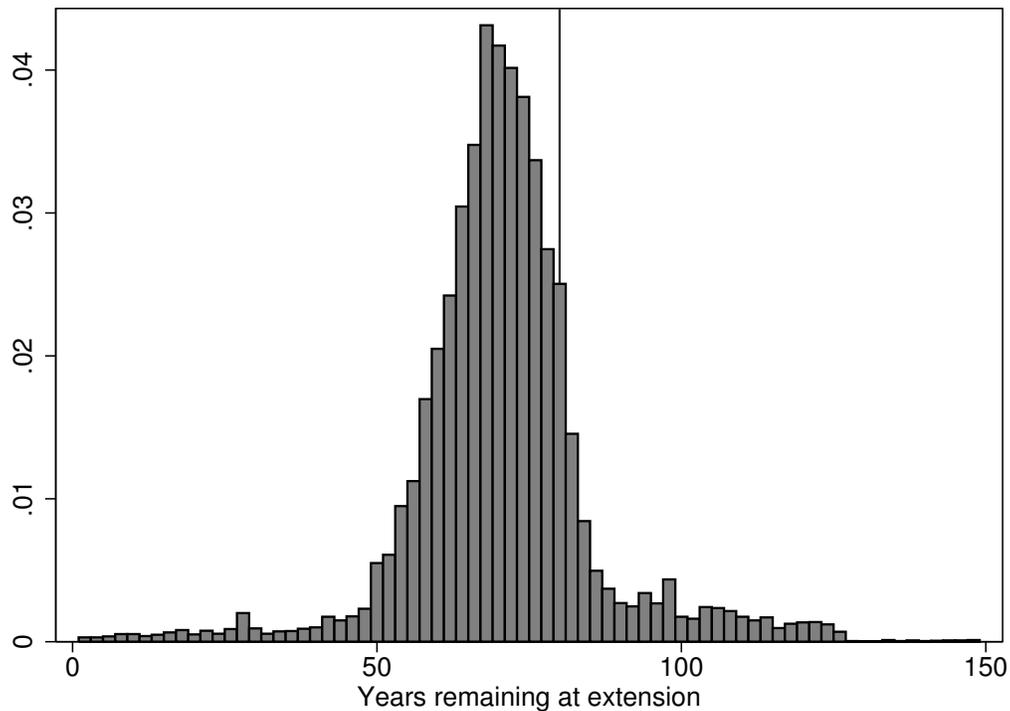
Note: The figure shows the distribution of the number of years remaining at the time of a lease extension for the subsample of 21,974 properties for which we can identify the time of the extension. The extensions are identified by finding all cases in which the same property transacts at least twice, in which different contracts appear across the various transactions. The years remaining at extension are the number of years remaining on the oldest contract as of the starting date of the next contract. The vertical bar corresponds to 80 years remaining.

Figure A.2: Distribution of covenants by initial lease length
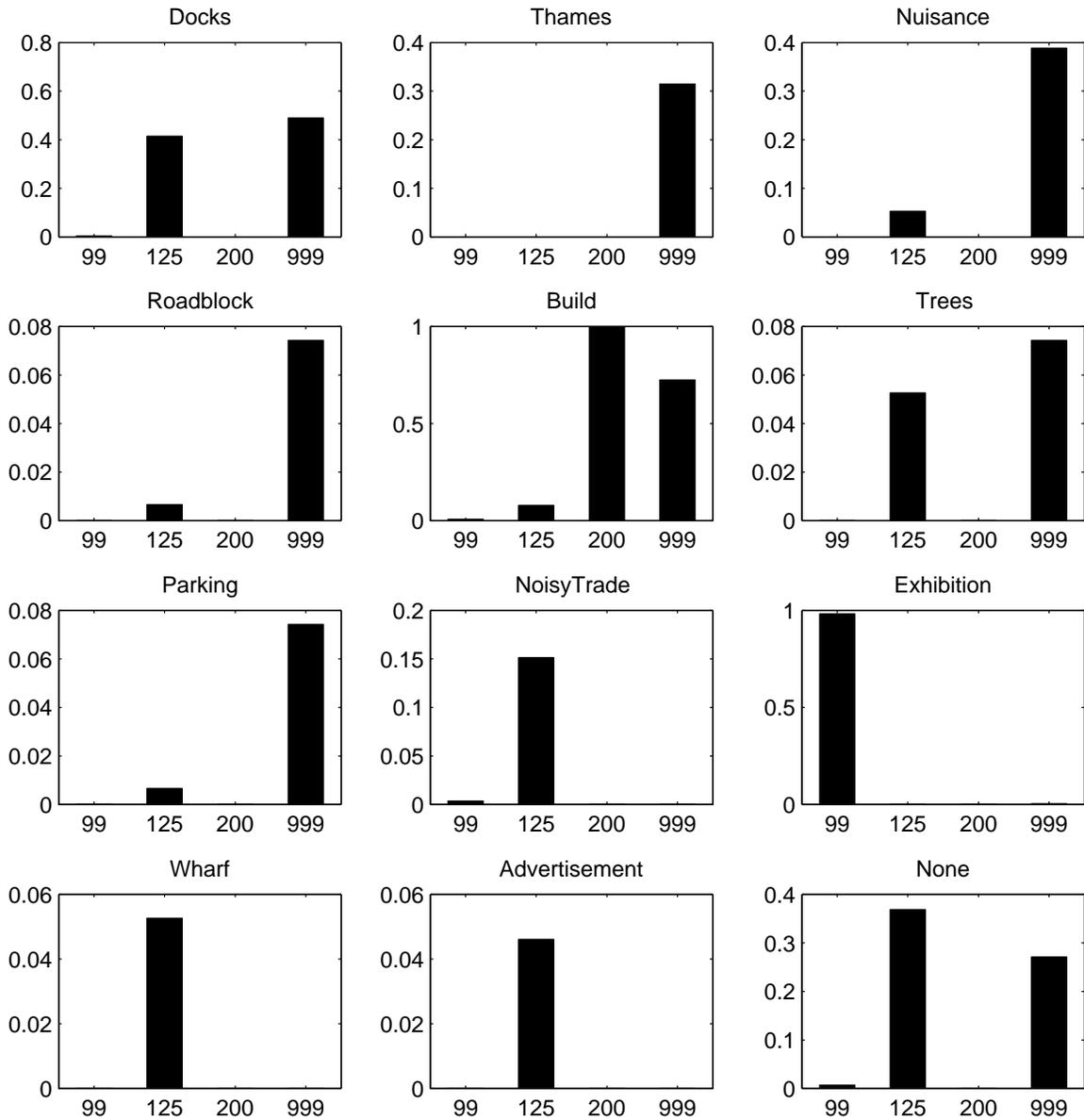
Note: The figure shows the share of leasehold contracts with a certain initial lease length in postcode E16 with the relevant covenant, defined in Appendix Section A.1.7.1. 273 transactions were of leaseholds with an initial length of 99 years, 152 with an initial length of 125 years, 147 with an initial length of 200 years and 229 with an initial length of 999 years.

Figure A.3: U.K. Flats: Fraction of Freeholds
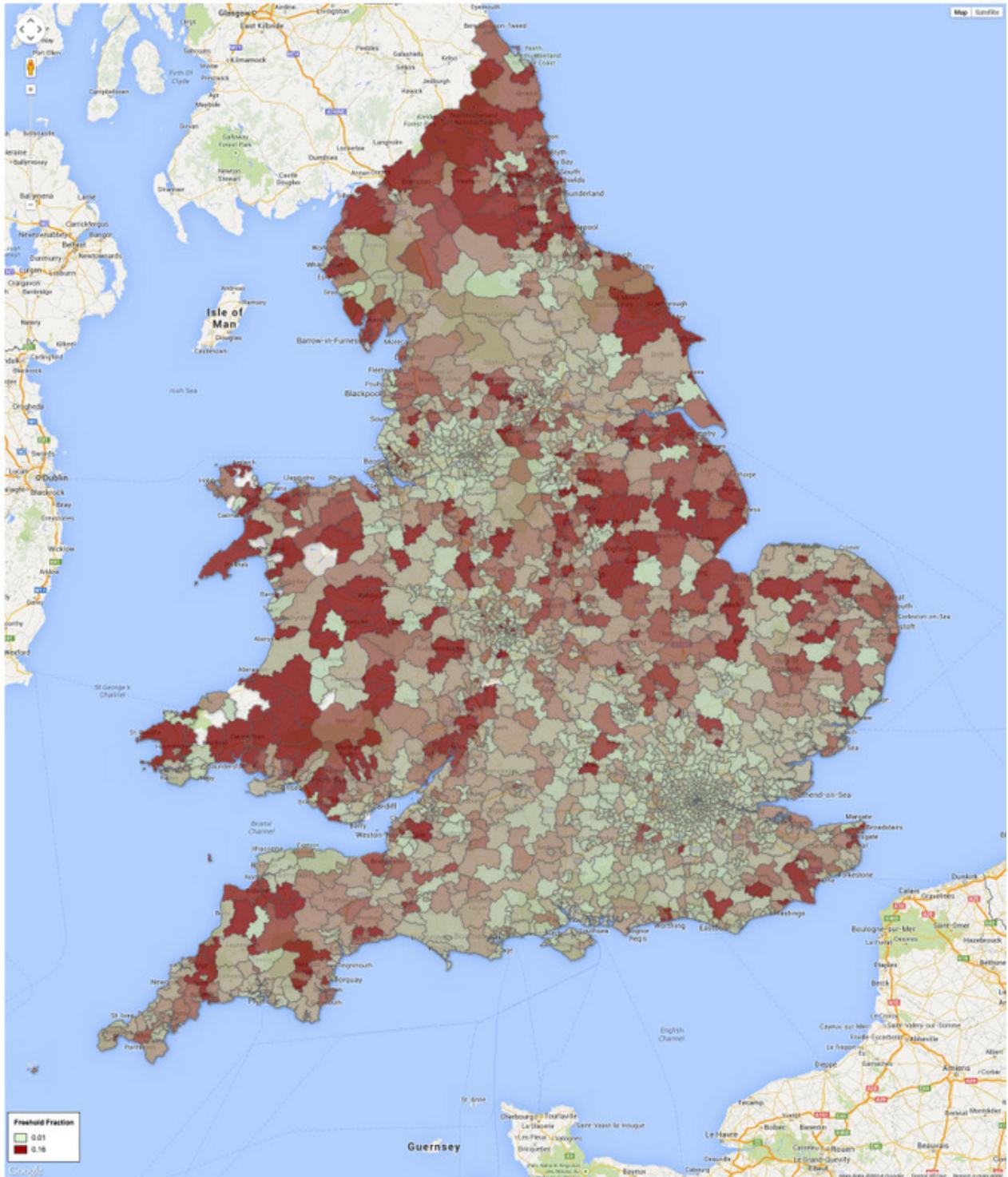
Note: The figure shows the fraction of freehold flats in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.4: London Flats: Fraction of Freeholds
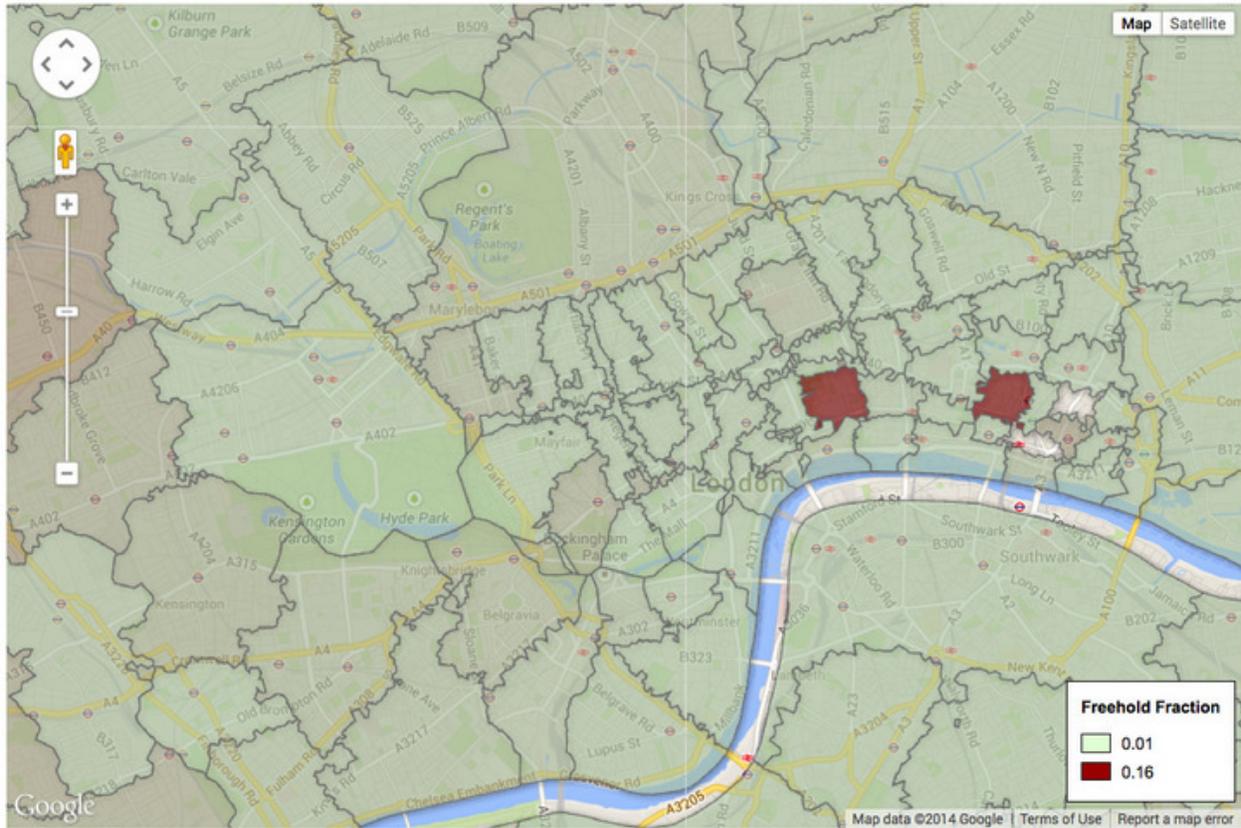
**Note:** The figure shows the fraction of freehold flats in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

Figure A.5: U.K. Flats: Fraction of 80-99 years leaseholds
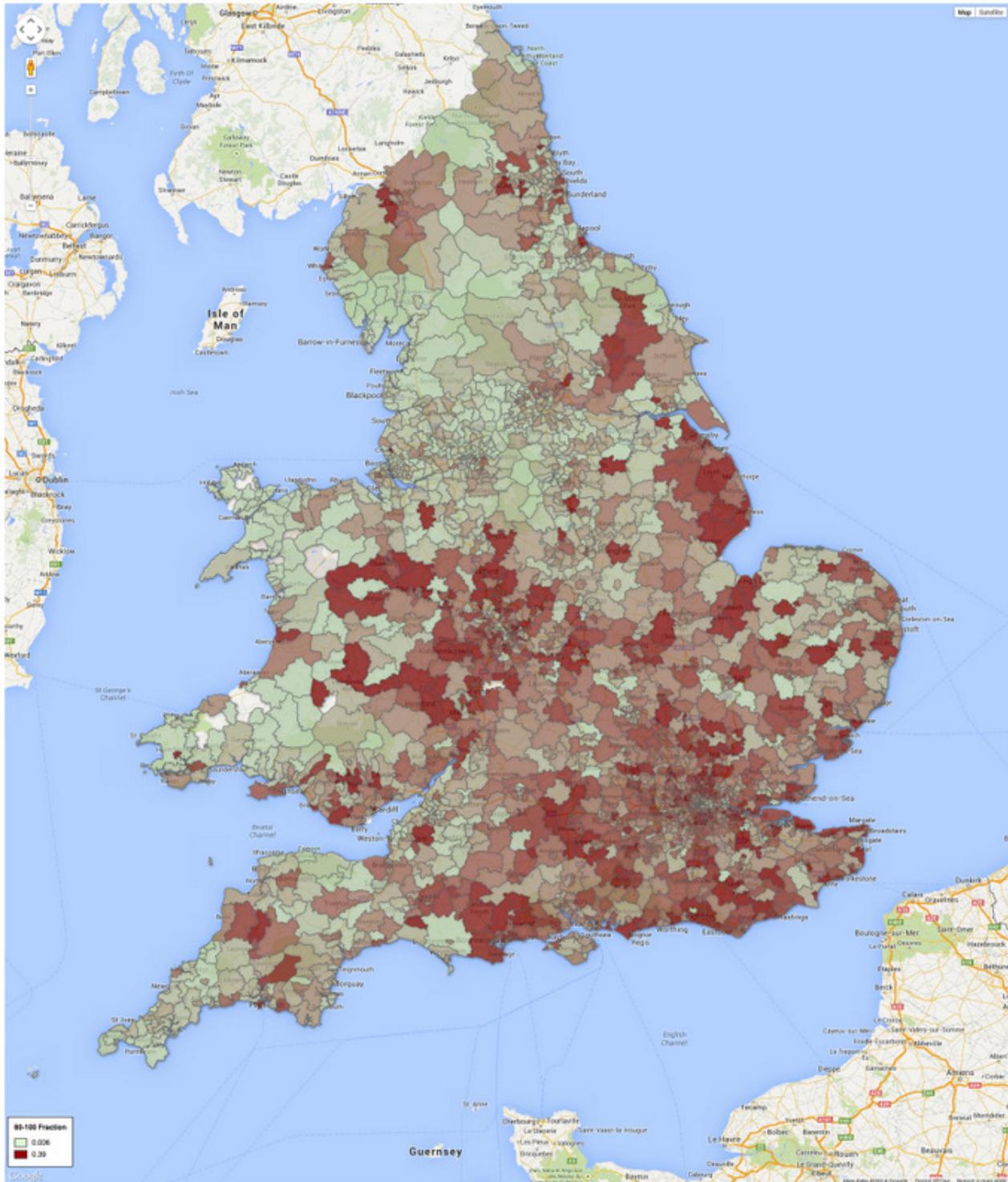
Note: The figure shows the fraction of flat transactions with 80-99 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.6: London Flats: Fraction of 80-99 years leaseholds
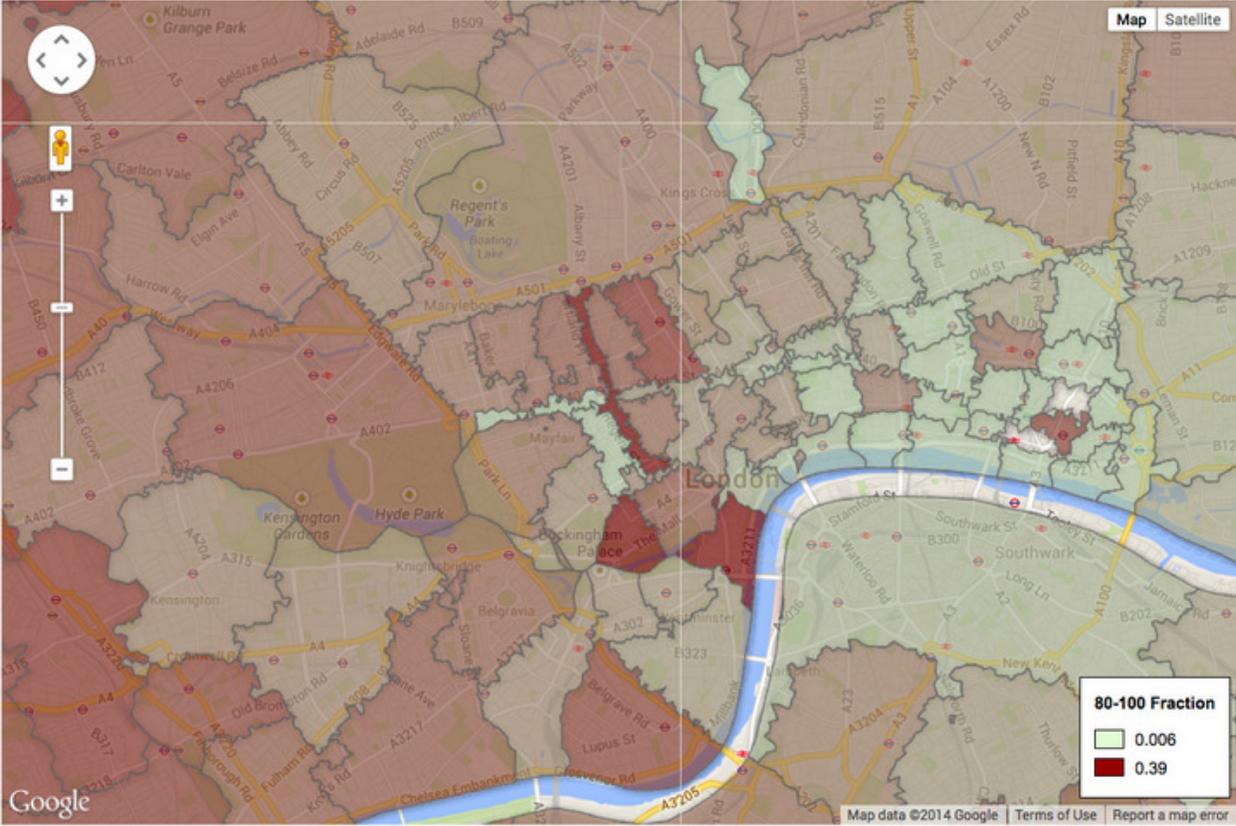
Note: The figure shows the fraction of flat transactions with 80-99 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

**Figure A.7:** U.K. Flats: Fraction of 100-124 years leaseholds
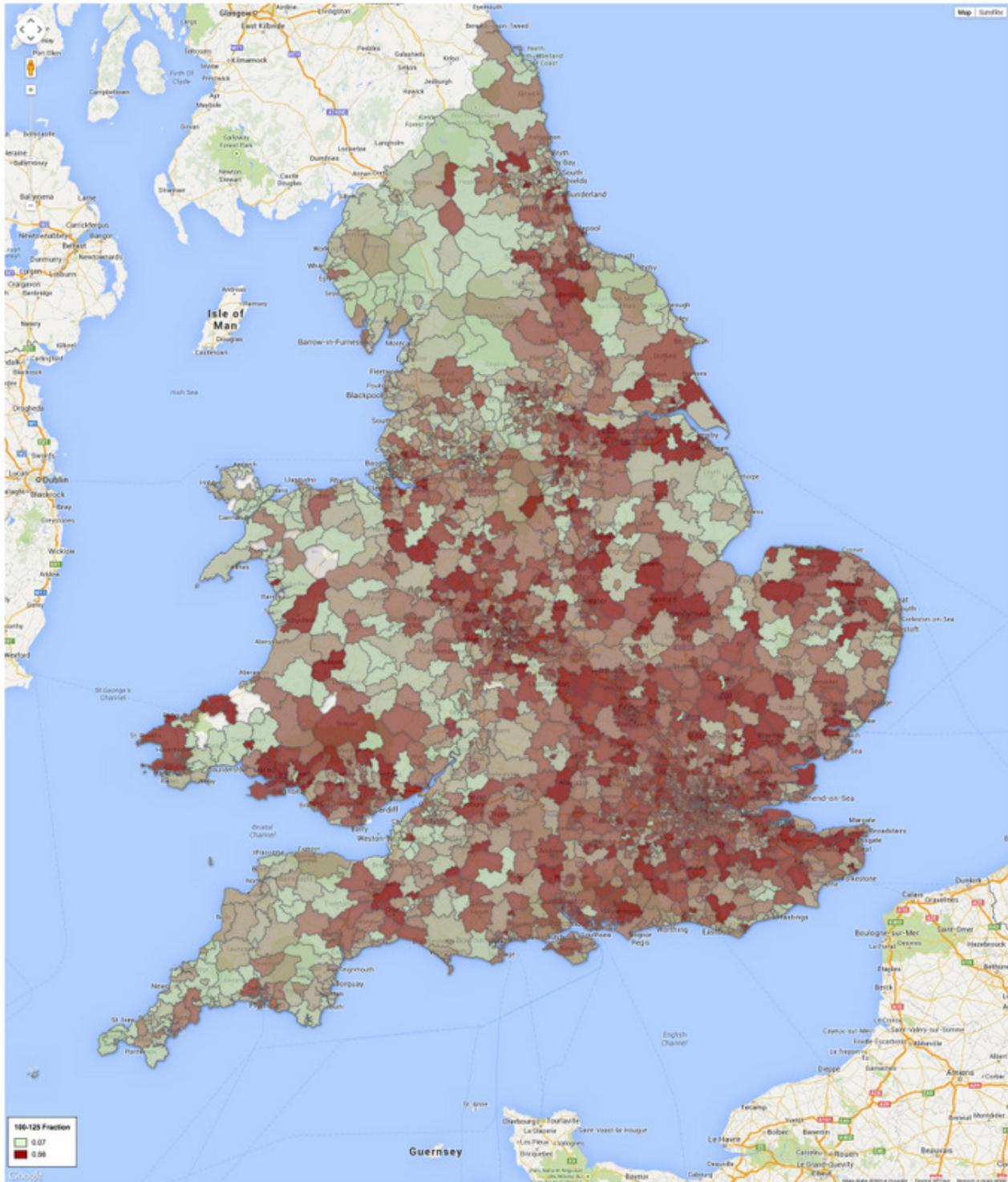
Note: The figure shows the fraction of flat transactions with 100-124 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.8: London Flats: Fraction of 100-124 years leaseholds
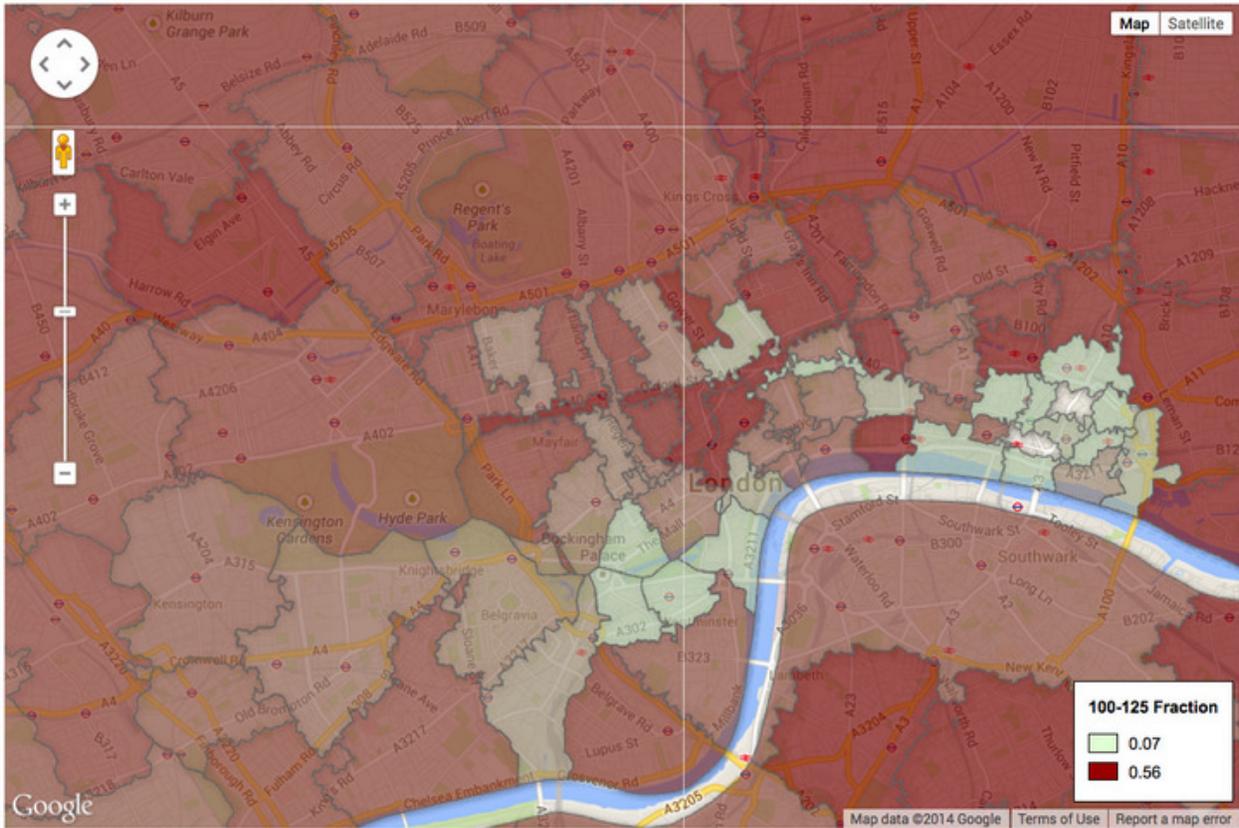
Note: The figure shows the fraction of flat transactions with 100-124 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

**Figure A.9:** U.K. Flats: Fraction of 125-149 years leaseholds
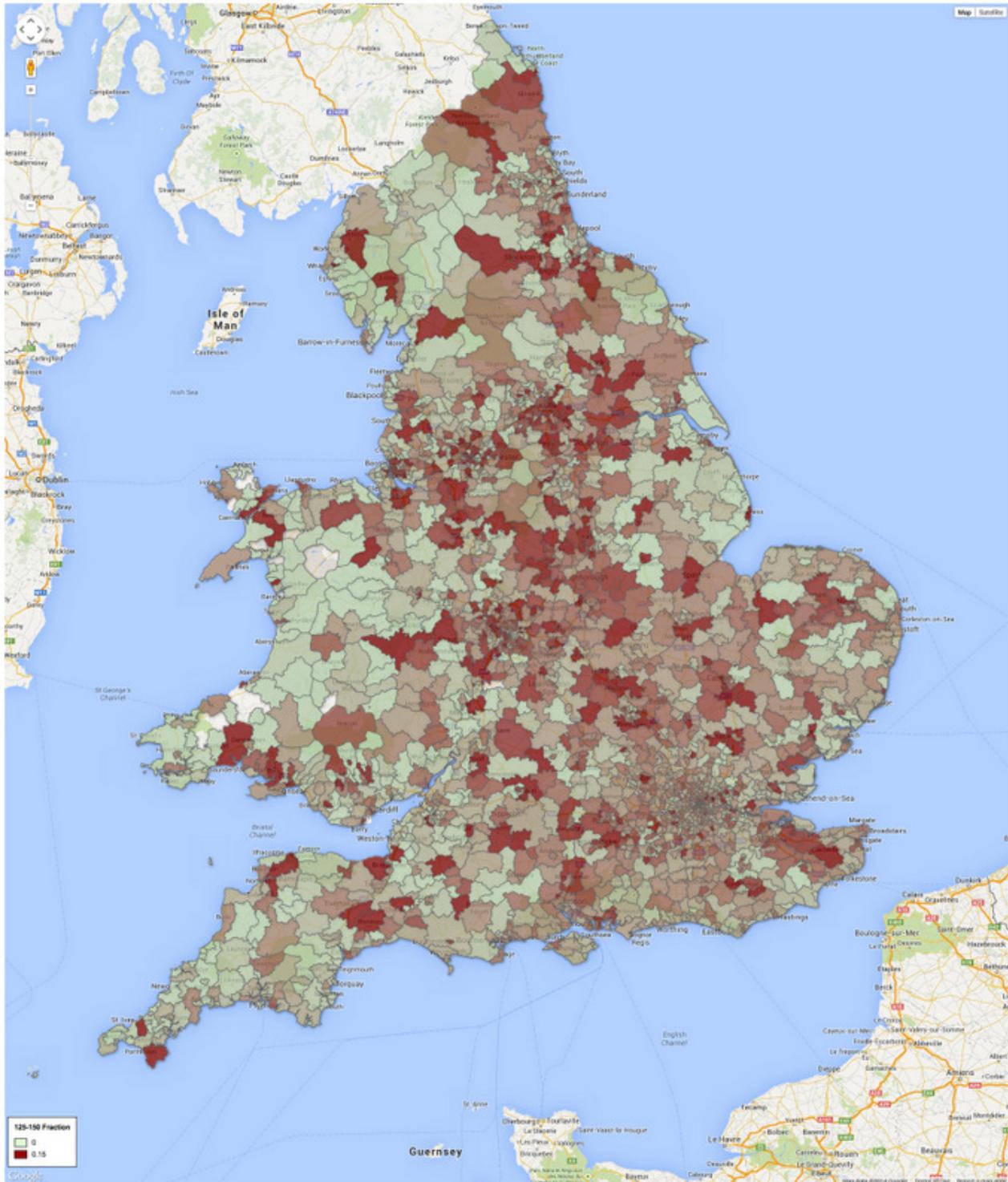
Note: The figure shows the fraction of flat transactions with 125-149 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

**Figure A.10:** London Flats: Fraction of 125-149 years leaseholds
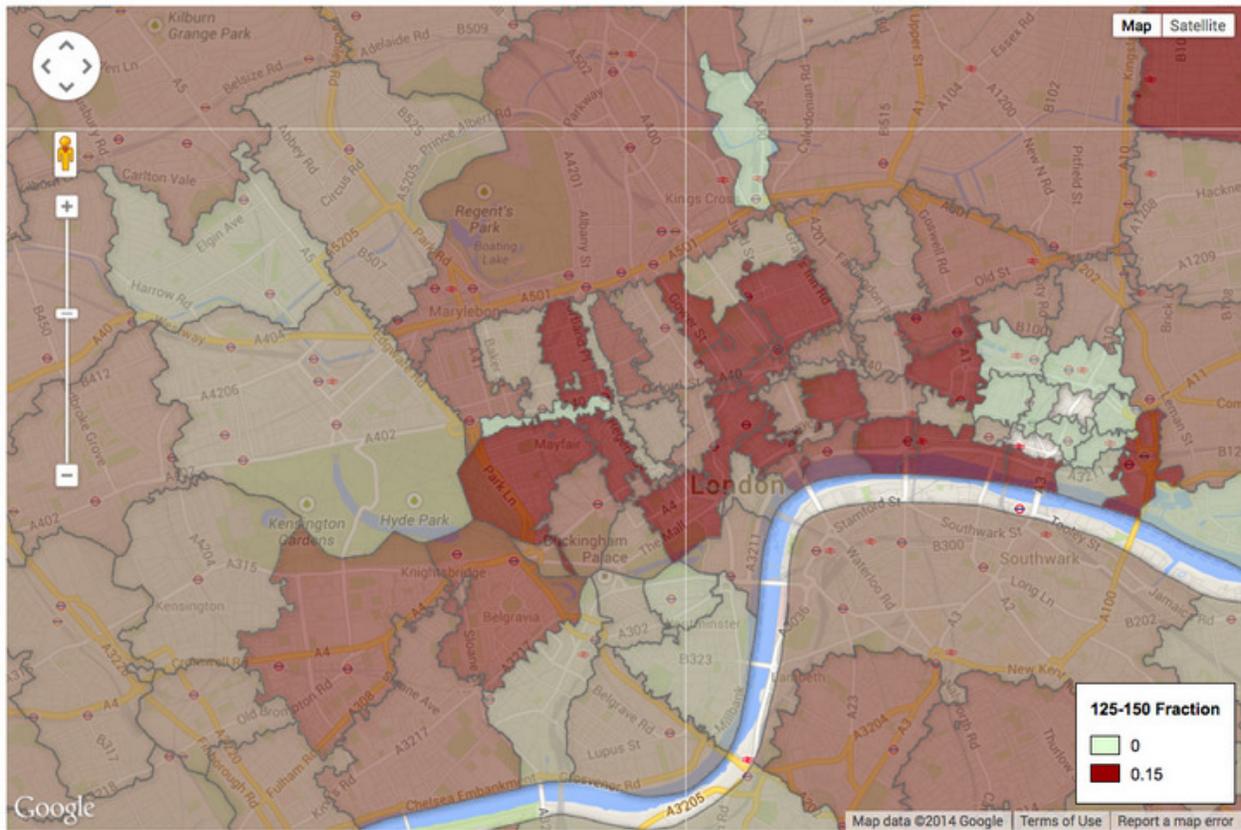
Note: The figure shows the fraction of flat transactions with 125-149 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

Figure A.11: U.K. Flats: Fraction of 150-300 years leaseholds
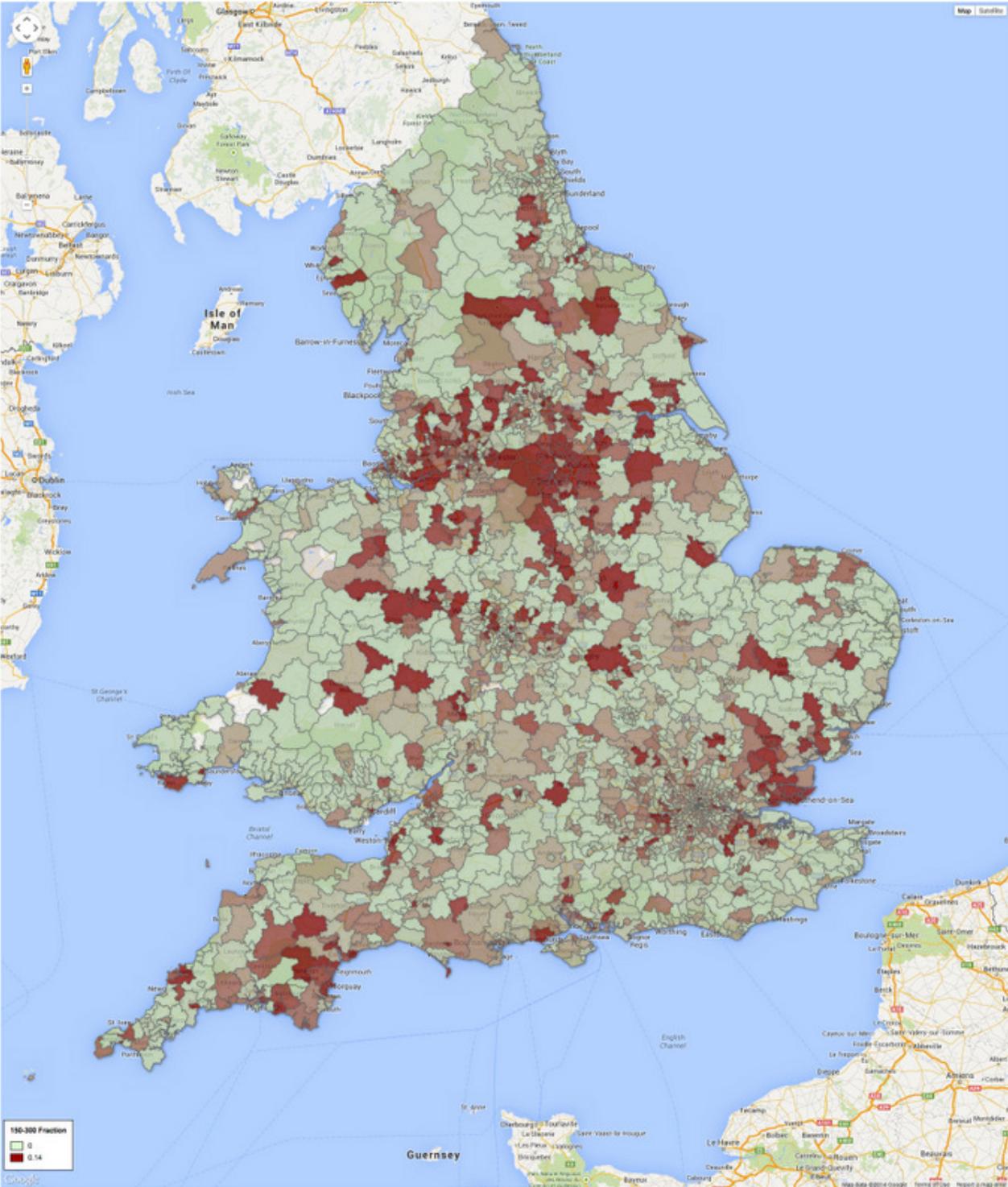
Note: The figure shows the fraction of flat transactions with 150-300 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

**Figure A.12:** London Flats: Fraction of 150-300 years leaseholds
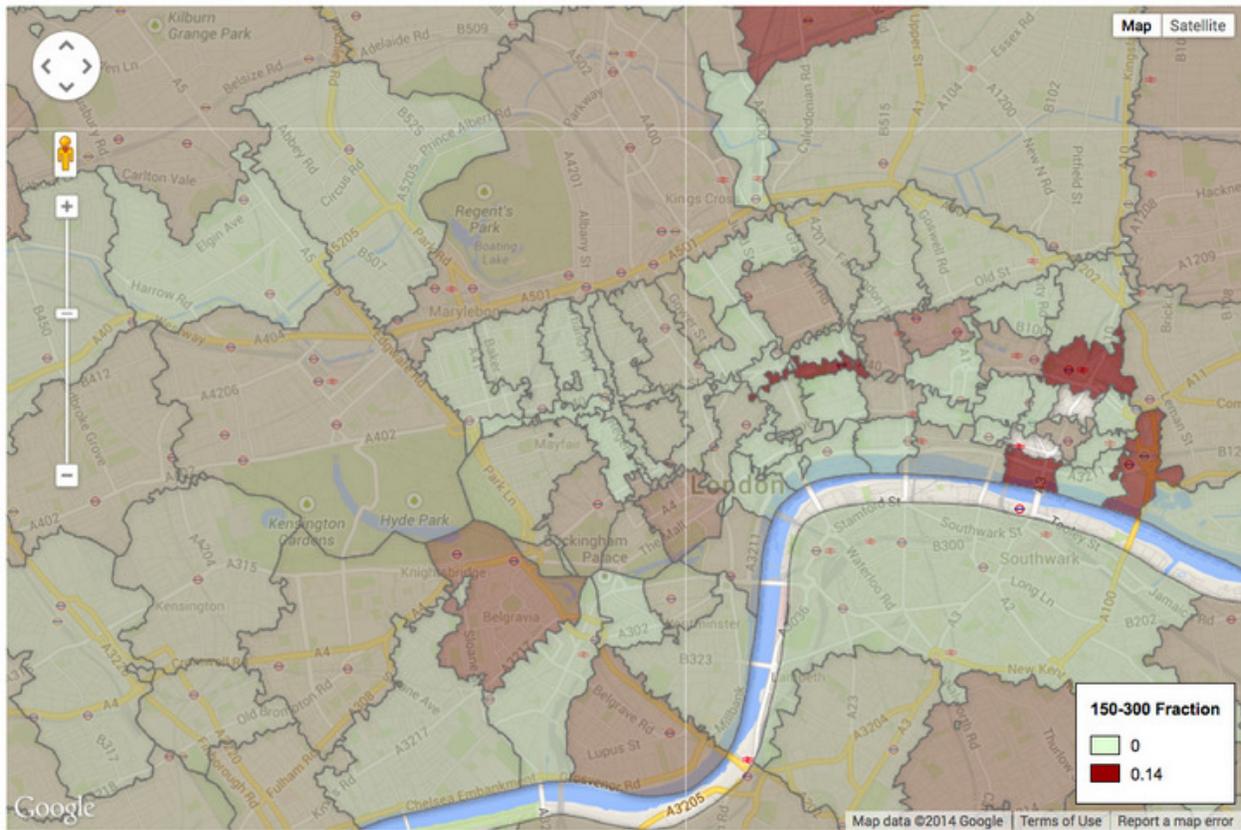
Note: The figure shows the fraction of flat transactions with 150-300 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

**Figure A.13:** U.K. Flats: Fraction of 700+ years leaseholds
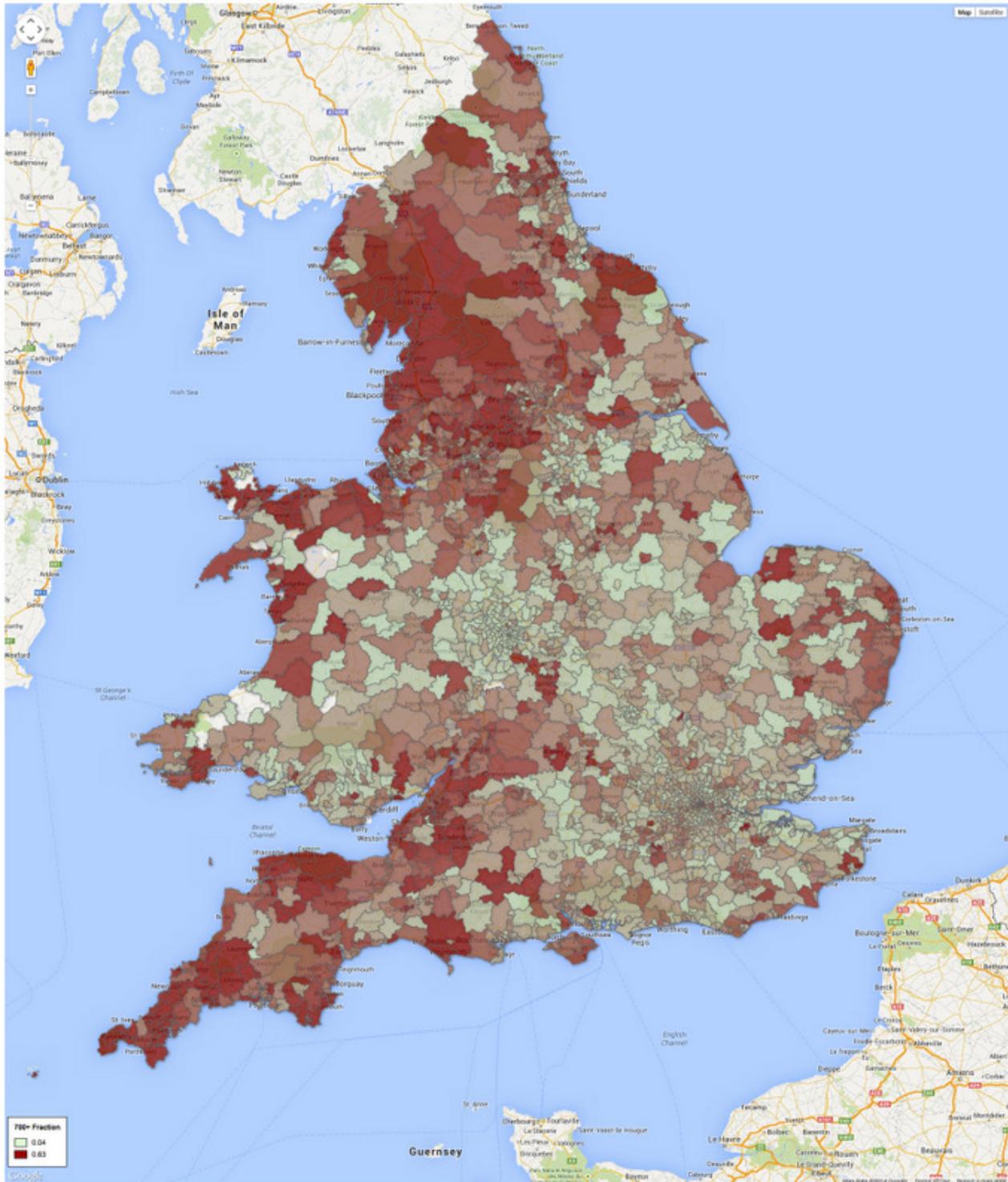
Note: The figure shows the fraction of flat transactions with 700+ years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

**Figure A.14:** London Flats: Fraction of 700+ years leaseholds
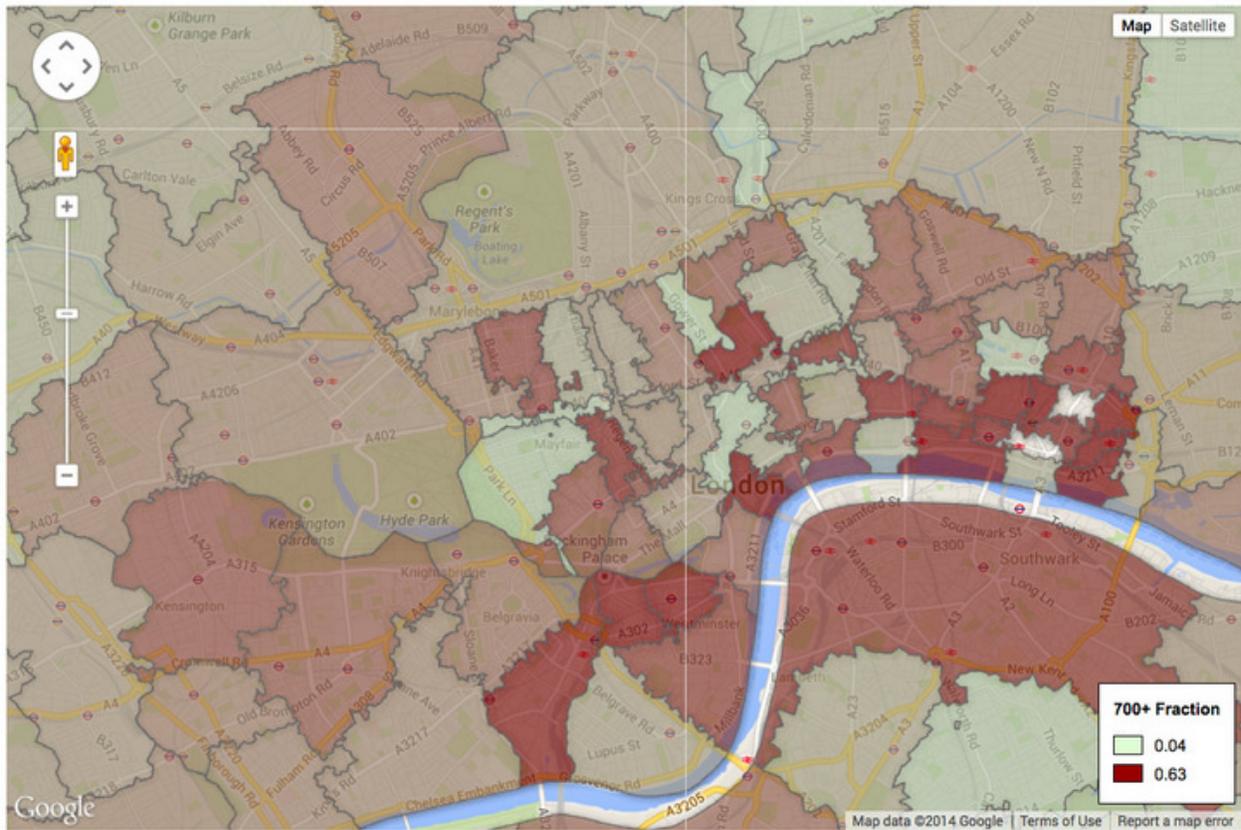
Note: The figure shows the fraction of flat transactions with 700+ years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

Figure A.15: U.K. Houses: Fraction of Freeholds
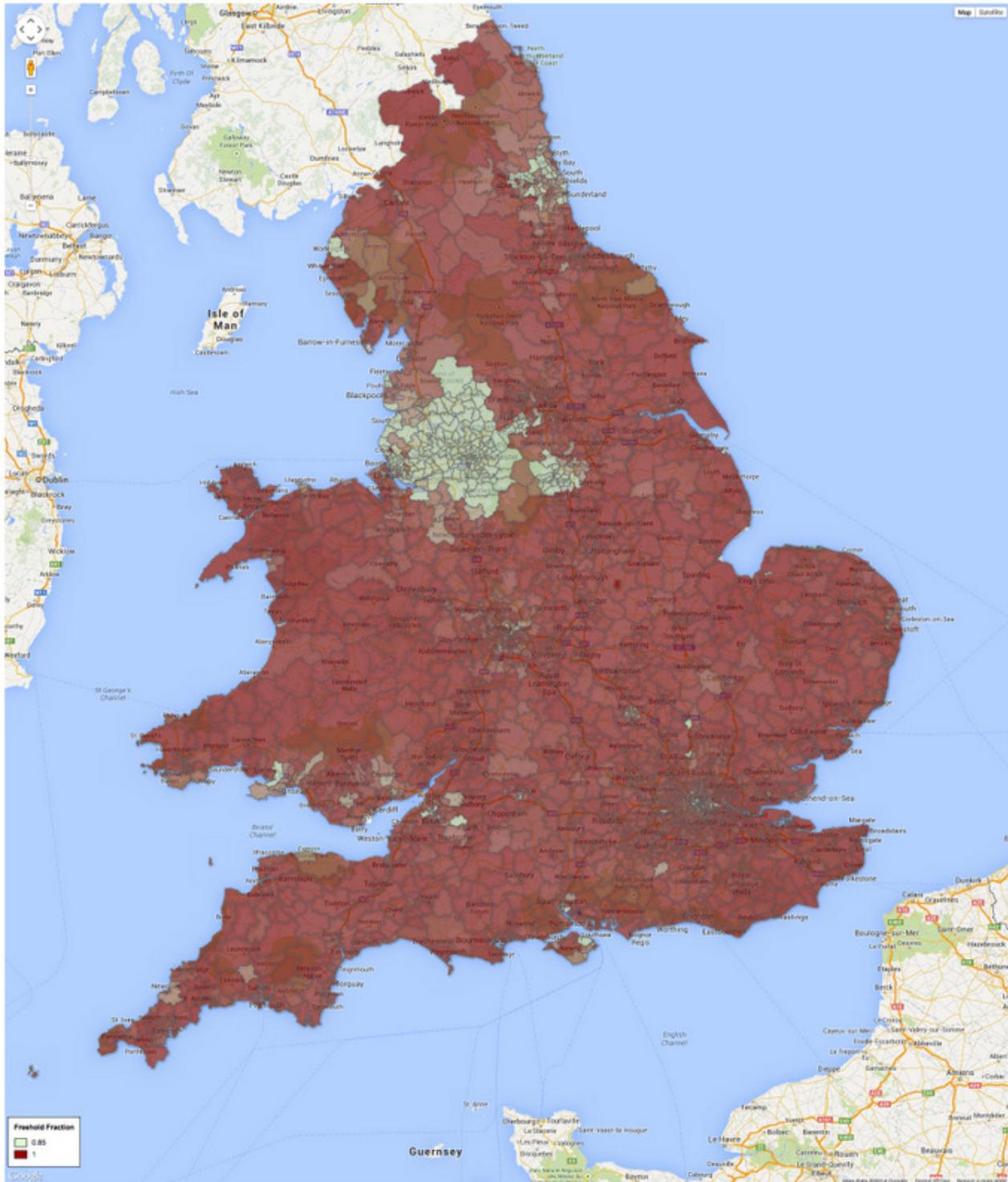
Note: The figure shows the fraction of freehold house transactions in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.16: London Houses: Fraction of Freeholds
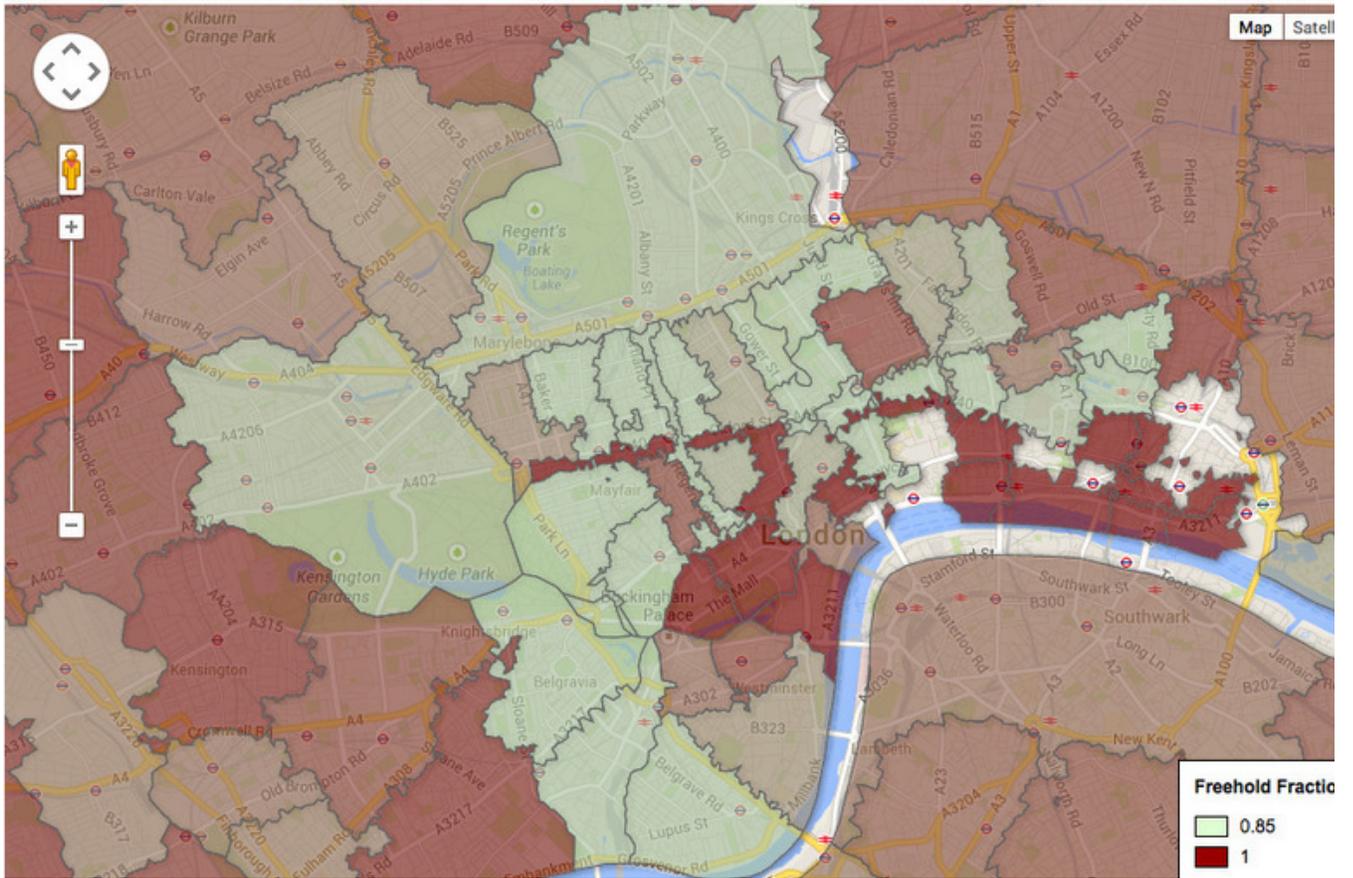
Note: The figure shows the fraction of freehold house transactions in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

**Figure A.17:** U.K. Houses: Fraction of 80-99 years leaseholds
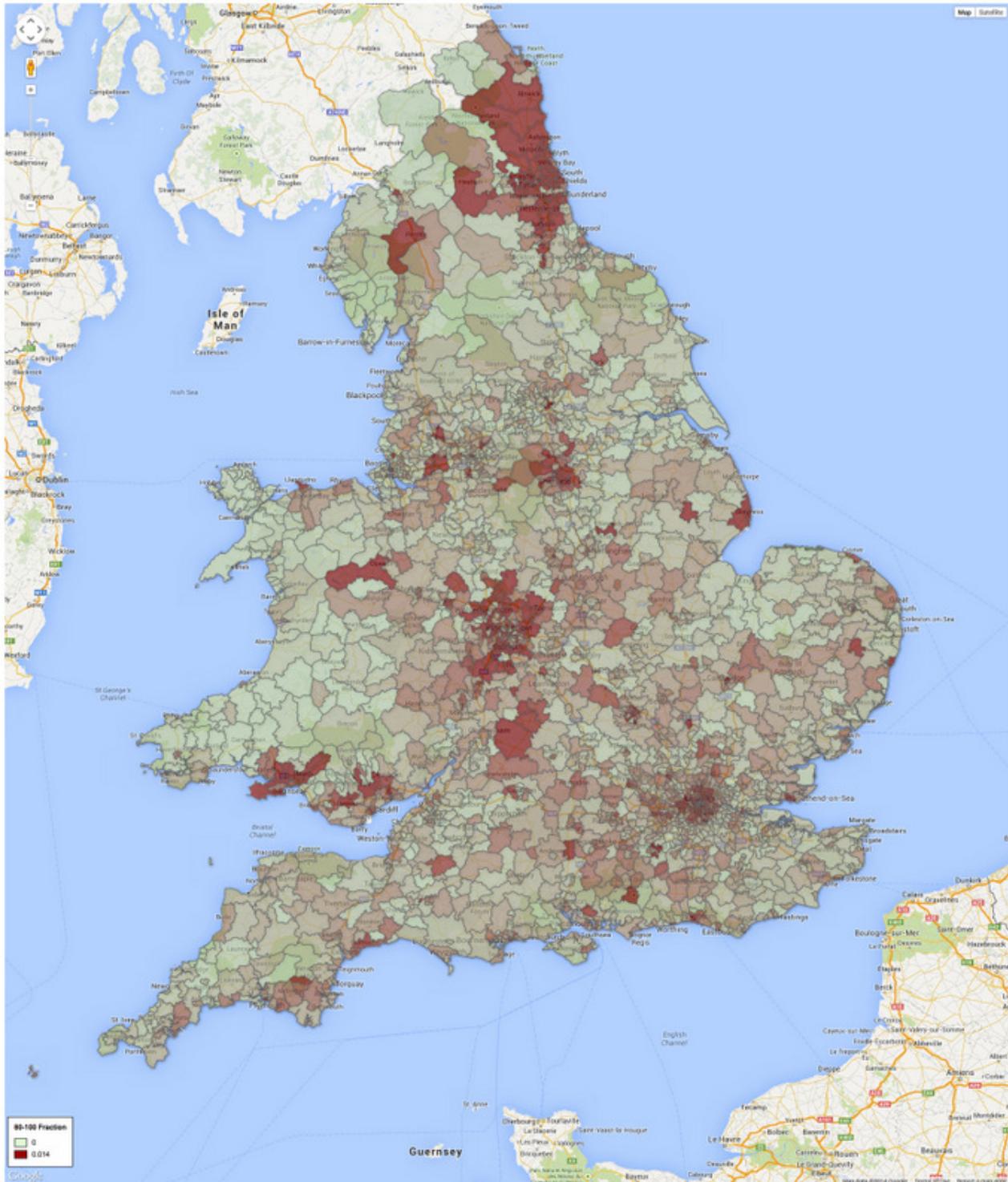
Note: The figure shows the fraction of house transactions with 80-99 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

**Figure A.18:** London Houses: Fraction of 80-99 years leaseholds
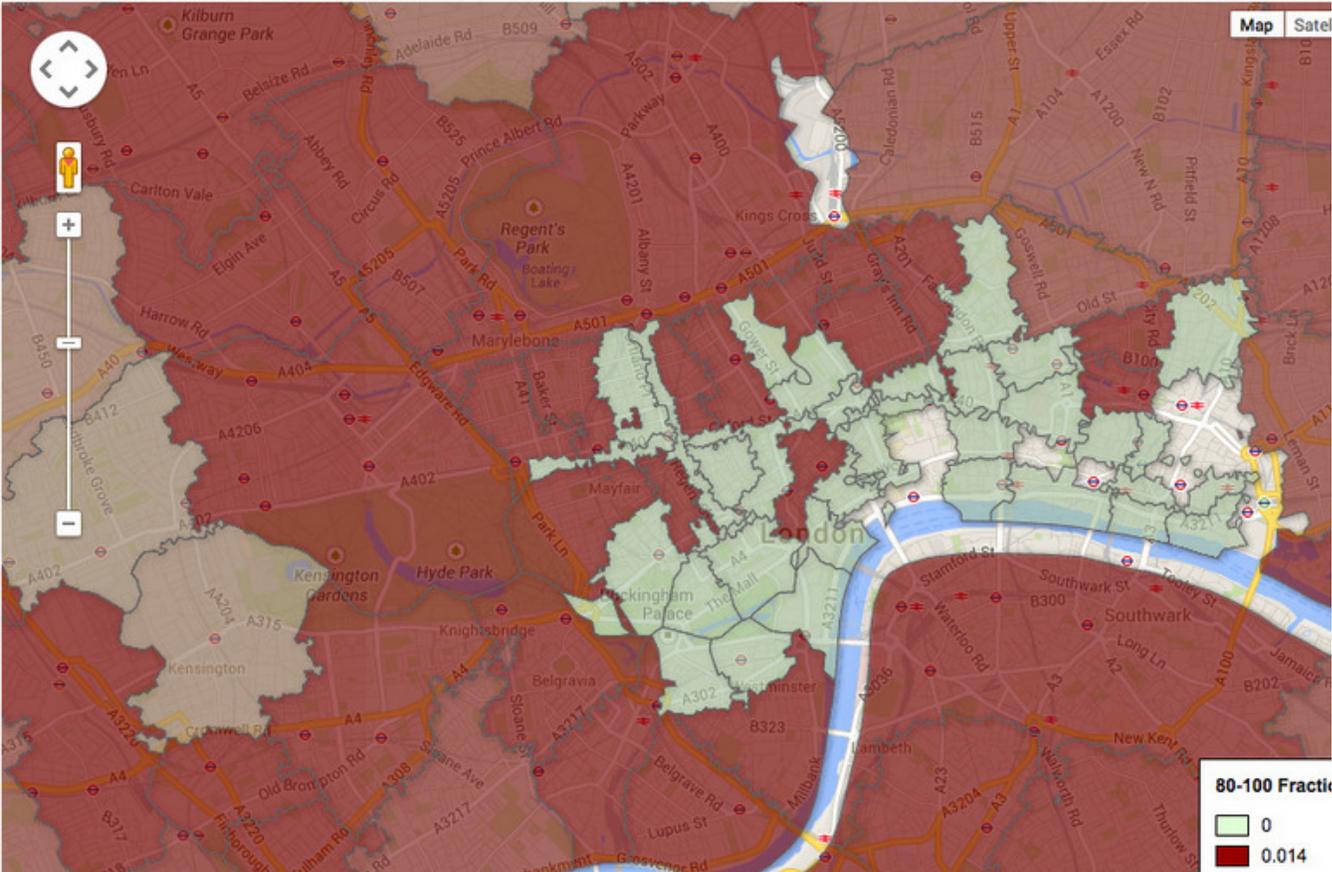
Note: The figure shows the fraction of house transactions with 80-99 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

**Figure A.19:** U.K. Houses: Fraction of 100-124 years leaseholds
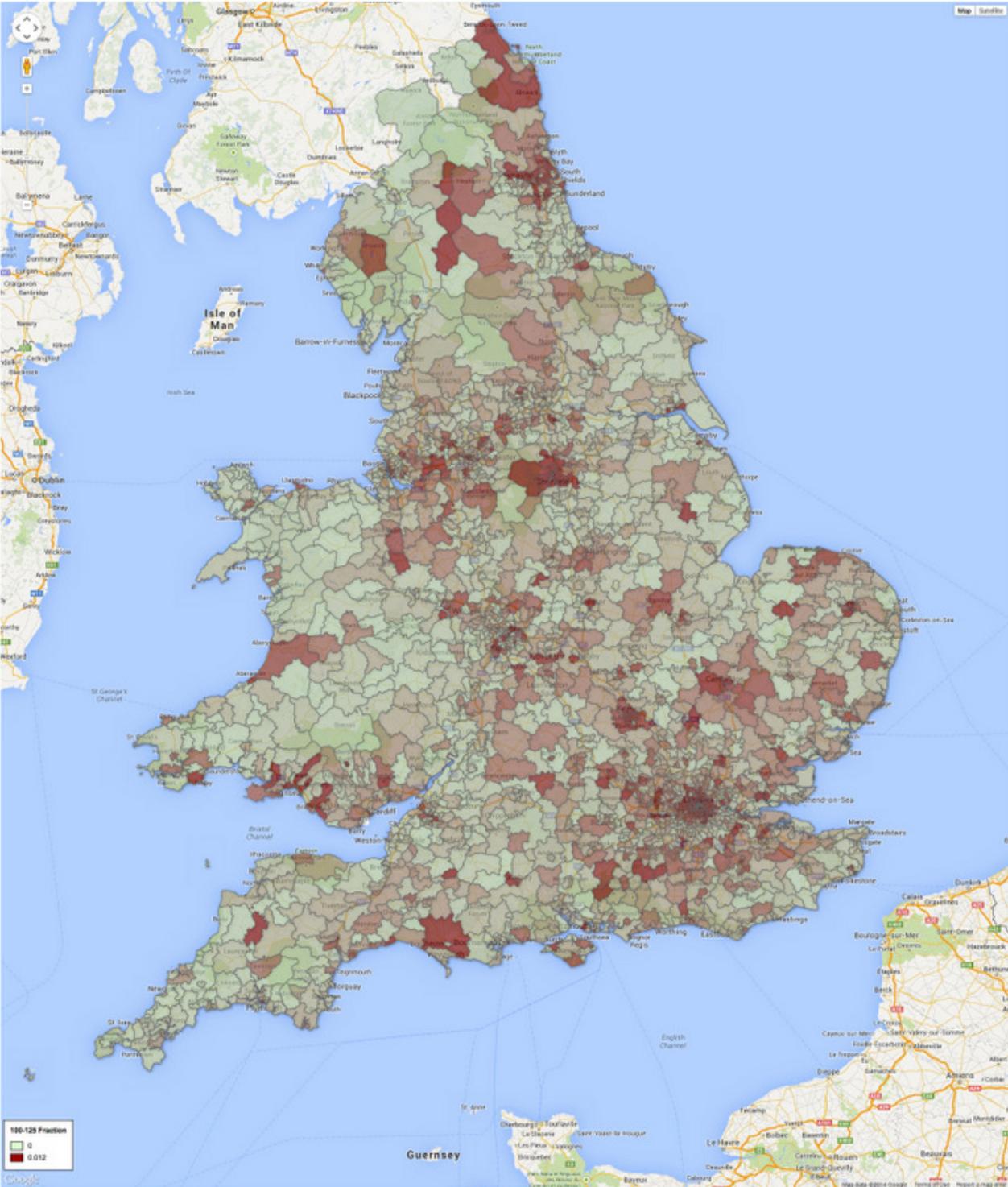
Note: The figure shows the fraction of house transactions with 100-124 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.20: London Houses: Fraction of 100-124 years leaseholds
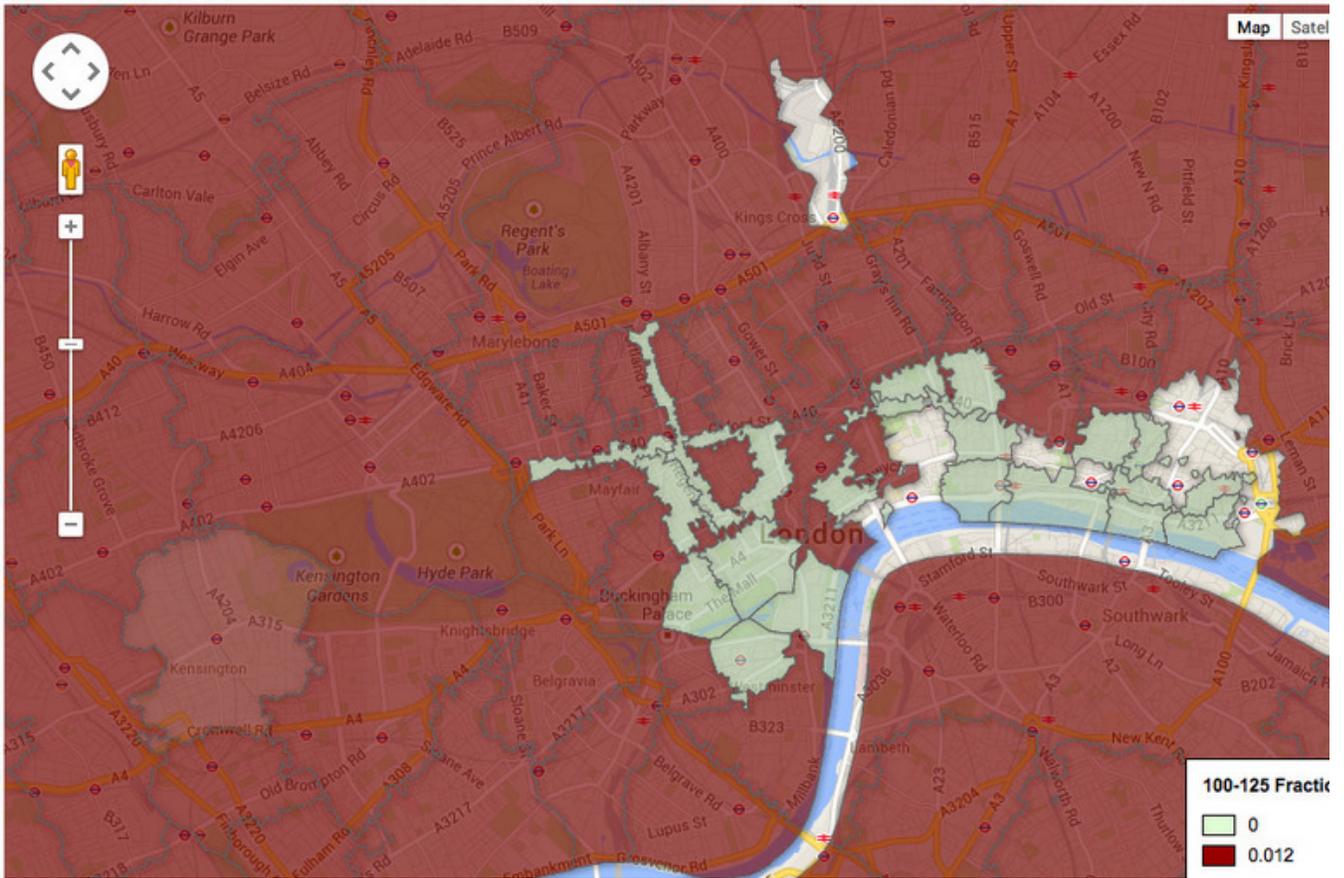
Note: The figure shows the fraction of house transactions with 100-124 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

Figure A.21: U.K. Houses: Fraction of 125-200 years leaseholds
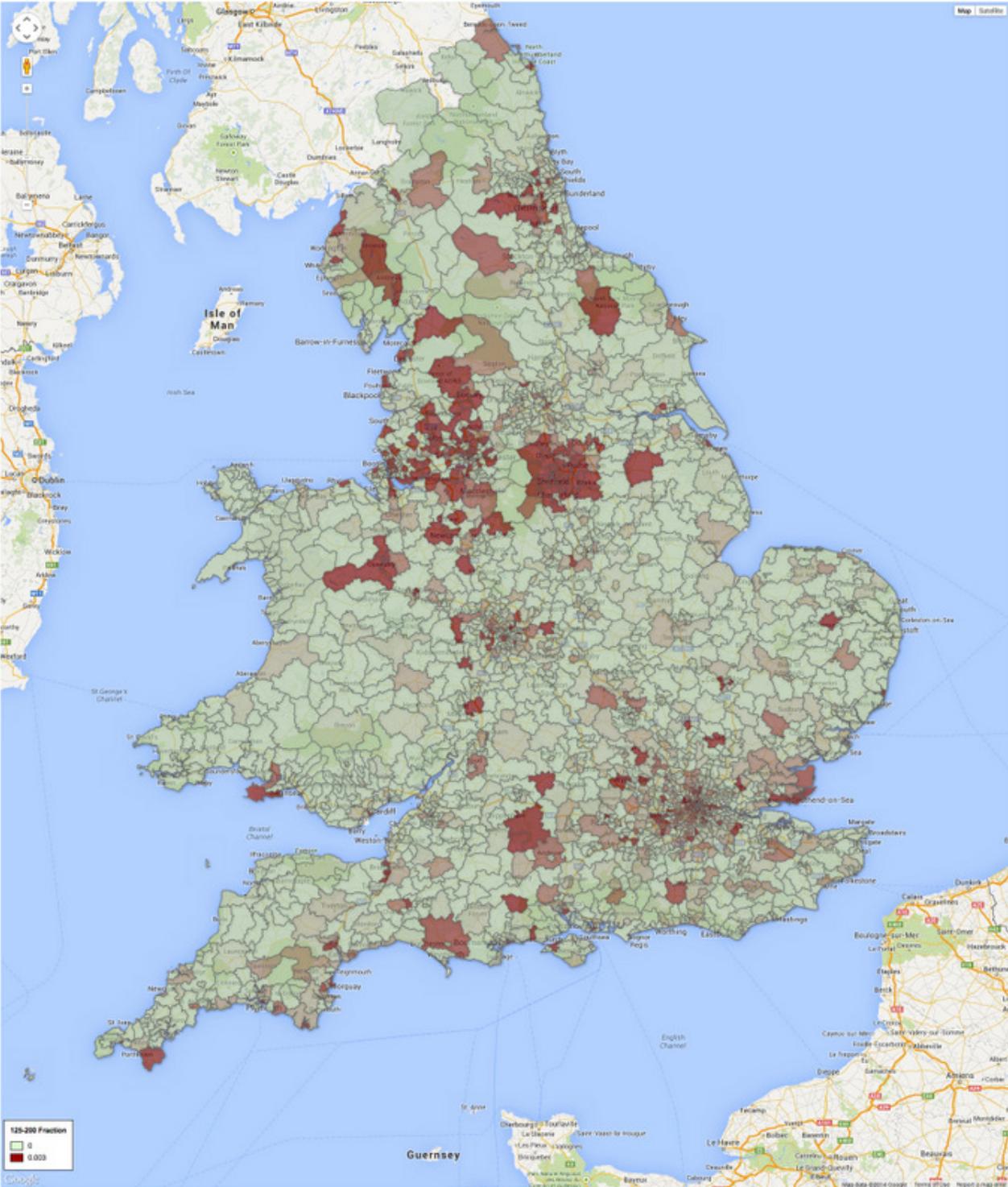

Note: The figure shows the fraction of house transactions with 125-200 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.22: London Houses: Fraction of 125-200 years leaseholds
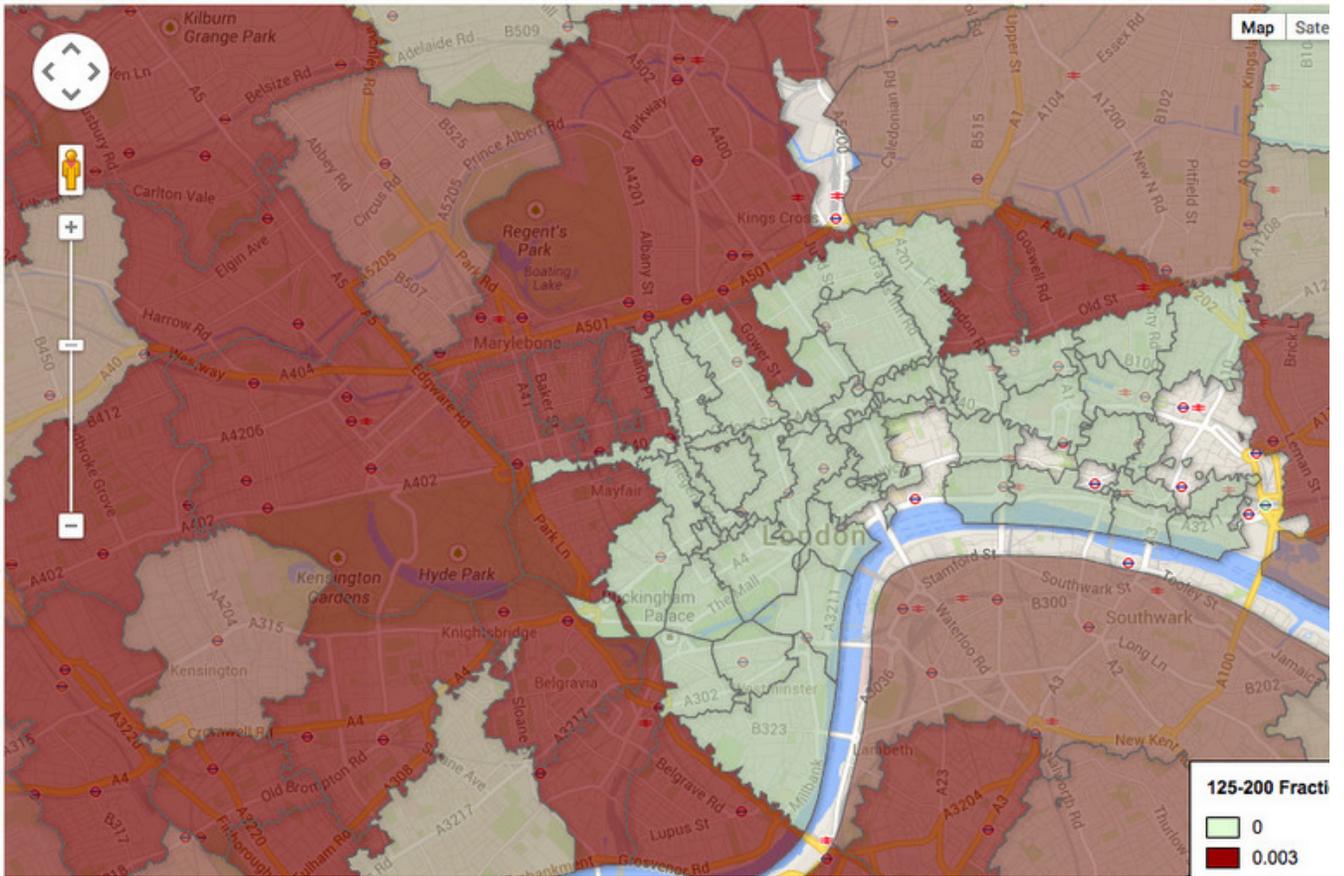
Note: The figure shows the fraction of house transactions with 125-200 years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

**Figure A.23:** U.K. Houses: Fraction of 700+ years leaseholds
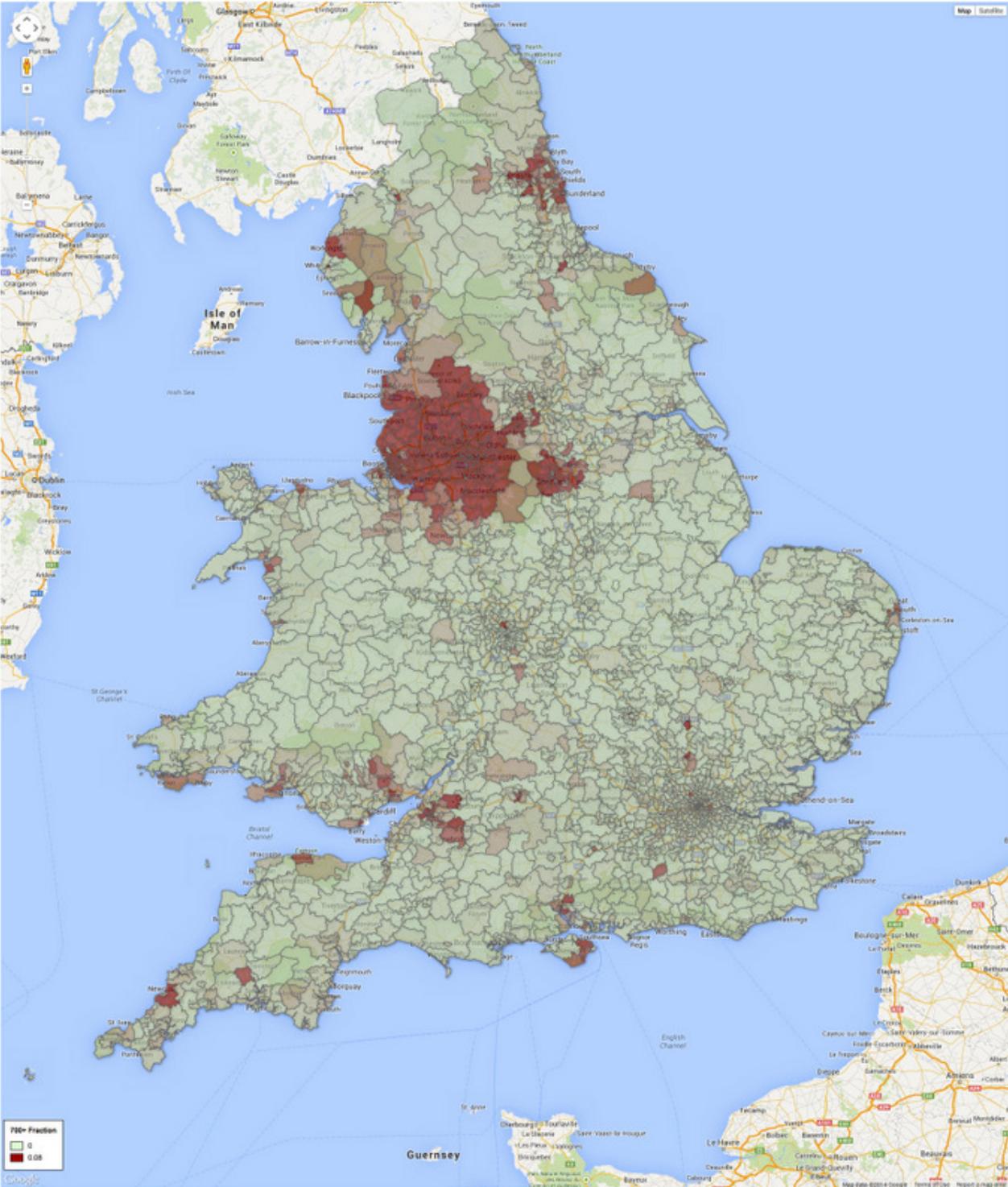
Note: The figure shows the fraction of house transactions with 700+ years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes.

Figure A.24: London Houses: Fraction of 700+ years leaseholds
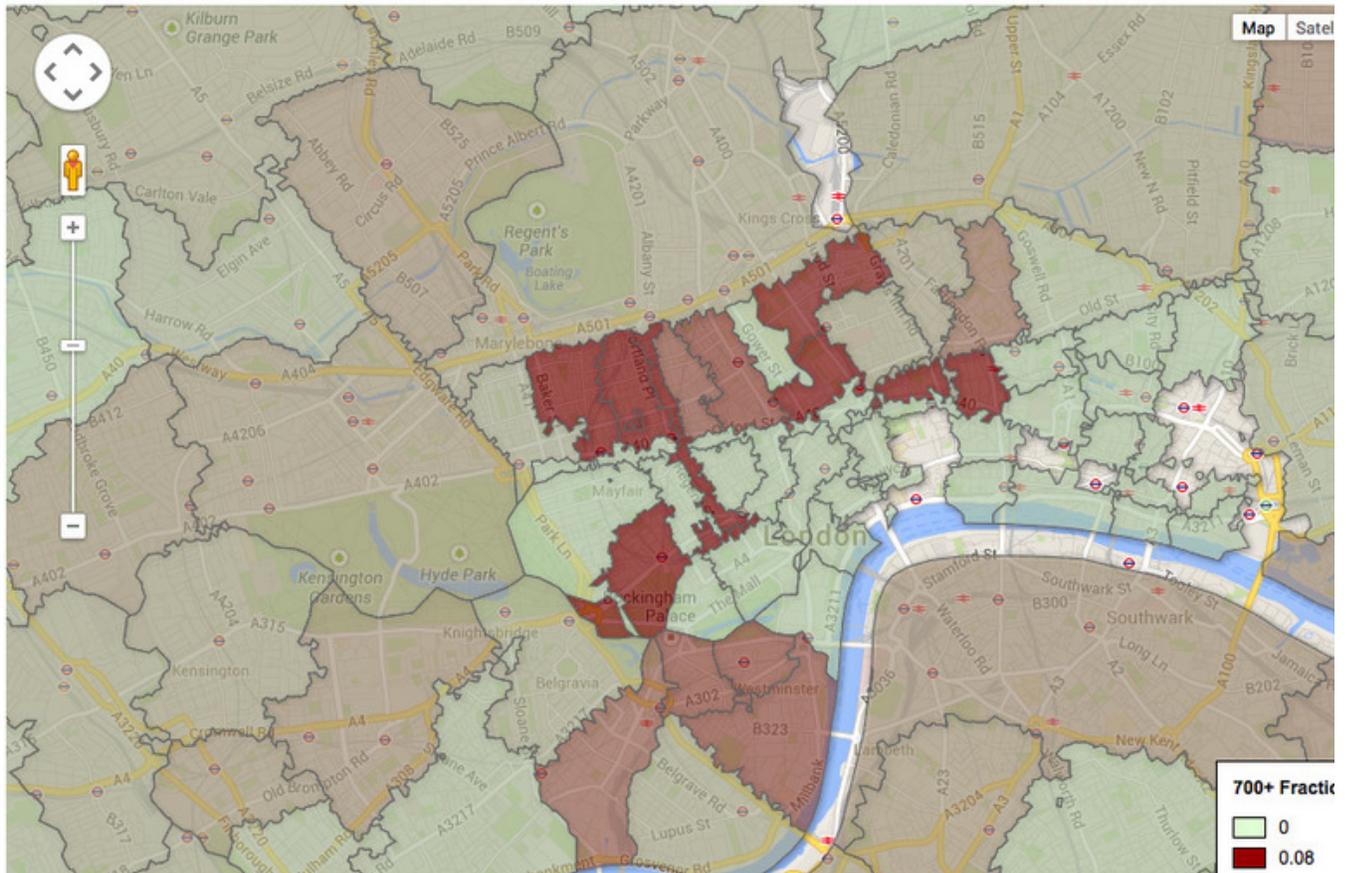
Note: The figure shows the fraction of house transactions with 700+ years remaining in each UK 3-digit postcode. Green and red correspond to the 10th and 90th percentile of the distribution of the fraction across postcodes. The figure zooms in on London.

Figure A.25: U.K. Houses: Fraction of 700+ years leaseholds
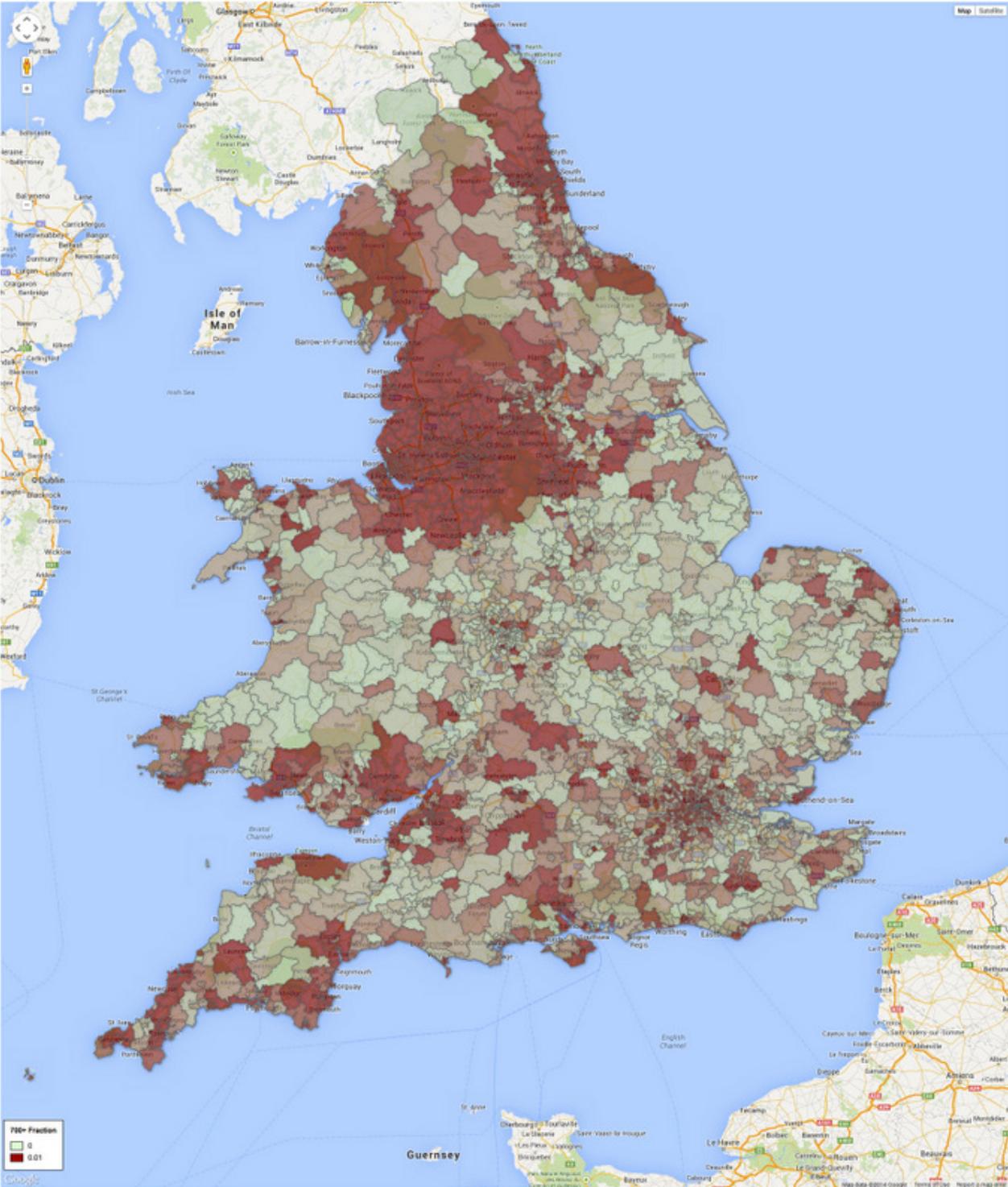
Note: The figure shows the fraction of house transactions with 700+ years remaining in each UK 3-digit postcode. White corresponds to the 10th percentile of the distribution of the fraction across postcodes, while black corresponds to 1%

Figure A.26: London Houses: Fraction of 700+ years leaseholds
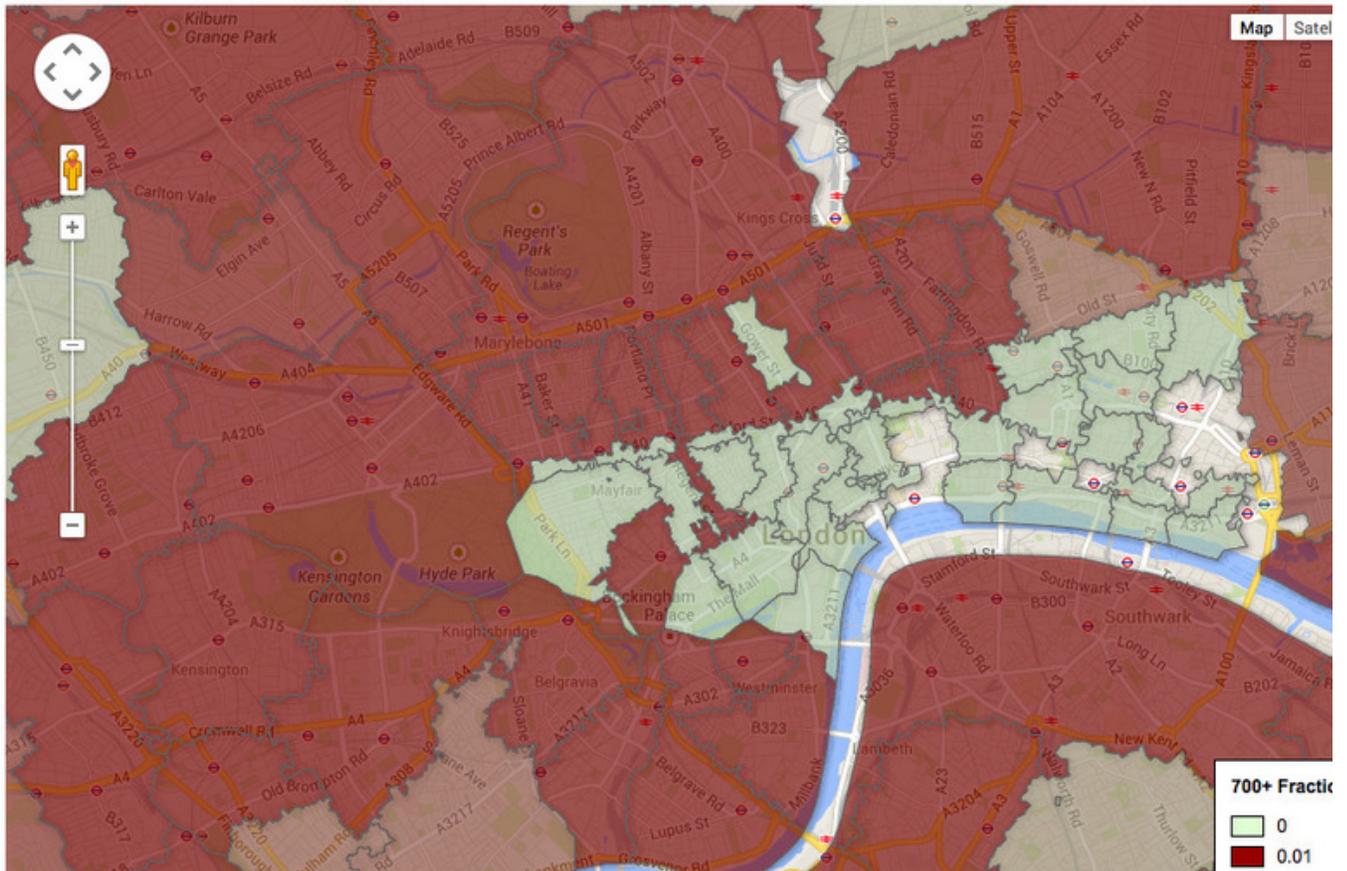
Note: The figure shows the fraction of house transactions with 700+ years remaining in each UK 3-digit postcode. White corresponds to the 10th percentile of the distribution of the fraction across postcodes, while black corresponds to 1%. The figure zooms in on London.

Figure A.27: Hedonic Characteristics by Leasetype - UK Flats
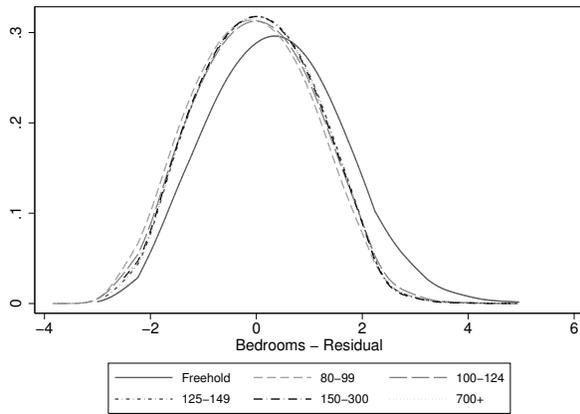
(a) Bedrooms, flats
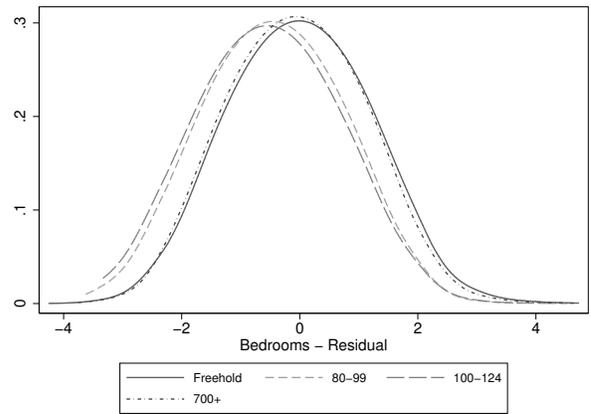
(b) Bedrooms, houses
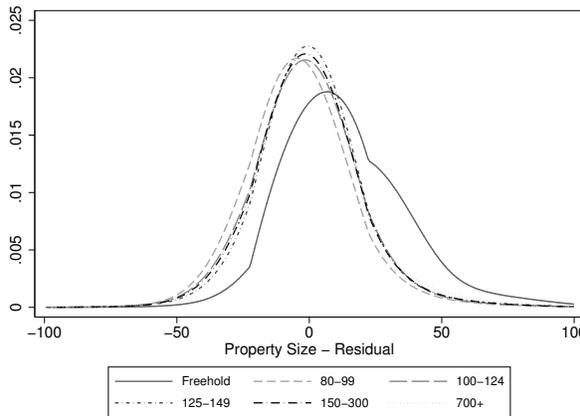
(c) Size, flats
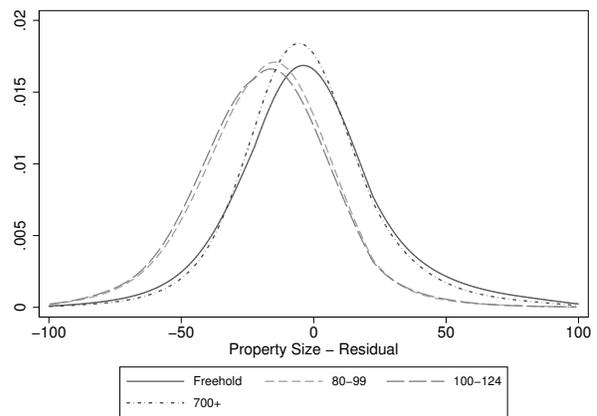
(d) Size, houses
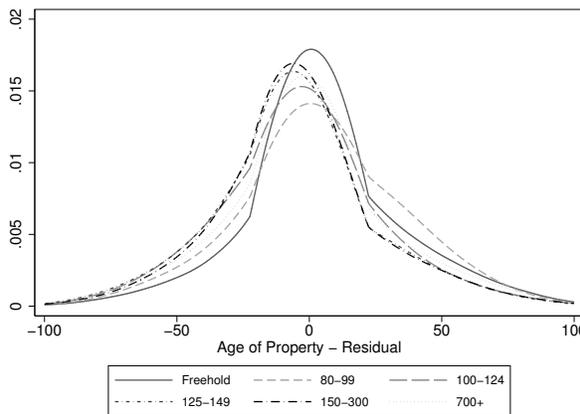
(e) Property Age, flats
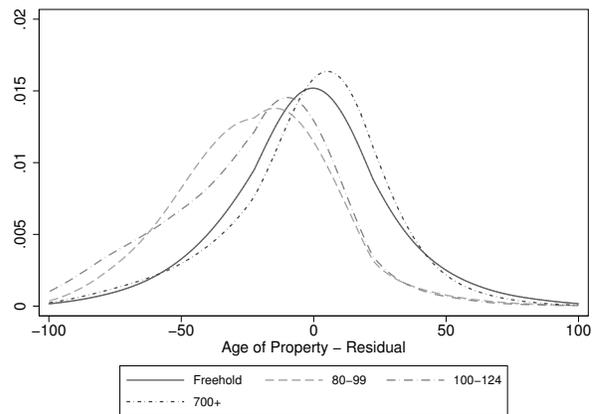
(f) Property Age, houses

Note: This figure shows the distribution of residuals from a regression of property characteristics on 3-digit postcode × property type × year fixed effects for leaseholds with different remaining lease maturity and freeholds. The characteristics plotted are: number of bedrooms, size of the property in square meters, age of the property

A.89

Figure A.28: Hedonic Characteristics by Leasetype - Singapore
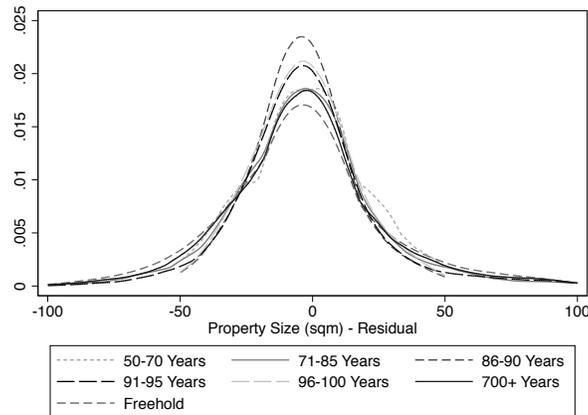

(a) Property Size
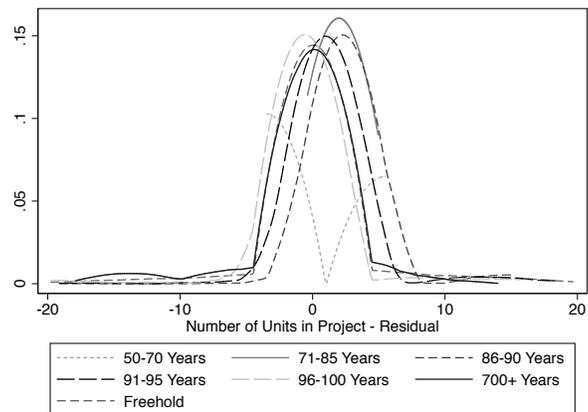
(b) Development Size
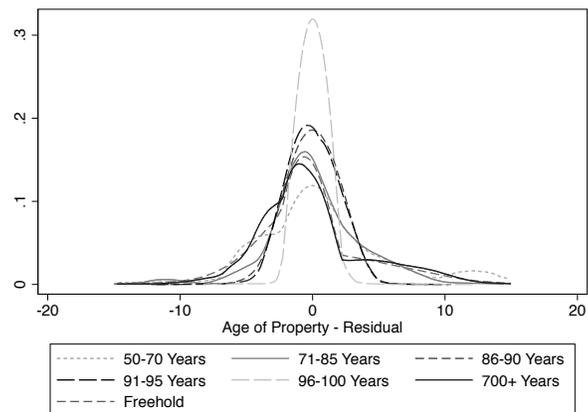
(c) Property Age

Note: This figure shows the distribution of residuals from a regression of property characteristics on property type × title type (strata or land) × 5-digit postcode × year fixed effects for leaseholds with different remaining lease maturity. The top panel shows residuals for the total area in square meters, the middle panel for the total project size and the bottom panel for property age.

Figure A.29: Price Discount by Remaining Lease Length - Within Leasehold Estimates

Leasehold Discounts - Log(Price)
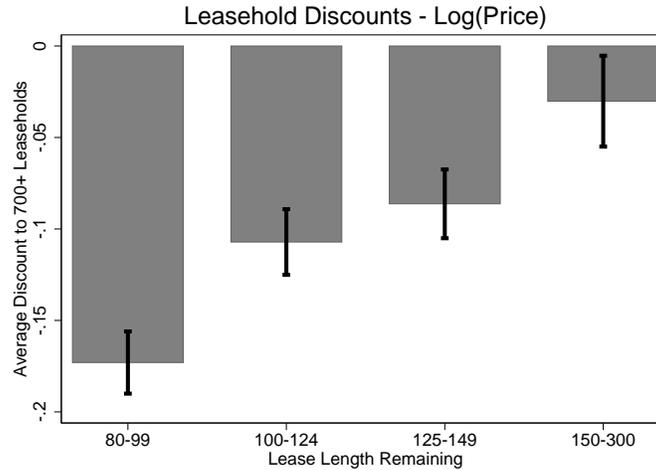
(a) U.K. - Relative to Leases with 700+ years remaining
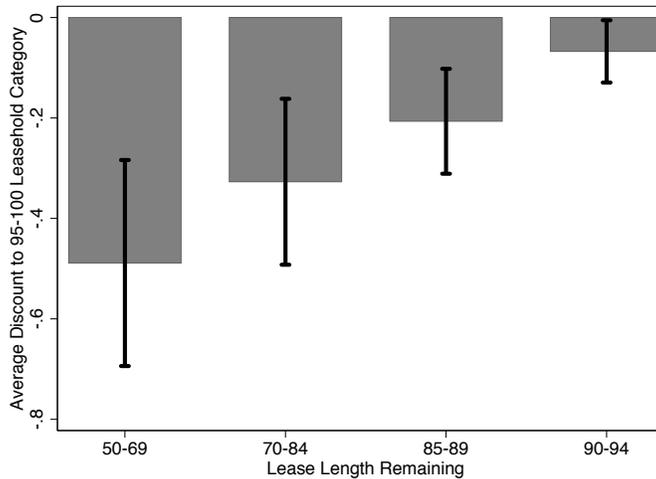
(b) Singapore - Relative to Leases with 96-100 years remaining

Note: This figure shows β_j coefficients from regression (1) in the top panel and (2) in the bottom panel. In the top panel, which focusses on U.K. transactions, the price discounts are relative to leaseholds with more than 700 years remaining, and correspond to column (1) of Appendix Table A.3. The analysis is conducted only within flats. We include 3-digit post code by transaction year-month fixed effects. We also control for the size, number of bedrooms, bathrooms, property age, property condition, whether there is parking, and the type of heating. The bars indicate the 95% confidence interval of the estimate using standard errors double clustered at the 3-digit postcode and at the year level. In the bottom panel, which focusses on Singapore Transactions, we restrict to an estimation within strata leases with initial lease length of 99 years; the excluded category are those leaseholds with 95-100 years remaining. The dependent variable is the log price foot paid for properties sold by private parties in Singapore between 1995 and 2013, corresponding to Column (2) in Appendix Table 4. We include fixed effect for the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction month. We control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size) and the total number of units in the property. The bars indicate the 95% confidence interval of the estimate using standard errors double clustered by 5-digit postcode and by year.

**Figure A.30:** Distribution Time on Market
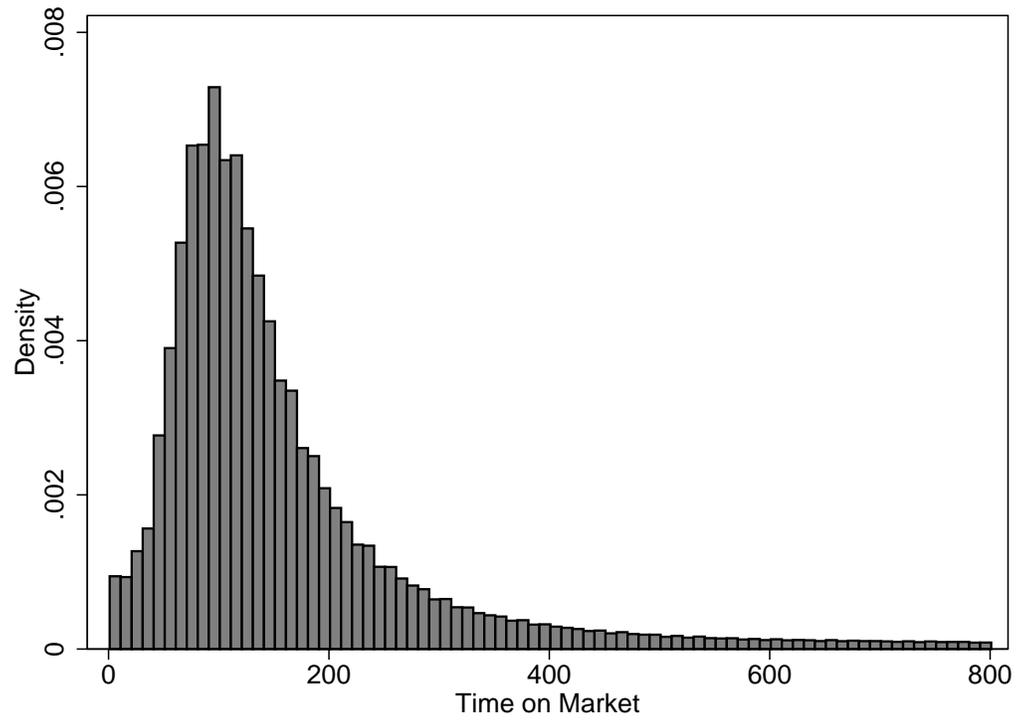
Note: The figure shows the distribution of the time on market observed in our data.

Figure A.31: Cross-Sectional Distribution of Price-Rent Ratio in the U.S.
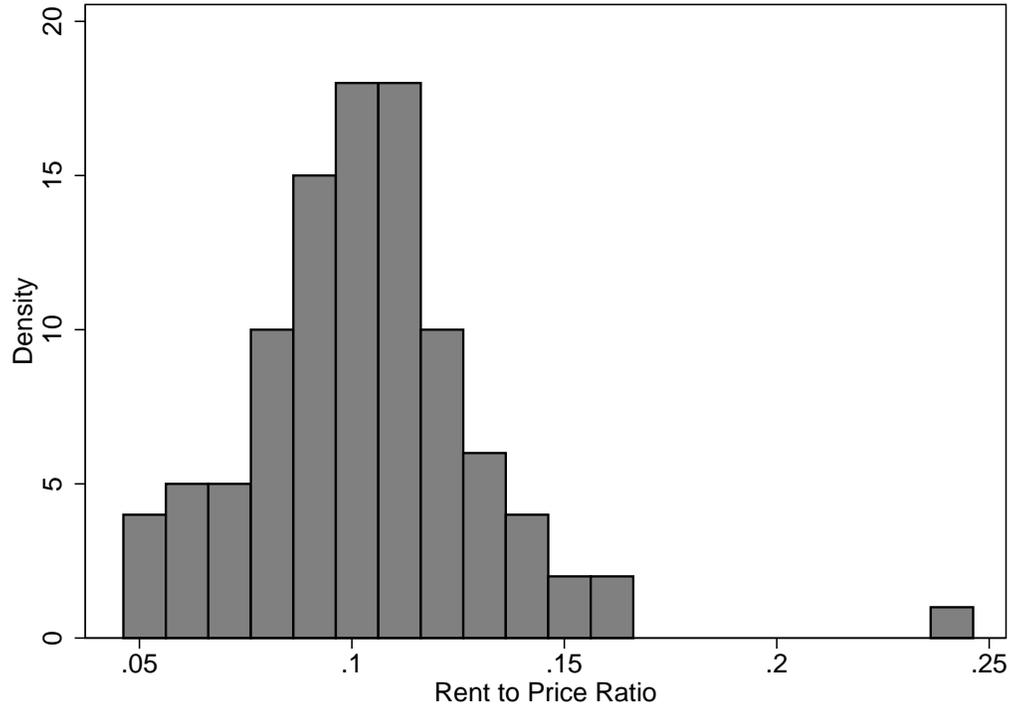

Note: The figure shows the distribution of the rent-to-price ratio for the 100 largest MSAs in the U.S. in September 2013 as constructed by Trulia, which observes a large set of both for-sale and for-rent listings. It is constructed using a metro-level hedonic regression of ln(*price*) on property attributes, zipcode fixed effects, and a dummy for whether the unit is for rent. The rent-to-price ratio is constructed by taking the exponent of the coefficient on this dummy variable.

Figure A.32: Price-Rent Ratio Timeseries in the U.S.
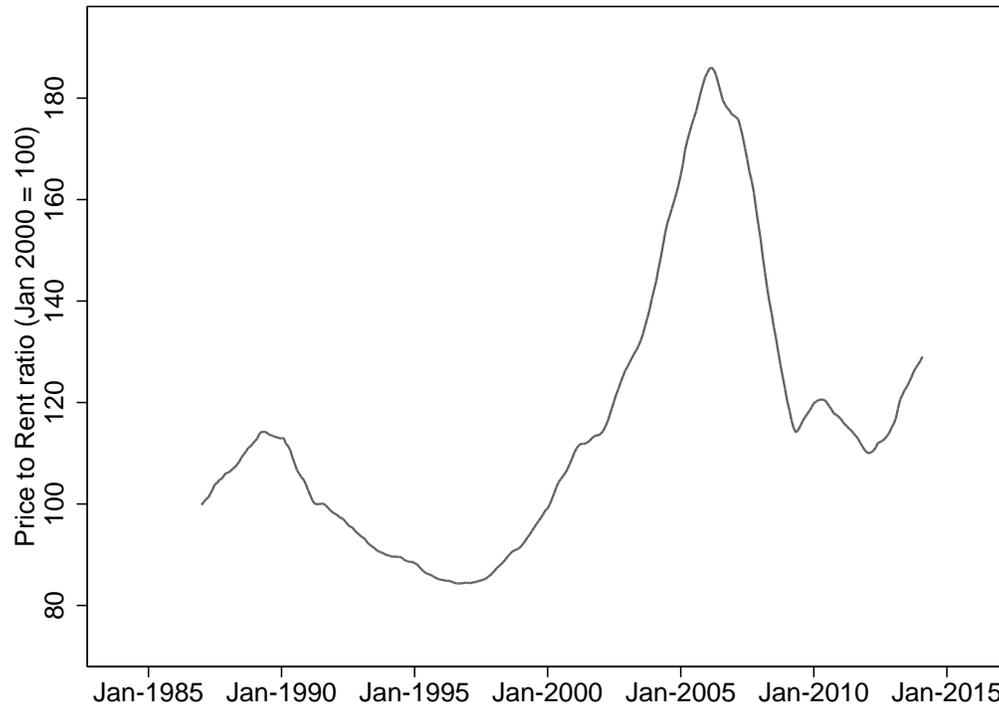

Note: The figure shows the time series of the price-rent ratio in the U.S., constructed as the ratio of the 10-city Case-Shiller House Price Index (Fred Series: SPCS10RSA) and the Bureau of Labor Statistic's Consumer Price Index for All Urban Consumers: Owners' equivalent rent of residences (Fred Series: CUSR0000SEHC). The index ratio is normalized to 100 in January 2000.

Figure A.33: House Price Riskiness II

(a) U.K.
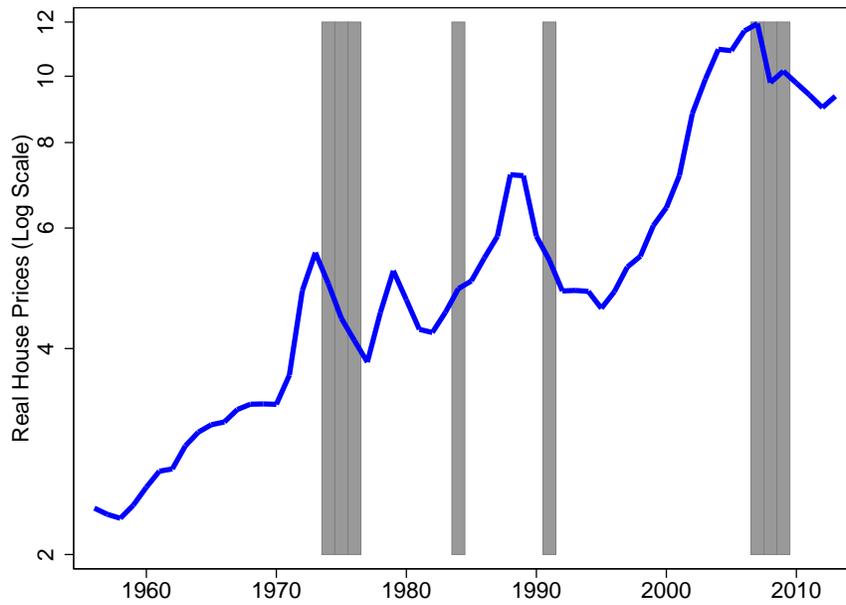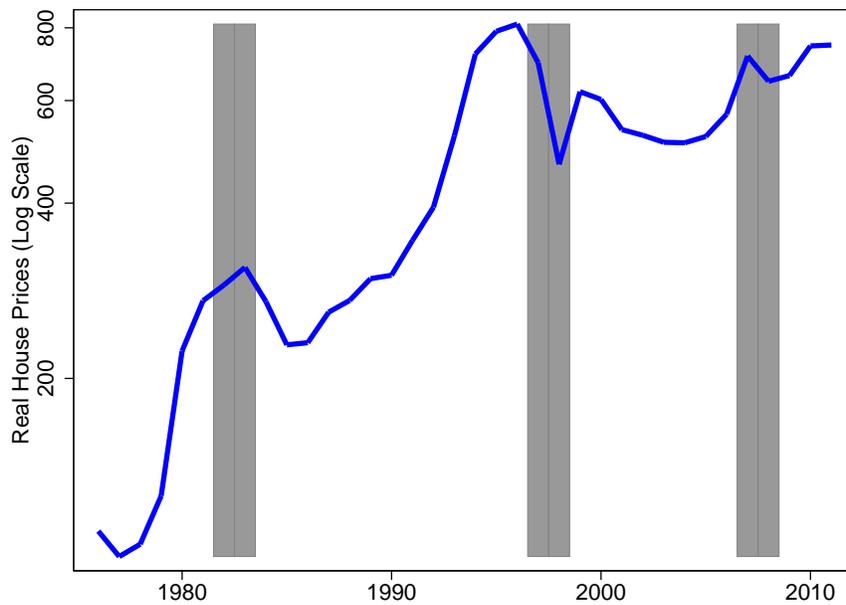
(b) Singapore

Note: The figures show the evolution of real house prices in the U.K. (top panel) and Singapore (bottom panel). Shaded regions for the U.K. are financial crises identified by Reinhart and Rogoff (2009): 1974-1976, 1984, 1991 and 2008-2009. Shaded regions for Singapore include the 1982-1983 financial crisis identified by Reinhart and Rogoff (2009), as well as the Asian financial crisis (1997-1998), and the 2007-2008 global financial crisis. See Appendix A.3.3 for a description of the data series studied here.

Figure A.34: House Price Riskiness III
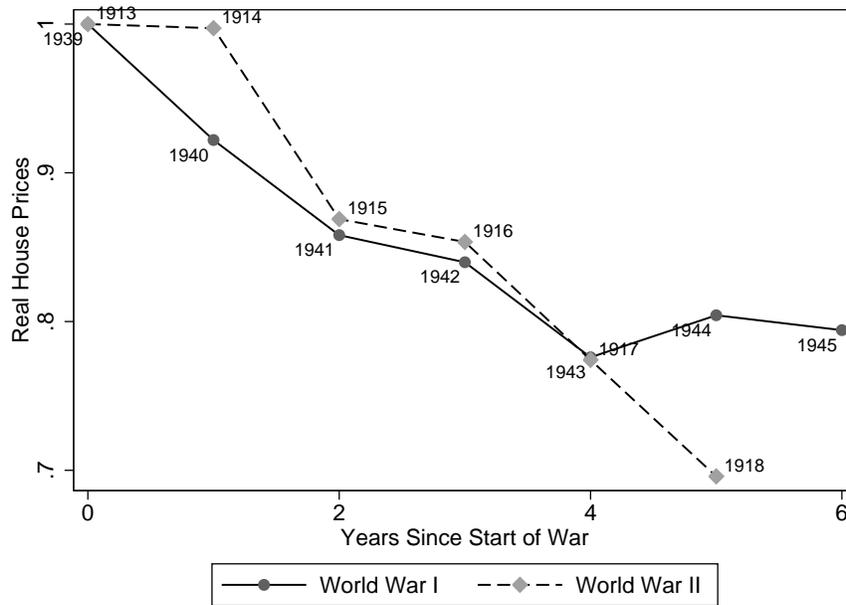
(a) Housing During World Wars

Note: The top panel shows the evolution of real house prices for countries with available house-price time series during World War I (Australia, France, Netherlands, Norway, United States) and World War II (Australia, France, Netherlands, Norway, Switzerland, United States). See Appendix A.3.3 for a description of the data series studied here.

Figure A.35: Rent Growth vs. PCE Growth in U.S.
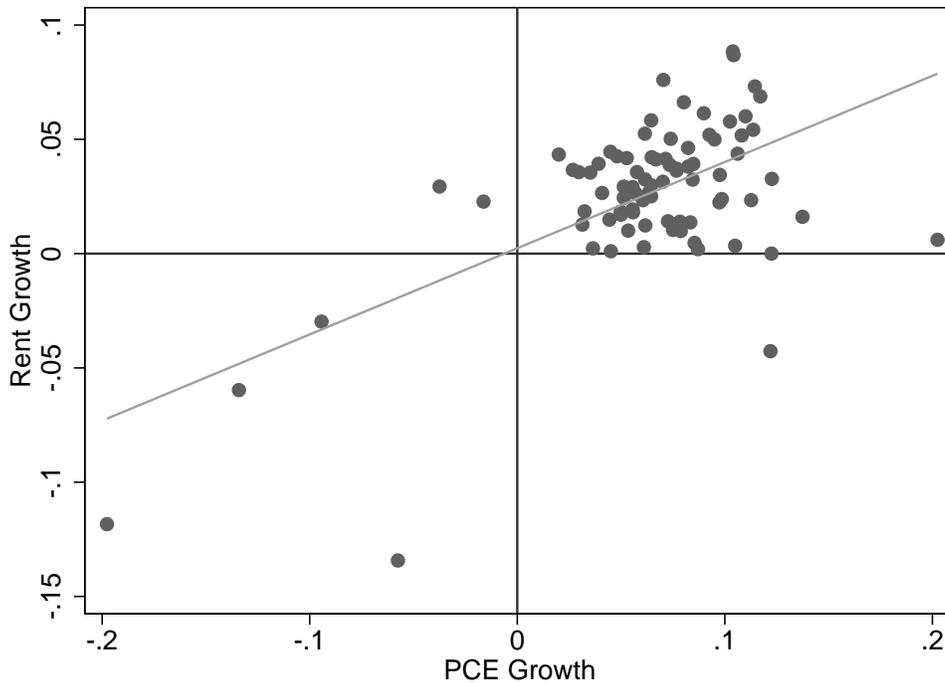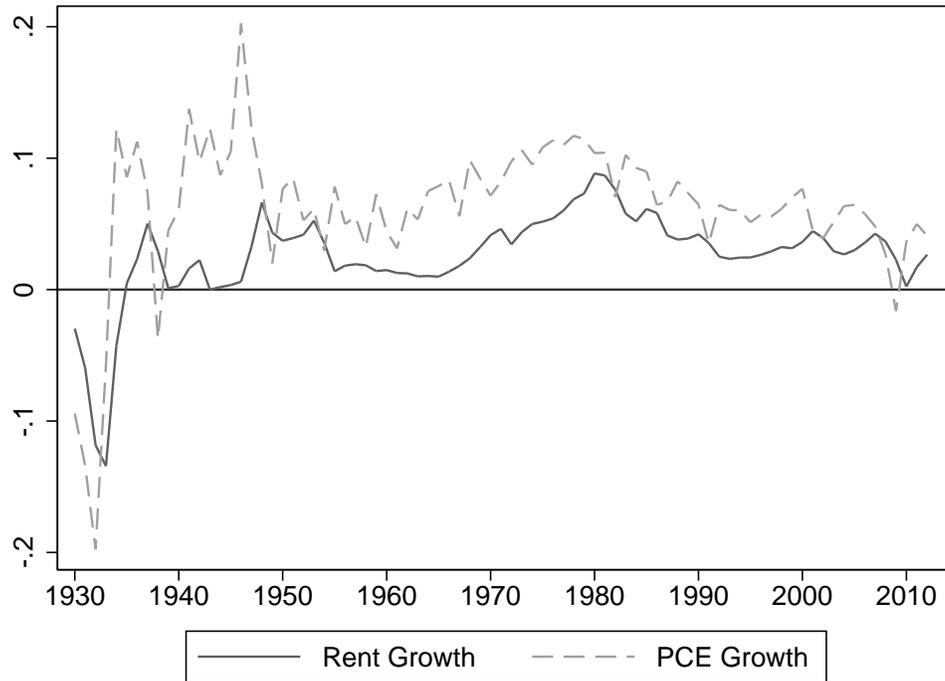
Note: The figure shows the annual growth rates of the "Consumer Price Index for All Urban Consumers: Rent of primary residence" (FRED ID: CUUR0000SEHA) and "Personal Consumption Expenditure" (FRED ID: PCDGA) since 1929.

Figure A.36: House Prices and Rents in Prime Central London Areas during the 2007-09 Financial Crisis
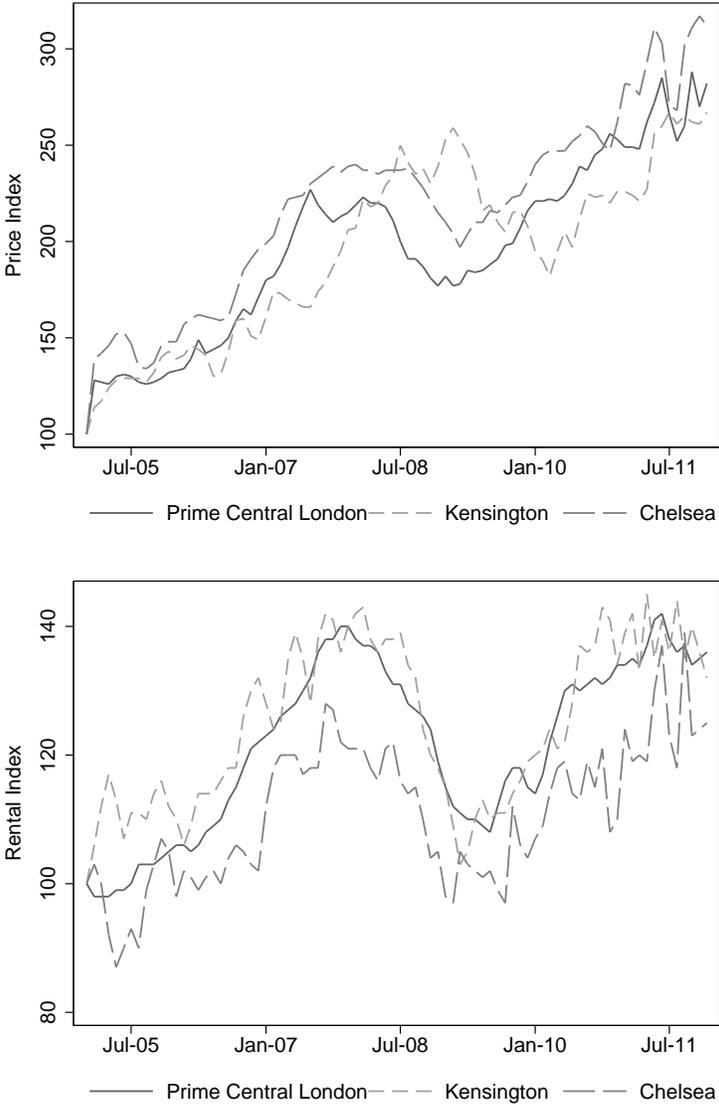
Note: The figure shows the time series of house prices and rents for Prime Central London, Kensington, and Chelsea for the period January 2005 to January 2012. The series are monthly and available from John D Wood & Co. at http://www.johndwood.co.uk/content/indices/london-property-prices/

Figure A.37: UK Gilts Real Yield Curve
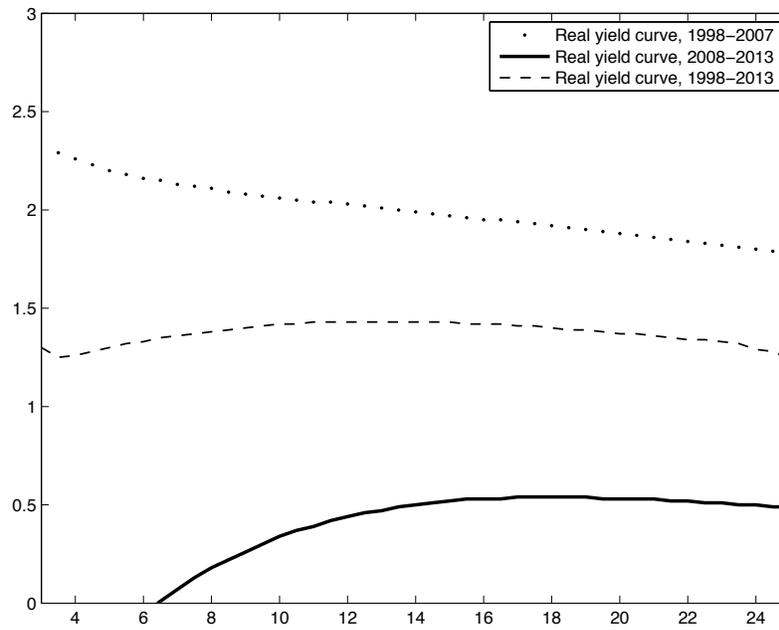
Note: The figure plots the real yield curve for UK gilt bonds as computed by the Bank of England.

Figure A.38: Asset Pricing Models: Model-Implied Discounts vs. Data
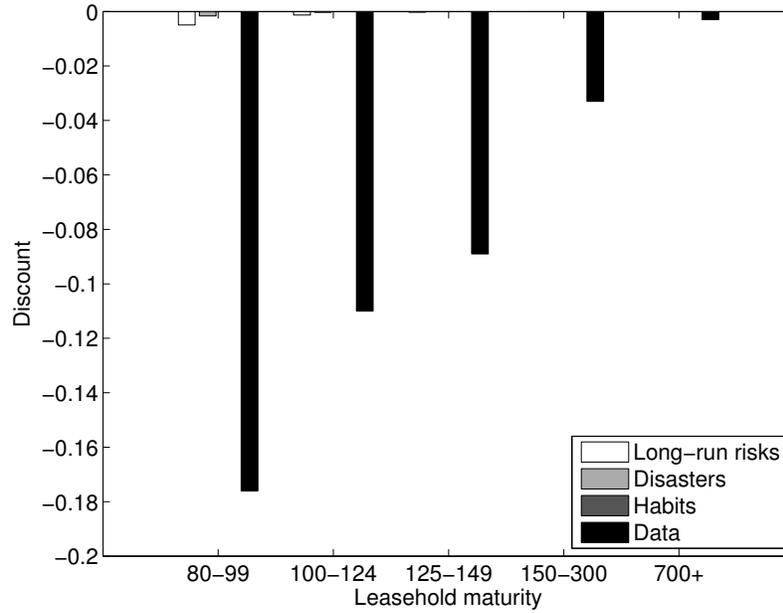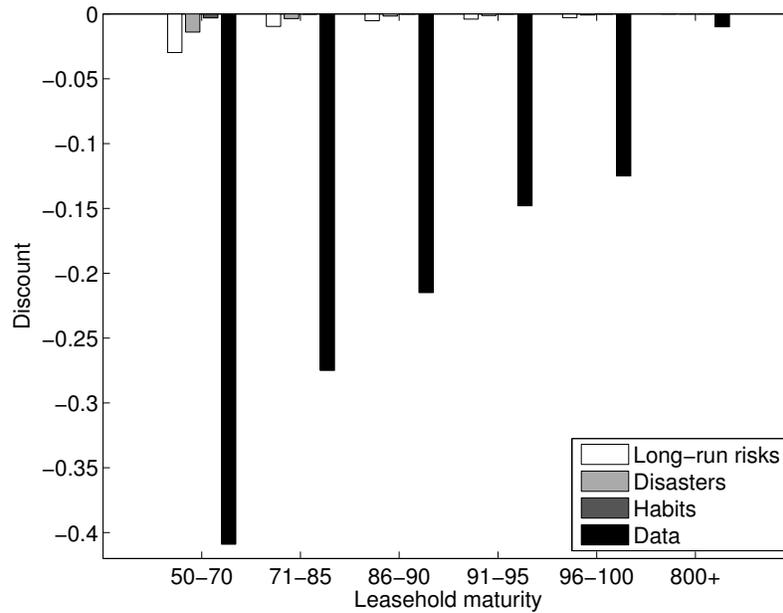
(a) U.K.

(b) Singapore

Note: The figure shows the discounts for leaseholds observed in the U.K. (top panel) and Singapore (bottom panel) together with discounts implied by the long-run risk model, the variable rare-disaster model, and the habit-formation model. The calibrations impose that housing has expected return of 6.5% and growth rate of rents of 0.7%.

Figure A.39: Asset Pricing Models: Discount Rates by Maturity

(a) Long-run risks

(b) Disasters

(c) Habits
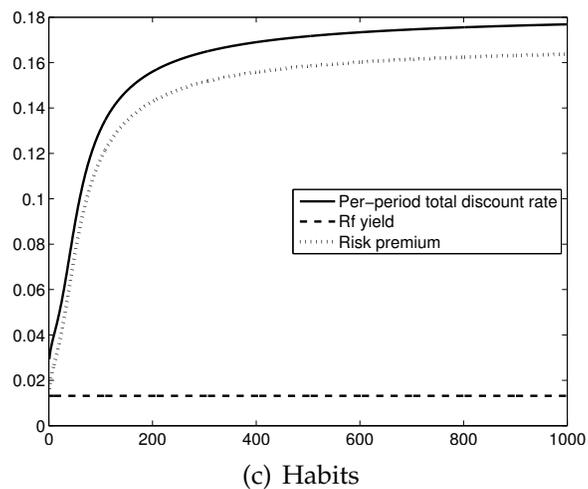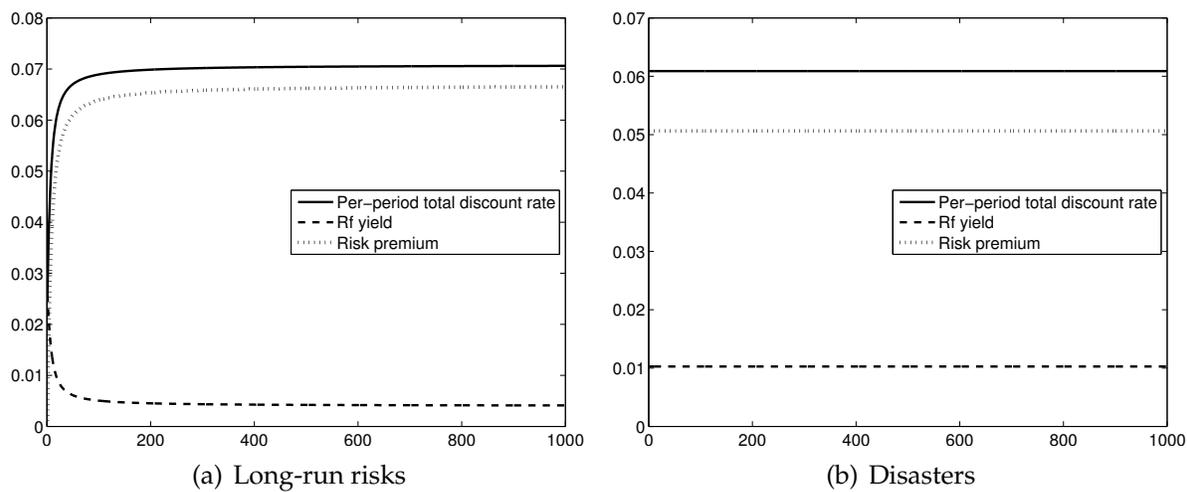
Note: Total per-period discount rates, risk-free yields, and risk premia for leading asset pricing models.

Figure A.40: Financing-Frictions Reduced-Form Model: Model-Implied Discounts vs. Data
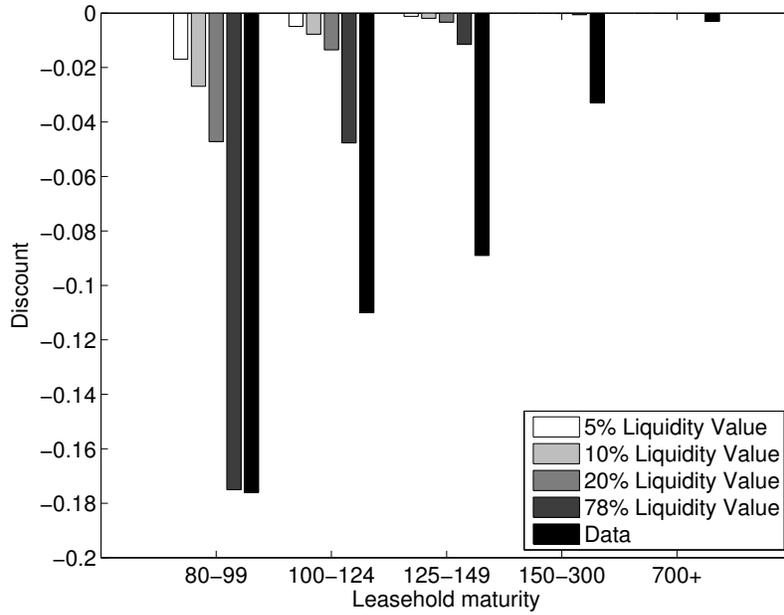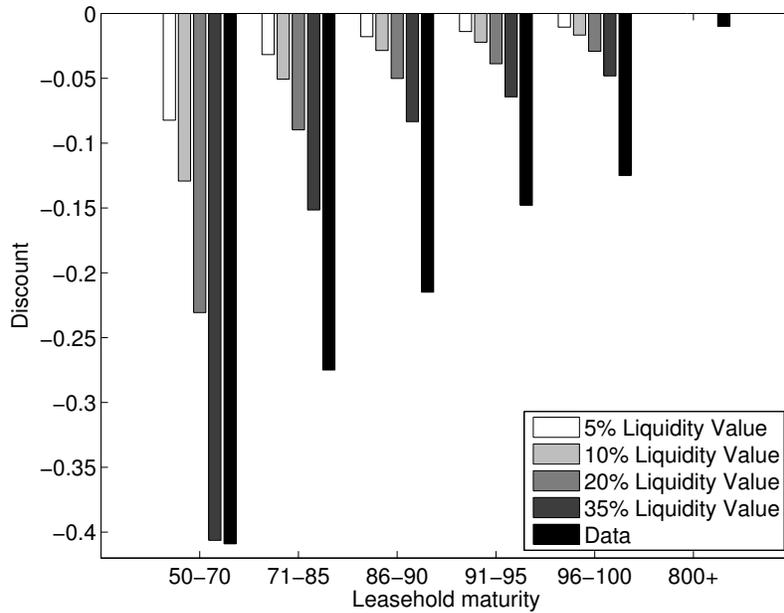
(a) U.K.

(b) Singapore

Note: The figure shows the discounts for leaseholds observed in the U.K. (top panel) and Singapore (bottom panel) together with discounts implied by a parameterizations of the financing-friction reduced form model in Section 3.5 using $r = 6.5\%$, $g = 0.7\%$, and different values of α. The last value of α is chosen so that the shorter-maturity discount is matched exactly.

Appendix Tables

Table A.1: Summary Statistics U.K.: Fraction of leaseholds and freeholds by postcode

| Houses	Mean	Std	10th perc	25th perc	Median	75th perc	90th perc
% Freeholds	0.943	0.128	0.857	0.966	0.986	0.993	0.997
% 80-99	0.008	0.021	0	0.001	0.003	0.007	0.015
% 100-124	0.008	0.032	0	0.001	0.003	0.006	0.012
% 125-200	0.003	0.020	0	0	0	0.001	0.003
% 700+	0.039	0.114	0	0.001	0.004	0.014	0.078

Flats	Mean	Std	10th perc	25th perc	Median	75th perc	90th perc
% Freeholds	0.076	0.133	0.009	0.018	0.038	0.076	0.164
% 80-99	0.177	0.157	0.006	0.052	0.145	0.263	0.388
% 100-124	0.322	0.191	0.070	0.182	0.322	0.440	0.562
% 125-149	0.069	0.077	0	0.021	0.051	0.088	0.153
% 150-300	0.052	0.094	0	0	0.017	0.058	0.138
% 700+	0.304	0.229	0.042	0.126	0.259	0.443	0.636

Note: This table shows descriptive statistics on the fractions of freeholds and leaseholds of each maturity bucket by 3-digit U.K. postcode.

Table A.2: U.K.: Impact of Lease Type on Prices - Houses

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Lease Length Remaining							
80-99 Years	-0.375***	-0.378***	-0.379***	-0.375***	-0.364***	-0.332***	-0.374***
	(0.026)	(0.025)	(0.025)	(0.025)	(0.024)	(0.022)	(0.027)
100-124 Years	-0.296***	-0.298***	-0.298***	-0.297***	-0.286***	-0.273***	-0.290***
	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.017)
> 700 Years	-0.012**	-0.012**	-0.012**	-0.012**	-0.011*	-0.007	-0.011*
	(0.006)	(0.006)	(0.006)	(0.006)	(0.006)	(0.005)	(0.006)
Fixed Effects	PC × M	PC × Q	PC × Y	PC × M	PC × M	PC × M	PC × M
Controls	✓	✓	✓	✓, × year	✓	✓	✓
Restrictions	.	.	.	.	Wisorize Price	Nonmiss. Hedonics	Exclude London
R-squared	0.783	0.781	0.777	0.785	0.790	0.820	0.765
N	6,628,133	6,628,133	6,628,133	6,628,133	6,628,133	5,453,962	6,393,099

Note: This table shows results from regression (1) estimated for houses. The dependent variable is log price, for properties sold in England and Wales between 2004 and 2013. To convert into percentage discounts for leasehold properties we compute $e^{\beta_j} - 1$. We include 3-digit postcode by transaction time fixed effects. In columns (2) and (3) the transaction time is the transaction quarter and year, respectively, in the other columns the transaction month. In column (4) we interact the controls with the transaction year. In column (5) we winsorize the price at the 1st and 99th percentile, in column (6) we only include properties for which characteristics are not missing, and in column (7) we exclude transactions in London. We also control for the size, number of bedrooms, bathrooms, property age, property condition, whether there is parking, and the type of heating. Standard errors are clustered at the level of the fixed effect. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Table A.3: U.K.: Impact of Lease Type on Prices - Flats, relative to 700+ year leaseholds

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Lease Length Remaining							
80-99 Years	-0.173***	-0.173***	-0.174***	-0.172***	-0.165***	-0.144***	-0.170***
	(0.009)	(0.008)	(0.008)	(0.008)	(0.008)	(0.009)	(0.010)
100-124 Years	-0.107***	-0.104***	-0.101***	-0.105***	-0.100***	-0.099***	-0.069***
	(0.009)	(0.008)	(0.008)	(0.009)	(0.009)	(0.010)	(0.007)
125-149 Years	-0.086***	-0.084***	-0.081***	-0.085***	-0.078***	-0.074***	-0.056***
	(0.010)	(0.009)	(0.009)	(0.009)	(0.009)	(0.009)	(0.008)
150-300 Years	-0.030**	-0.031***	-0.030**	-0.029***	-0.024*	-0.015	-0.008
	(0.013)	(0.012)	(0.011)	(0.0012)	(0.012)	(0.012)	(0.012)
Fixed Effects	PC × M	PC × Q	PC × Y	PC × M	PC × M	PC × M	PC × M
Controls	✓	✓	✓	✓, × year	✓	✓	✓
Restrictions	.	.	.	.	Wisorize Price	Nonmiss. Hedonics	Exclude London
R-squared	0.732	0.724	0.714	0.733	0.740	0.778	0.620
N	1,338,244	1,338,244	1,338,244	1,338,244	1,338,244	931,198	996,907

Note: This table shows results from regression (1) estimated for flats, excluding freeholds and computing the discounts relative to the 700+ leaseholds. The dependent variable is log price, for properties sold in England and Wales between 2004 and 2013. To convert into percentage discounts for leasehold properties we compute $e^{\beta_j} - 1$. We include 3-digit postcode by transaction time fixed effects. In columns (2) and (3) the transaction time is the transaction quarter and year, respectively, in the other columns the transaction month. In column (4) we interact the controls with the transaction year. In column (5) we winsorize the price at the 1st and 99th percentile, in column (6) we only include properties for which characteristics are not missing, and in column (7) we exclude transactions in London. We also control for the size, number of bedrooms, bathrooms, property age, property condition, whether there is parking, and the type of heating. Standard errors are double clustered by 3-digit postcode and by year. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Table A.4: Impact of Lease Type on Log(Price) - Singapore, Relative to 96-100 Year Lease

| | RELATIVE TO 96-100 YEAR LEASES | | | |
	(1)	(2)	(3)	(4)
Lease Length Remaining				
50-70 Years	-0.376***	-0.469***	-0.489***	-0.599***
	(0.105)	(0.098)	(0.105)	(0.151)
71-85 Years	-0.348***	-0.358***	-0.327***	-0.325***
	(0.075)	(0.084)	(0.084)	(0.092)
86-90 Years	-0.221***	-0.223***	-0.207***	-0.196***
	(0.046)	(0.047)	(0.053)	(0.054)
91-95 Years	-0.058	-0.072**	-0.067**	-0.056
	(0.039)	(0.028)	(0.032)	(0.043)
Fixed Effects	PC × Y × Prop Type × Title Type	PC × Q × Prop Type × Title Type	PC × M × Prop Type × Title Type	PC × M × Prop Type × Title Type
Controls	✓	✓	✓	✓
Restrictions	.	.	.	New Only
R-squared	0.955	0.967	0.970	0.969
N	172,690	172,690	172,690	82,408

Note: This table shows results from regression (2). To convert into percentage discounts for leasehold properties relatives to freeholds, construct $e^{\beta_j} - 1$. The dependent variable is the price paid for strata properties with initially 99-year leases sold by private parties in Singapore between 1995 and 2013. The excluded category is for transactions with leases with 96-100 years remaining. We include fixed effect at the 5-digit postcode by property type (apartment, condominium, detached house, executive condominium, semi-detached house and terrace house) by title type (Strata or Land) by transaction date. In column (1), the transaction date interaction is for the transaction quarter, in columns (2) - (6) the transaction month. We control for the age of the property (by including a dummy variable for every possible age in years), the size of the property (by including a dummy for each of 40 equally sized groups capturing property size), and the total number of units in the property. In column (3) we only focus on properties that were bought by a private individual (and not the HDB); in column (4) we only focus on properties that were built within the last 3 years of our transaction date. In columns (5) and (6) we conduct the analysis for Strata and non-Strata titles separately. Standard errors are double clustered by 5-digit postcode and by year. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Table A.5: Rent-Price Ratio Singapore - 2012

	(1)	(2)	(3)	(4)
For-Rent Dummy	-3.095***	-3.131***	-3.123***	-3.107***
	(0.044)	(0.019)	(0.014)	(0.025)
Fixed Effects	Quarter × Postal Code	Quarter × Postal Code	Month × Postal Code	Month × Postal Code × Bedrooms
Controls	.	✓	✓	✓
Implied Rent-Price Ratio	4.5%	4.4%	4.4%	4.5%
R-squared	0.804	0.873	0.872	0.872
N	106,145	105,189	105,189	105,189

Note: This table shows results from regression (A.2). To convert into rent-price ratios, we take e^β. The dependent variables is the price (for-sale price or annualized for-rent price) for properties listed on iProperty.com in Singapore in 2012. Fixed effects are included as indicated. In columns (2) and (4) we also control for characteristics of the property: we include dummy variables for the type of property (condo, house, etc.), indicators for the number of bedrooms and bathrooms, property age, property size (by adding dummy variables for 50 equal-sized buckets), information on the kitchen (ceramic, granite, etc.), which floor the property is on and the tenure type for leaseholds. Standard errors are clustered at the level of the fixed effect. Significance Levels: * (p<0.10), ** (p<0.05), *** (p<0.01).

Table A.6: House Prices, Banking Crises, Rare Disasters

	House Price Index Time Period	Banking Crises	Rare Disasters
Australia	1880 - 2013	1893, 1989	1918, 1932, 1944
Belgium	1975 - 2012	2008	
Canada	1975 - 2012		
Denmark	1975 - 2012	1987	
Finland	1975 - 2012	1991	1993
France	1840 - 2010	1882, 1889, 1907, 1930, 2008	1871, 1915, 1943
Germany	1975 - 2012	2008	
Japan	1975 - 2012	1992	
Italy	1975 - 2012	11990, 2008	
Netherlands	1649 - 2009	1893, 1907, 1921, 1939, 2008	1893, 1918, 1944
New Zealand	1975 - 2012	1987	
Norway	1819 - 2013	1899, 1922, 1931, 1988	1918, 1921, 1944
Singapore	1975 - 2012	1982	
South Africa	1975 - 2012	1977, 1989	NA
South Korea	1975 - 2012	1986, 1997	1998
Spain	1975 - 2012	1978, 2008	
Sweden	1952 - 2013	1991, 2008	
Switzerland	1937 - 2012	2008	1945
U.K.	1952 - 2013	1974, 1984, 1991, 2007	
U.S.	1890 - 2012	1893, 1907, 1929, 1984, 2007	1921, 1933

Note: The table shows time series availability for house price indices in the second column. The third and fourth column report dates of banking crises or consumption rare disasters if any occur for the country in the time period provided in the first column. Banking crises dates for all countries, except Singapore, Belgium, Finland, New Zealand, South Korea, and South Africa, are from Schularick and Taylor (2012). Banking crises dates for the countries not covered by Schularick and Taylor (2012) are from Reinhart and Rogoff (2009). Rare disasters dates are the year of the trough in consumption during a consumption disaster as reported by Barro and Ursua (2008). NA means that the country is not covered by the source dataset.

Table A.7: Time Series Properties of Real House Price Growth

| | Real HP Growth | | Real PDI Growth | | |
	Mean	Std. Dev.	Mean	Std. Dev.	Correlation
Australia	3.20%	6.89%	1.43%	2.77%	0.093
Belgium	2.80%	5.87%	1.17%	2.27%	0.436
Canada	2.51%	7.63%	1.37%	2.10%	0.489
Switzerland	0.94%	4.73%	1.12%	1.63%	0.445
Germany	-0.29%	2.31%	1.27%	1.70%	0.288
Denmark	1.57%	8.99%	1.09%	2.29%	0.211
Spain	2.05%	8.26%	0.83%	2.46%	0.631
Finland	2.04%	8.19%	2.07%	3.21%	0.482
France	2.52%	5.23%	1.22%	1.58%	0.358
U.K.	3.53%	8.54%	2.20%	2.74%	0.355
Italy	0.60%	8.28%	0.82%	2.44%	0.325
Japan	-0.24%	4.28%	1.55%	1.40%	0.587
S. Korea	0.59%	7.70%	3.95%	4.58%	0.235
Luxembourg	3.94%	6.68%	2.84%	3.75%	0.054
Netherlands	2.32%	9.43%	0.48%	3.25%	0.472
Norway	2.76%	7.23%	2.22%	2.52%	0.064
New Zealand	2.20%	7.73%	0.98%	3.45%	0.530
Sweden	1.50%	7.27%	1.34%	2.28%	0.431
U.S.	1.13%	3.89%	1.60%	1.56%	0.371
S. Africa	0.88%	9.65%	0.53%	3.05%	0.373

Note: This table shows time series properties of quarterly frequency annual growth rats of real house prices and personal disposable income between 1975 and Q2 2013, as collected by Mack and Martínez-García (2011). Columns (1) and (2) show the mean and standard deviation of real house price growth. Columns (3) and (4) the mean and standard deviation of real personal disposable income growth. Column (5) shows the correlation of real house price growth with real personal disposable income growth.