Online Appendix to History Remembered:
Optimal Sovereign Default on Domestic and External Debt

by

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This Appendix is divided in nine sections. Section A-1 presents a Table with summary indicators of the fiscal situation of the main Eurozone countries in 2011. Section A-2 contains a detailed description of the data sources and transformations for the various macro variables used in the analysis. Section A-3 describes the solution method used to solve for the model's Recursive Markov Equilibrium. Section A-4 offer additional details on the default event analysis. Section A-5 offers an analysis of the model's time-series dynamics between two representative default events. Section A-6 provides further analysis of the recursive equilibrium functions, particularly the individual welfare gains of default and the optimal debt decision rule. Section A-7 contains a more detailed comparison of the welfare weights versus the average bond distribution, looking at marginal distributions over different income levels. Section A-8 discusses the results under a calibration to Spain. Finally, Section A-9 presents the algorithm used to solve the model with endogenous partial default.

A-1 Eurozone Fiscal Situation in 2011

Table A.1: Eurozone Fiscal Situation in 2011

Moment in (%)	Gov. Debt	Gov. Debt Held by Residents	Gov. Exp.	Gov. Rev.	Primary Balance	Sov. Spreads
France	62.73	46.17	24.48	50.60	-2.51	0.71
Germany	51.49	44.47	19.27	44.50	1.69	0.00
Greece	133.10	29.68	17.38	42.40	-2.43	13.14
Ireland	64.97	45.35	18.38	34.90	-9.85	6.99
Italy	100.23	64.33	20.42	46.20	1.22	2.81
Portugal	75.84	37.36	20.05	45.00	-0.29	7.63
Spain	45.60	66.00	20.95	35.70	-7.04	2.83
Austria	52.92	38.03	19.78	48.55	-0.45	0.71
Belgium	83.58	54.73	23.77	50.31	-0.91	1.62
Finland	-48.79	23.90	23.62	53.34	-1.02	0.40
Netherlands	37.19	44.76	25.99	42.68	-3.04	0.38
Avg	59.90	44.98	21.28	44.93	-2.24	3.38
Median	62.73	44.76	20.42	45.00	-1.02	1.62
Avg (GDP w)	62.93	50.14	21.45	45.25	-1.20	2.37

Note: Author's calculations based on OECD Statistics, Eurostat and European Central Bank (ECB).“Gov. Debt” corresponds to total general government net financial liabilities as a fraction of GDP; “Gov. Debt Held by Residents” refers to fraction of gross government debt held by domestic non-financial corporations, financial institutions, other government sectors, households and non-profit institutions; “Gov. Exp.” is general government final consumption as a fraction of GDP; “Gov. Rev.” corresponds to general government revenues as a fraction of GDP. “Prim. Balance” corresponds to the primary balance (total expenditures net of interest payments minus total revenue) as a fraction of GDP; and “Sov Spreads” correspond to the difference between interest rates of the given country and Germany (for bonds of similar maturity). For a given country i, spreads are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$. See Appendix A-2 for a detailed explanation of variables and sources.

A-2 Data Description and Sources

This Appendix describes the variables we gathered from the data and the sources. Most data cover the 1981-2015 period, but for some variables the sample starts in 2002. Most of the moments used for the calibration correspond to GDP-weighted averages of country specific moments. The countries we use for this calibration are Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain. We calculate the weights using real GDP data from 2007 (the year prior to the start of the crisis). The weights for Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal, Spain are 0.031, 0.038, 0.021, 0.212, 0.273, 0.026, 0.019, 0.177, 0.067, 0.019, and 0.117, respectively.

The details are as follows:

1. Government debt: total general government net financial liabilities as a fraction of GDP, from OECD Statistics for the period 1981−2015.[35]

2. Fraction of government debt held by residents (also referenced in the paper as fraction of domestic debt): corresponds to fraction of general government gross debt held by domestic investors in the IMF dataset put together by Arslanalp and Tsuda [10]. We extended the data when necessary to complete the 1981−2015 sample using information from OECD Statistics on the fraction of marketable debt held by residents as a fraction of total marketable debt. The correlation between both series when they overlap is equal to 0.84.

3. Government expenditures: general government final consumption as a fraction of GDP from World Development Indicators for the period 1981-2015.

4. Government revenue: Total general government revenue as a fraction of GDP from OECD statistics for the period 1981-2015.

5. Sovereign spreads: constructed using EMU convergence criterion bond yields from Eurostat for the period 2002-2015. For a given country i, spreads are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1.$, where r^{Ger} is the yield on German bonds. Data before 2002, prior to the introduction of the euro, are excluded because spreads were heavily influenced by currency risk, and not just sovereign risk. The GDP-weighted average in this case re-normalizes weights because the average is computed without Germany (the country use as reference for the risk-free rate).

6. Cross sectional variance of log-wages (needed to calibrate the income process) obtained from the cross-sectional variance of residual log-earnings in Germany, Italy and Spain as reported by Fuchs-Schundeln, Krueger and Sommer [28] (Germany), Japelli and Pistaferri [32] (Italy), and Pijoan-Mas and Sanchez Marcos [47] (Spain).

7. Income net of fixed investment (μ_y): constructed as GDP minus gross capital formation (formerly gross domestic investment) as a ratio of GDP, from World Development Indicators for the period 1981-2015.

[35] At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

8. Maturity adjusted debt ratio: computed using the Macaulay duration rate. The Macaulay duration for a consol is $D = \frac{1+r^*}{r^*+\delta}$, where r^* is the consol's constant annual yield. Denoting the observed outstanding debt as $\overline{B}$ and the equivalent one-period debt at the beginning of the period (i.e. the maturity-adjusted debt) as B, we use δ to express $\overline{B}$ as the present value of outstanding coupon claims $\overline{B} = \sum_{s=1}^{\infty} \frac{B(1-\delta)^{s-1}}{(1+r^*)^{s-1}}$, which then reduces to the expression noted in the text:

$$\overline{B} = \frac{B(1+r^*)}{(r^*+\delta)}.$$

   Duration is calibrated to average term to maturity of central government debt. Source: OECD statistics for the period 2002-2010. OECD stopped updating this dataset after 2010.

9. Tax revenue: defined to include only effective labor taxes levied on individuals, accruing to both individual labor income and consumption taxes, and excluding all forms of capital income taxation. Consumption tax revenues and the split of labor and capital components of individual income taxes are obtained using the effective tax rates constructed by Mendoza, Tesar, and Zang [41]) using OECD data for the period 1995-2015.

10. Government transfers: measured as a residual using the government budget constraint. Hence, transfers are equal to transfer and entitlement payments, plus other outlays (total outlays minus current expenditures, debt service and transfers), minus tax revenue other than effective labor taxes, plus the difference between net lending in the general government national accounts and the change in reported net general government financial liabilities. Data from OECD Statistics for the period 1995-2015.

11. Household disposable income: corresponds to gross household disposable income at constant 2010 prices from OECD Statistics (downloaded from Bloomberg) for the period 1981-2015.

12. Trade balance: external balance on goods and services as a fraction of GDP, from World Development Indicators for the period 1981−2015.

Table A.2: Country Specific Moments (averages)

	GDP-weights	G/Y	μ_y	ρ_g	σ_e	B^d/B
France	0.212	22.60	77.96	0.87	0.019	56.45
Germany	0.273	19.24	77.98	0.80	0.021	50.43
Greece	0.026	18.79	76.49	0.88	0.045	25.47
Ireland	0.019	17.72	78.02	0.93	0.061	40.09
Italy	0.177	18.85	79.27	0.82	0.024	62.50
Portugal	0.019	17.46	75.56	0.94	0.033	35.34
Spain	0.117	17.27	75.77	0.94	0.026	73.23
Austria	0.031	18.95	75.07	0.85	0.019	54.30
Belgium	0.038	22.19	77.76	0.92	0.023	52.43
Finland	0.021	21.52	75.98	0.89	0.038	23.33
Netherlands	0.067	22.99	78.27	0.94	0.030	58.39
GDP-weighted avg		19.98	77.74	0.86	0.024	55.53
	Spreads†	B/Y^*	D	$(B/Y)/D$	Tax Rev	τ
France	0.33	31.18	6.52	4.78	33.57	10.02
Germany	0.00	38.47	6.11	6.29	30.33	9.82
Greece	4.88	90.07	7.13	12.63	25.70	10.55
Ireland	1.55	33.04	5.62	5.88	22.78	5.61
Italy	1.21	90.40	6.59	13.72	27.95	4.96
Portugal	2.21	60.52	5.24	11.54	24.41	4.83
Spain	1.15	37.25	6.42	5.80	25.18	6.11
Austria	0.34	43.84	7.46	5.87	35.14	14.23
Belgium	0.53	101.60	6.29	16.14	33.10	4.28
Finland	0.22	-35.31	4.06	-8.71	36.01	12.17
Netherlands	0.21	31.20	6.38	4.89	30.53	6.27
GDP-weighted avg	0.92	48.23	6.35	7.45	30.00	8.15

Note: † GDP-weighted spreads use GDP-weights re-normalized for a sample that excludes Germany. * At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

Table A.3: Country Specific Moments (Peak-Crisis)

	GDP-weights	B/Y^*	D	$(B/Y)/D$	B^d/B	Tax Rev	τ	G/Y	Spreads†
France	0.21	67.44	6.60	10.22	47.59	32.64	15.13	23.93	1.04
Germany	0.27	49.34	5.89	8.38	51.46	29.69	13.86	19.56	0.00
Greece	0.03	101.78	7.10	14.34	29.69	22.80	24.60	23.31	21.00
Ireland	0.02	79.28	4.31	18.40	45.35	21.13	22.62	20.19	6.99
Italy	0.18	111.77	6.80	16.44	67.65	28.19	20.54	20.63	3.99
Portugal	0.02	90.55	5.77	15.69	40.40	23.74	25.26	21.43	9.05
Spain	0.12	59.12	6.40	9.24	73.74	22.93	15.88	20.52	4.35
Austria	0.03	57.90	8.30	6.98	38.30	34.33	19.16	20.58	0.87
Belgium	0.04	92.22	5.94	15.53	55.88	31.99	14.48	24.26	1.62
Finland	0.02	48.79	3.90	12.51	25.41	34.37	28.06	20.70	0.52
Netherlands	0.07	39.69	6.60	6.01	47.32	30.70	15.02	22.06	0.47
GDP-weighted avg		69.36	6.34	10.94	54.15	29.20	16.78	21.34	3.34

Note: † GDP-weighted spreads use GDP-weights re-normalized for a sample that excludes Germany. Peak-Crisis duration refers to the minimum value during 2008-2010. * At present, and as opposed to other countries in our sample, the financial assets of Finland's private pension system are included in the balance sheet of the general government. For this reason, its net financial liabilities are negative.

Table A.4: Country Specific Moments (Business Cycle Correlations)

	France	Ger.	Greece	Ireland	Italy	Port.	Spain	Austria	Belgium	Finland	Neth.	Avg.	Min	Max
Standard Dev.														
$hhdi$	0.85	0.65	n.a.	2.62	1.07	1.57	1.44	0.86	1.03	1.62	1.89	1.05	0.65	2.62
Consumption	0.82	1.12	1.18	1.26	0.98	0.99	0.83	0.36	0.47	0.69	0.37	0.89	0.36	1.26
TB/GDP	0.55	1.05	0.56	0.90	0.55	0.67	0.57	0.51	0.55	0.48	0.29	0.68	0.29	1.05
Spreads	0.16	n.a.	1.85	0.40	0.47	0.92	0.40	0.14	0.21	0.05	0.04	0.35	0.04	1.85
B/GDP	3.42	3.49	4.07	0.06	2.60	2.73	1.60	2.22	2.50	3.57	1.39	2.82	0.06	4.07
B^d/GDP	2.87	2.14	1.11	0.03	2.54	1.13	1.61	1.87	3.14	0.74	1.09	2.15	0.03	3.14
Correl(x,$hhdi$)														
Consumption	0.76	0.56	n.a.	0.32	0.67	0.82	0.46	0.48	0.45	0.49	0.51	0.60	0.32	0.82
TB/GDP	-0.27	-0.01	n.a.	0.36	-0.50	-0.74	-0.33	0.11	-0.09	-0.48	0.08	-0.21	-0.74	0.36
Spreads	-0.24	n.a.	n.a.	-0.23	-0.60	-0.52	-0.61	-0.04	-0.25	-0.20	0.39	-0.33	-0.61	0.39
B/GDP	-0.24	-0.25	n.a.	-0.58	-0.47	-0.18	-0.21	-0.04	-0.18	-0.24	-0.04	-0.26	-0.58	-0.04
B^d/GDP	-0.32	-0.15	n.a.	-0.57	-0.53	0.32	-0.36	0.10	-0.57	0.23	0.02	-0.27	-0.57	0.32
Correl(x,g/GDP)														
$hhdi$	-0.33	-0.19	n.a.	-0.13	0.04	0.05	0.41	-0.10	0.11	0.08	-0.26	-0.09	-0.33	0.41
Consumption	-0.69	-0.31	0.02	-0.47	-0.17	0.18	-0.19	-0.13	-0.15	-0.41	-0.13	-0.31	-0.69	0.18
TB/GDP	0.40	-0.11	0.01	-0.20	-0.11	-0.07	0.27	-0.11	0.15	-0.19	-0.44	0.03	-0.44	0.40
Spreads	0.11	n.a.	0.07	-0.14	-0.21	-0.45	-0.23	0.27	0.20	0.45	-0.34	-0.07	-0.45	0.45
B/GDP	0.47	0.47	0.33	0.31	0.46	0.23	0.20	0.48	0.66	0.01	0.09	0.40	0.01	0.66
B^d/GDP	0.24	0.71	0.25	0.02	0.32	0.33	-0.19	0.38	0.44	0.20	-0.03	0.32	-0.19	0.71

Note: TB denotes trade-balance. $hhdi$ denotes household disposable income. In the model, $hhdi = (1 - \tau^y)Y + \tau$ and $TB = Y - C - g$. $hhdi$ and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations (except that for $hhdi$) are ratios to the standard deviations of $hhdi$ ($hhdi$ data for Greece is not available, so in this case we provide the ratio to the standard deviation of GDP). GDP-weighted moments for spreads use GDP-weights re-normalized for a sample that excludes Germany.

A-3 Computational Algorithm

This Appendix describes the algorithm we constructed to solve for the model's CRME and RME. The algorithm performs a global solution using value function iteration. We approximate the solution of the infinite horizon economy by solving for the equilibrium of a finite-horizon version of the model for which the finite number of periods (T) is set to a number large enough such that the distance between value functions, government policies and bond prices in the first and second periods are the same up to a convergence criterion. The corresponding first-period functions are then treated as representative of the solution of the infinite-horizon economy.

The algorithm has a backward-recursive structure with the following steps:

1. Define a discrete state space of values for the aggregate states $\{B, g\}$ and individual states $\{b, y\}$

2. Solve for date-T recursive functions for each $\{b, y\}$ and $\{B, g\}$:

   - Government debt choice: $B'_T(B, g) = 0$, because T is the final period of the economy.
   - Price Debt: $q_T(B', g) = 0$, also because T is the final period.
   - The lump-sum tax under repayment follows from the government budget constraint:

     $$\tau_T(B', B, g) = B + g - \tau^y Y.$$

   - Using the agents' budget constraint under repayment, we obtain the agents' value function for arbitrary debt choice (note that at T it is actually independent of $\tilde{B}$ since $q_T(B', g) = 0$)

     $$\tilde{V}_T^{d=0}(\tilde{B}, y, b, B, g) = u((1 - \tau^y)y + b - g - B + \tau^y Y)$$

   - The agents' value functions under repayment and default can then be solved for as:

     $$V_T^{d=0}(y, b, B, g) = \tilde{V}_T^{d=0}(0, y, b, B, g).$$

     $$V_T^{d=1}(y, g) = u((1 - \tau^y)y(1 - \phi(g)) - g + \tau^y Y).$$

   - Given the above, the social welfare functions under repayment and default are:

     $$W_T^{d=0}(B, g) = \int_{Y \times B} V_T^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_T^{d=1}(g) = \int_{Y\times B} V_T^{d=1}(y,g)d\omega(b,y).$$

- The default decision rule can then be obtained as:

$$d_T(B,g) = \arg\max_{d=\{0,1\}} \{W_T^{d=0}(B,g), W_T^{d=1}(g)\}.$$

- The agents' ex-ante value function (before the default decision is made) is:

$$V_T(y,b,B,g) = (1-d_T)V_T^{d=0}(y,b,B,g) + d_T V_T^{d=1}(y,g).$$

3. Obtain the solution for periods $t = T-1,\ldots,1$.

   (a) Set $t = T-1$.

   (b) Obtain the default probability for all $\{B', g\}$ as:

$$p_t(B',g) = \sum_{g'} d_{t+1}(B',g')F(g',g).$$

   (c) Solve for the pricing function $q_t(B',g)$:

$$q_t(B',g) = \frac{1-p_t(B',g)}{1+r}.$$

   (d) Given the above, the lump-sum tax under repayment for an initial (B,g) pair and a given B' is:

$$\tau_t(B',B,g) = B + g - q_t(B',g)B' - \tau^y Y.$$

   (e) Solve the agents' optimization problem for each agent with bonds and income b, y and each triple $\{\tilde{B}, B, g\}$:

$$\tilde{V}_t^{d=0}(\tilde{B},y,b,B,g) = \max_{b'} u(c) + \beta E_{g'}[V_{t+1}(b',y',\tilde{B},g')]$$

   s.t.

$$c = (1-\tau^y)y + b - q_t(\tilde{B},g)b' - \tau_t(\tilde{B},B,g).$$

   (f) Given the solution to the above problem, solve for the optimal debt choice of the government:

$$B_t'(B,g) = \arg\max_{\tilde{B}} \int \tilde{V}_t^{d=0}(\tilde{B},y,b,B,g)d\omega(b,y).$$

(g) The agents' continuation value under repayment is:

$$V_t^{d=0}(y, b, B, g) = \tilde{V}_t^{d=0}(B_t'(B, g), y, b, B, g).$$

(h) The agents' continuation value under default is:

$$V_t^{d=1}(y, g) = u((1 - \tau^y)y(1 - \phi(g)) - g + \tau^y Y) + \beta E_{g'}[V_{t+1}^{d=0}(y', 0, 0, g')]$$

(i) Given the above, the social welfare functions under repayment and default are:

$$W_t^{d=0}(B, g) = \int_{Y \times B} V_t^{d=0}(y, b, B, g) d\omega(b, y)$$

$$W_t^{d=1}(g) = \int_{Y \times B} V_t^{d=1}(y, g) d\omega(b, y).$$

(j) Compute the government's default decision as:

$$d_t(B, g) = \arg \max_{d=\{0,1\}} \{W_t^{d=0}(B, g), W_t^{d=1}(g)\}.$$

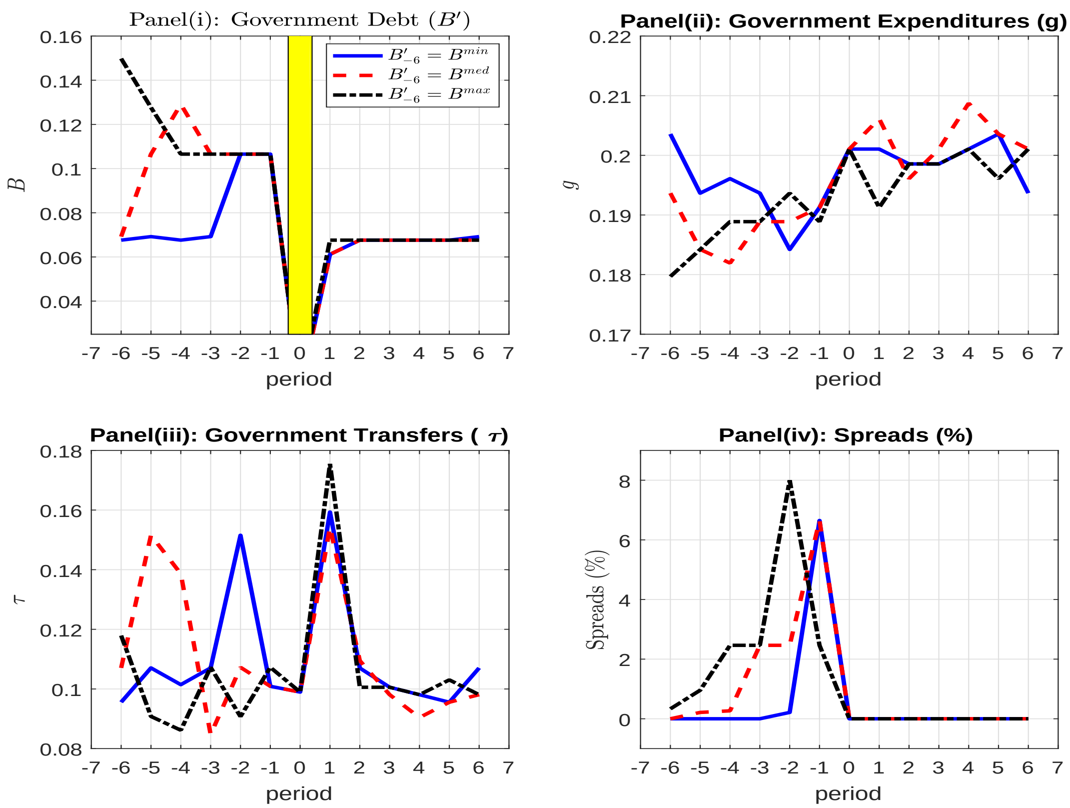
(k) If $t > 1$, set $t = t - 1$ and return to point 3b. If $t = 1$ continue.

4. Check whether value functions, government decision rules, and bond prices in periods $t = 1$ and $t = 2$ satisfy a convergence criterion. If they do, the functions in period $t = 1$ are the solution of the RME and the algorithm stops. If the convergence criterion fails, increase T and return to Step 2.

A-4 Default Event Analysis Extended

Figure A.1 presents the evolution of debt, government expenditures, transfers, and spreads across three different default events: one with the maximum level of debt at the beginning of the default event window (denoted by $B_6 = B^{max}$), other with median level of debt in period $t = -6$ (denoted by $B_6 = B^{med}$ is the same event presented in Figure 3 in the body of the paper), and one with the lowest debt level observed at the beginning of the default window (denoted by $B_5 = B^{min}$).

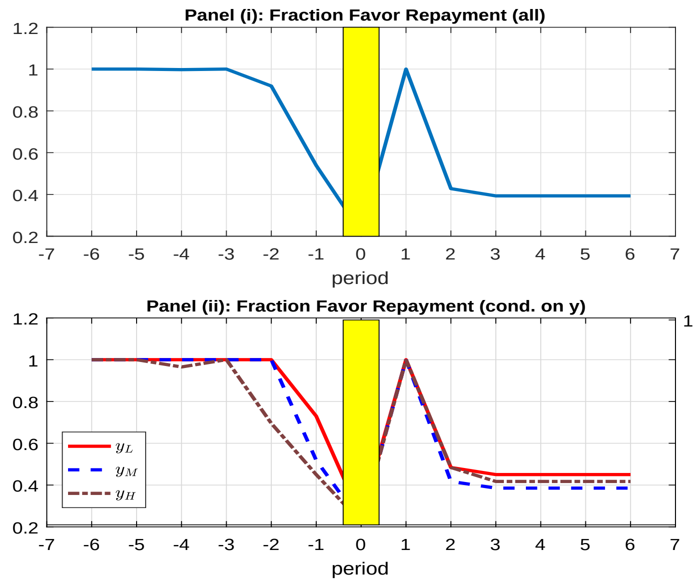
Figure A.1: Default Event Analysis



We observe the same pattern across default events. As government expenditures decrease, the government has more room to redistribute and that results in an increase in the debt level and lump-sum transfer.

Figure A.2 shows event windows for the government's perceived fraction of agents who prefer repayment (i.e., the fraction of agents for whom $\alpha(b, y, B, g) < 0$ obtained by aggregating using the social welfare weights $\omega(b, y)$), again using medians across each of the 121 defaults events for each of the 13 periods in the windows. Panel (i) aggregates across all (b, y) and Panel (ii) splits the results into low, mean and high income levels.

Figure A.2: Preferences over Repayment



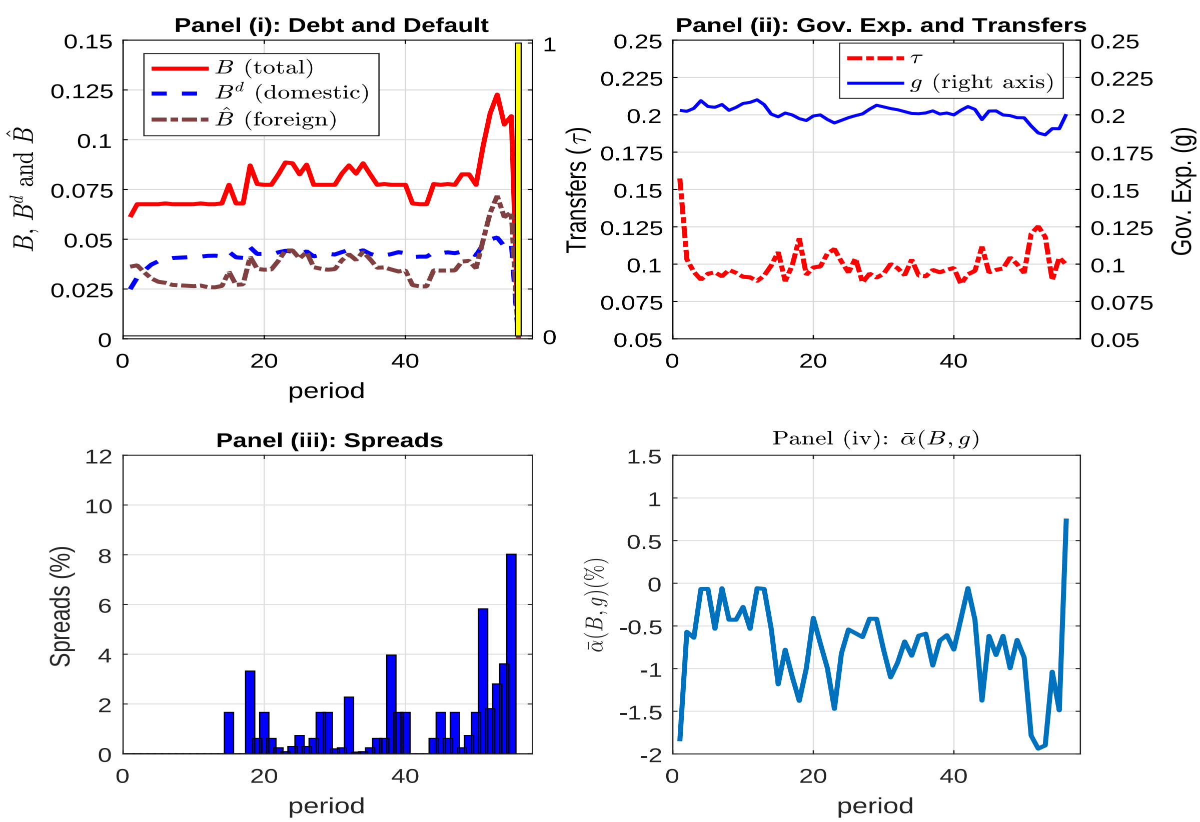
Panel (i) shows that the perceived fraction of agents that prefer repayment remains close to 100 percent until 3 years before the default. It then declines in periods $t = -3, -2, -1$, when Figure 3 shows that default risk rise. Since debt is stable prior to the default, these movements reflect mainly the effects of changes in government expenditures and transfers (a reduction in government redistribution). Then in year 0, the increase in g is sufficient to make default optimal even though debt did not increase in the previous two years.

Panel (ii) shows interesting dynamics in the perceived fractions of agents who prefer repayment across income levels. The fraction is highest for low-income agents who value lump-sum transfers and the liquidity benefits of debt the most. The fraction of low-income agents who favor repayment drops only in years $t = -2, -1$. The fraction of mid-income and high-income agents who prefer repayment follows a similar pattern, but the decline starts a year earlier in the case of high income households. Mid-income and high-income agents value the liquidity services of debt but rely less on lump-sum transfers that can be sustained with debt. Interestingly, the fraction of agents who favor repayment is above zero in all years before and after the default and for all income levels. This is because there are sufficiently wealthy individuals with very low income that still favor repayment.

A-5 Dynamics Between Default Events

In the text, we illustrated the time series dynamics of the model using an event analysis with 13-year event windows centered on default events. In this appendix, we follow an alternative approach by studying time series dynamics across two default events. Figure A.3 shows the time-series dynamics between two defaults that are separated by a number of years equal to the mode duration of the non default or repayment period in the simulated data set, which is 57 years (the mode of the distribution of periods between default events). This long mode repayment period is in line with the result that defaults occur with a long-run frequency of only 1.2 percent. The figure is divided in the same four panels as the event analysis plots in the text. Panel (i) shows total government bonds (B) and their aggregate domestic and foreign holdings (B^d and $\hat{B}$ respectively). Panel (ii) shows g and transfers (τ). Panel (iii) shows the bond spreads and Panel (iv), displays the social welfare gain of default $\overline{\alpha}$ (in %). These charts start just after the first of the two defaults occurred, and end right when the next default occurs, 57 years later.

Figure A.3: Time-Series Dynamics between Default Events



Panel (i) of Figure A.3 shows that public debt grows rapidly after the initial default but stays close to its mean (the value that maximizes the "Debt Laffer" curve) for a large portion of the sample, and then (around period 50) starts to grow at a faster pace, until it reaches about 12.5 percent of GDP and the second default occurs. In line with what we found in the event analysis, the initial rise in debt occurs with declining g, which makes default more costly due to the exogenous income cost of default, thus strengthening repayment incentives and allowing the government to sustain more debt. Also in line with what the event analysis showed, taxes are generally lower than government purchases when the debt is rising, generating a primary deficit(see Panel (ii)). Spreads are generally small (Panel (iii)), and the social welfare gain of default is negative and relatively large (Panel (iv)).

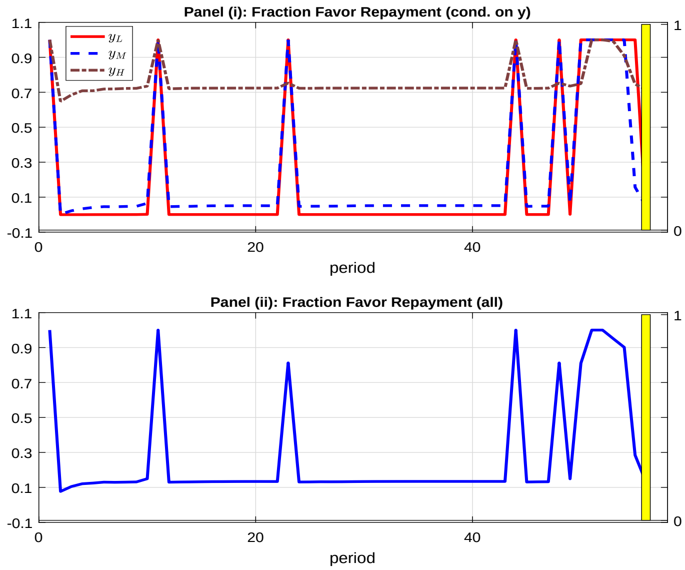
Panel (i) also shows that in the early years after the initial default, when the supply of public debt is increasing, domestic demand for risk-free assets is also rising, as the government is lowering taxes (which increases disposable income) and agents with relatively high-income realizations seek to replenish their buffer stock of savings. Domestic debt remains a higher fraction of total debt in most periods, as well as on average over the 57 years plotted. The ratio of domestic to external debt holdings, however, fluctuates, being smaller in the initial and final years than in the prolonged period in between.

In the last 10 years before the second default, domestic demand for risk-free assets increases but not as fast as total debt, which implies that the bulk of the new debt is placed abroad. With this creditor mix, and since foreign creditors do not enter in the social welfare function, default risk and spreads increase significantly. This pattern of spreads shifting suddenly from, on average, 1 percent to high levels is qualitatively consistent with standard predictions of external default models and with the stylized facts of debt crises. Still, default does not occur because the social welfare gain of default remains negative, until the 57th year arrives and the realization of g is sufficiently high to make default optimal at the existing outstanding debt since the relatively high level of debt in combination with the increase in expenditures forces the government to reduce lump-sum transfers.

The dynamics of the social gain of default in panel (iv) also capture the previous result showing that, even tough the welfare weights given by $\omega(b, y)$ are exogenous, the heterogeneity of agents plays a central role. The fraction of agents that the planner sees as benefiting from a default changes endogenously over time as debt, taxes, and spreads change, and the associated changes in the dispersion of individual gains of default affect the social welfare function, the default decision, and spreads.

We examine next the evolution of the fraction of agents in the economy who value repayment (i.e., those with $\alpha(b, y, B, g) < 0$ in the actual wealth distribution $\Gamma_t(b, y)$). Figure A.4 plots the evolution of this fraction for three income levels in Panel (i) and across all (b, y) in Panel (ii).

Figure A.4: Preferences over Repayment



With sufficiently large fraction of agents close to the borrowing limit, the faction of agents who favors repayment remains relatively low for most of the period. In fact, only close to the default event, the fraction that favors repayment reaches 1 for more than one period. This is due to the fact that as g declines the government issues more debt and increases transfers. As time goes by, the government starts to reduce the level of debt but a new g shock (period 55) results in a reduction in the fraction of agents in favor of repayment, since the government does not have room for further redistribution via debt at a relatively high initial debt and needs to cut transfers to pay, which induces a government default.

In line with the discussion of default payoffs in the text, the fraction of low-income agents who prefer repayment increases faster than the fraction of high-income agents who prefer repayment when confronted with government spending shocks. Interestingly, the fraction of agents with all levels of income, including the lowest, who favor repayment remains positive throughout. This is because, as we also noted in the text, there are sufficiently wealthy individuals with very low income that still favor repayment.

A-6 Details on Recursive Equilibrium Functions

This section of the Appendix provides further details on some of the implications of the recursive equilibrium functions. First we give a broader perspective on the cross-sectional properties of the individual welfare gains of default, which were examined in the paper using two-dimensional charts. Here we show that those properties are more general using intensity plots to illustrate three-dimensional variations. Figure A.5 shows two intensity plots of how $\alpha(b, y, B, g)$ varies over b and y with $g = \mu_g$. Panel (i) is for $B = B_L$ and Panel (ii) is for $B = B_H$.

The intuition for the features of these plots follows from the discussion of the threshold wealth that separates favoring repayment from favoring default, $\hat{b}(y, B, g)$, near the end of Section 2 in the main text. Comparing across panels (i) and (ii), $\alpha(b, y, B, g)$ is higher with the higher B for a given (b, y) pair, because $\hat{b}(y, B, g)$ is increasing in B. Consider next the variations along the b dimension. With $g = \mu_g$, only agents with very low b prefer default at both values of B. These agents benefit from the lower taxes associated with default and suffer negligible wealth losses. As b rises agents value increasingly more repayment for the opposite reason.

Explaining the variations along the y dimension is less straightforward, because both the repayment and default payoffs depend on y. $V^{d=1}(y, g)$ is increasing in y. $V^{d=0}(b, y, B, g)$ is increasing in "total resources," $y + b$, but is non-monotonic on b and y individually. In particular, while for a given b, $\alpha(b, y, B, g)$ is generally increasing in y, it decreases in y for high B and very low b. The reason for this follows from the discussion around Figure 4 in the paper.

Figure A.5: $\alpha(b, y, B, g)$ (for different B at $g = \mu_g$)

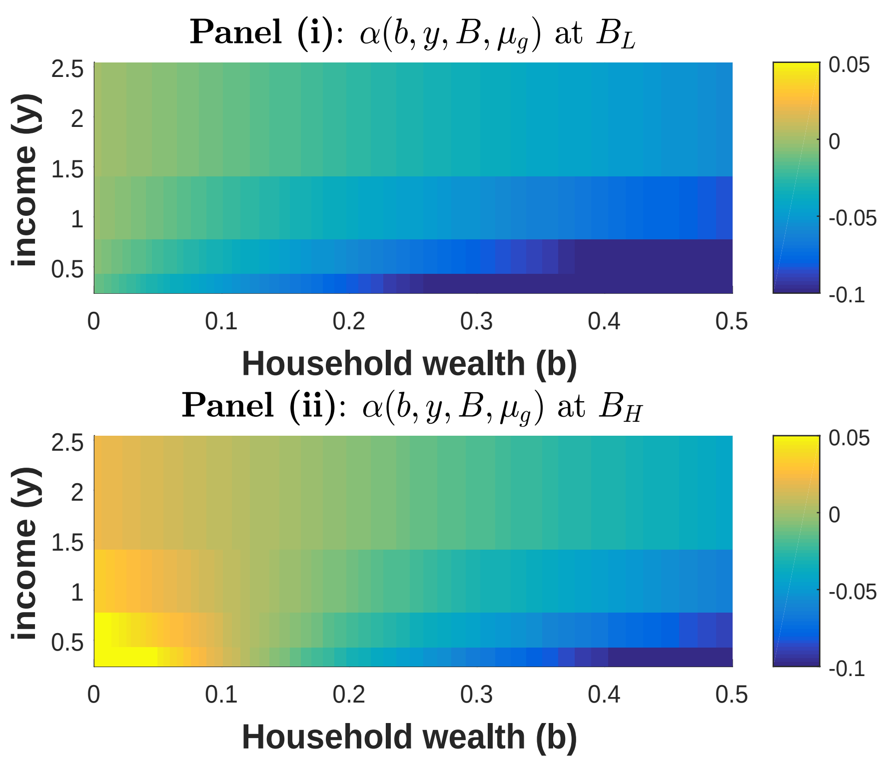


Figure A.6 presents $\hat{b}(y, B, g)$ for different values of y (Panel (i) for $y = y_L$, Panel (ii) for $y = y_M$, and Panel (iii) for $y = y_H$) and different values of g (lines within each panel) as a function of B.

Figure A.6: $\hat{b}(y, B, g)$ (for different g and y as a function of B)

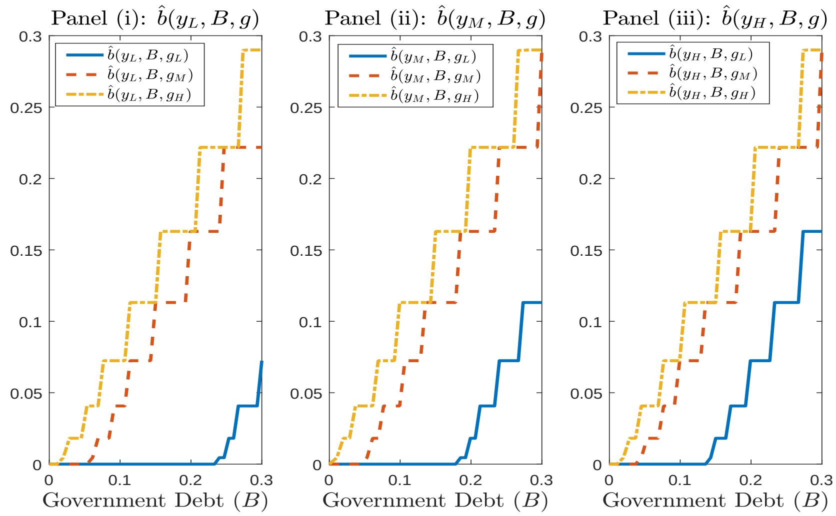


Figure A.6 corroborates than in our calibrated model $\hat{b}(y, B, g)$ is increasing in B for different values of y and g. Higher debt level reduces the level of transfers and limits the amount of redistribution that the government can implement.

Figure A.7: Optimal Debt $B'(B, g)$

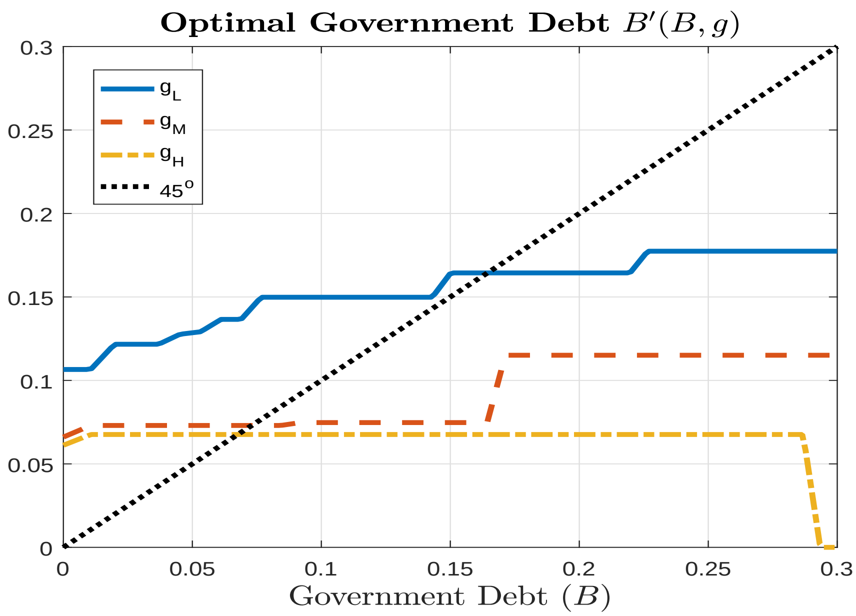


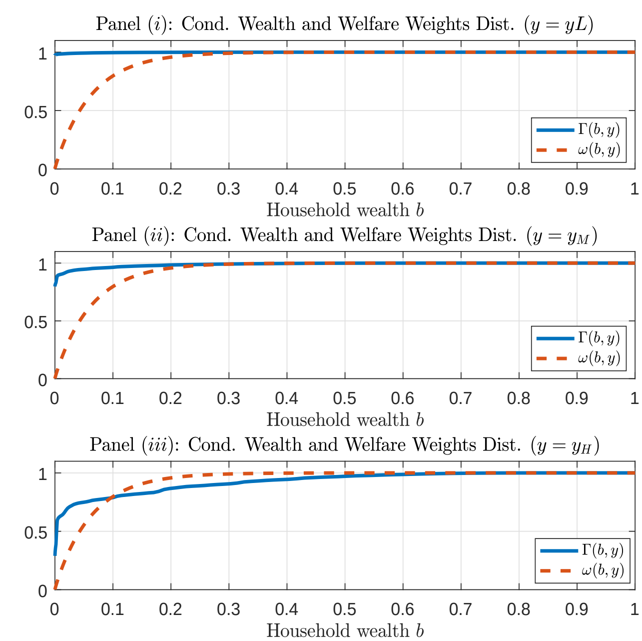
Figure A.7 shows that for high or average g, the optimal debt choice is independent of

B. In both cases, the government chooses the amount of debt that maximizes the Laffer curve regardless of the value of B (0.0106 for g_M and 0.0708 for g_H). Debt is risk-free but effectively "constrained" by the inability to commit to repay. For low g, the optimal debt rises with B and is always below the maximum of the Laffer curve (0.139).

A-7 Welfare Weights versus Wealth Distribution

Figure A.8 compares the weights of the social welfare function $\omega(b, y)$ with the distribution of wealth in the economy $\Gamma(b, y)$. The comparison is useful because, as explained in Section 3 of the main text, the distributional incentives to default are weaker the higher the relative weight of bond holders creditors in $\omega(b, y)$ v. $\Gamma(b, y)$. Since $\Gamma(b, y)$ is time- and state-contingent, we show the average $\bar{\Gamma}$ over the full time series simulation excluding default episodes. The plots show conditional distributions as functions of b for low, average, and high values of y in Panels (i), (ii), and (iii), respectively.

Figure A.8: "Average" Wealth Distribution $\bar{\Gamma}(b, y)$ and Welfare Weights $\omega(b, y)$



This figure shows the extent to which the fraction of agents with low b in the model economy exceeds their welfare weights. The differences are driven solely by differences in b because, by construction, $\bar{\Gamma}$ and ω have the same income distribution conditional on wealth ($\omega(b, y)$ was calibrated using $\pi^*(y)$ along the y dimension). Panels (i) and (ii) show that the majority of agents with income at the mean or lower are at the borrowing constraint or close to it (i.e., their bond holdings are zero or nearly 0), while bond holdings need to be equal to 0.30 and 0.10 to obtain the same fraction of agents using $\omega(b, y)$ for low income and mean income, respectively. For agents with high income, Panel (*iii*) shows that the fraction of agents with $b < 0.1$ is about the same under both distributions.

A-8 Calibration to Spain

This Appendix describes the calibration approach and the results of the model when calibrated to Spain. The first step of the calibration proceeds as follows: We set $\sigma = 1$ (i.e. log utility), which is in the range commonly used in macro models. The interest rate is set to $\bar{r} = 0.021$, which is the average annual return on German EMU-convergence criterion government bonds in the European Commission's Eurostat database for the period 2002−2012 (these are secondary market returns, gross of tax, with around 10 years' residual maturity).

To calibrate the individual income process, we set $\rho_y = 0.85$, which is a standard value in the heterogeneous-agents literature (e.g., Guvenen [31]). Then, we set σ_u to match Spain's cross-sectional variance of log-wages, which Pijoan-Mas and Sanchez Marcos [47] estimated at $\text{Var}(\log(y)) = 0.225$ on average for the period 1994−2001. Hence, $\sigma_u^2 = \text{Var}(\log(y))(1-\rho_y^2)$, which yields $\sigma_u = 0.2498$.[36] Average income is calibrated such that the aggregate resource constraint is consistent with national accounts data with GDP normalized to one. This implies that Y in the model must equal GDP net of fixed investment because the latter is not explicitly modeled. Investment averaged 24 percent of GDP during the period 1981-2012, which implies that $Y = \mu_y = 0.76$.

The g process is calibrated using data on government final consumption expenditures from National Accounts for the period 1981−2012 from the World Bank's World Development Indicators, and fitting an AR(1) process to the logged government expenditures-GDP ratio (controlling for a linear time trend). The results yield: $\rho_g = 0.88$, $\sigma_e = 0.017$ and $\mu_g = 0.18$. The value of τ^y is set to 35 percent following the estimates of the marginal labor tax in Spain (average for 2000-2002) reported by Conesa and Kehoe [18]. They studied the evolution of taxes in Spain from 1970 to 2002.

In the second calibration step, we use the SMM algorithm to set the values of β, $\overline{\omega}$, and ϕ_1 targeting these three data moments: the 1981−2012 average ratio of domestic public debt holdings to total public debt (74.43 percent), the 2002-2012 average bond spread relative to German bonds (0.94 percent), and the 1981-2012 average, maturity-adjusted public debt-GDP ratio (5.56 percent).[37]

[36] The data available for Spain consist of a sequence of cross sections, which prevented Pijoan-Mas and Sanchez-Marcos from estimating the autocorrelation of the income process.

[37] Total public debt refers to total general government net financial liabilities as a fraction of GDP. The

Table A.5 presents the targets and the parameter values.

Table A.5: Model Parameters and Targets

Calibrated from data or values in the literature			
Risk free rate (%)	$\bar{r}$	2.07	Real return German bonds
Risk aversion	σ	1.00	Standard value
Autocorrel. income	ρ_y	0.85	Guvenen [31]
Std. dev. error	σ_u	0.25	Spain wage data
Avg. income	μ_y	0.76	GDP net of fixed capital investment
Autocorrel. G	ρ_g	0.88	Autocorrel. government consumption
Std. dev. error	σ_e	0.02	Std. dev. government consumption
Avg. gov. consumption	μ_g	0.18	Avg. G/Y Spain
Proportional income tax	τ^y	0.35	Marginal labor income tax
Estimated using SMM to match target moments			
Discount factor	β	0.885	Avg. ratio domestic to total debt
Welfare weights	ω	0.051	Avg spread v. Germany
Default cost	ϕ_1	0.603	Avg. debt-GDP ratio (maturity adjusted)

Table A.6 shows the target data moments and the model's corresponding moments in the SMM calibration.

Table A.6: Results of SMM Calibration

Moments (%)	Model	Data
Avg. ratio domestic debt	74.31	74.43
Avg. spread Spain	0.94	0.94
Avg. debt to GDP Spain (maturity adjusted)	5.88	5.56

A-8.1 Equilibrium Time Series Properties

The quantitative analysis aims to answer two main questions. First, from the perspective of the theory, does the calibrated model support an equilibrium in which debt exposed to default risk can be sustained and default occurs along the equilibrium path? Second, from an empirical standpoint, to what extent are the model's time series properties in line with those observed in the data?

ratio of domestic to total debt corresponds to the fraction of general government gross debt held by domestic investors from Arslanalp and Tsuda [10], extended with the ratio of marketable debt held by residents to total marketable central government debt from Organization for Economic Co-operation and Development Statistics. See Appendix A-2 for further details.

To answer these questions, we study the model's dynamics using a time series simulation for 10,000 periods, truncating the first 2,000 to generate a sample of 8,000 years, large enough to capture the long-run properties of the model. This sample yields 73 default events, which implies an unconditional default probability of 0.91 percent. Thus, the model produces optimal domestic (and external, since the government cannot discriminate debtors) sovereign defaults as a low-probability equilibrium outcome, although still roughly twice Spain's historical domestic default frequency of 0.4 percent (Reinhart and Rogoff [48] show only one default episode in 216 years). In contrast with typical results from external default models, these defaults do not require costs of default in terms of exclusion from credit markets, permanently or for a random number of periods, and rely in part on endogenous default costs that reflect the social value of debt for self-insurance, liquidity, and risk-sharing.

Table A.7 compares moments from the model's simulation with data counterparts. Since Spain has not defaulted in the data sample period but its default risk spiked during the European debt crisis, we show model averages excluding default years to compare with data averages, and averages for the years before defaults occur ("prior default") to compare with the crisis peaks in the data (the "peak crisis" column, which shows the highest values observed during the 2008-2012 period).

Table A.7: Long-run and Pre-Crisis Moments: Data versus Model

	Data		Model	
Moment (%)	Avg.	Peak Crisis	Average	Prior Default
Gov. debt B	5.43*	7.43	5.88	7.95
Domestic debt B^d	4.04	4.85	4.29	4.84
Foreign debt $\widehat{B}$	1.39	2.58	1.59	3.11
Ratio B^d/B	74.34*	65.28	74.31	60.94
Tax revenues $\tau^y Y$	25.24	24.85	26.60	26.60
Gov. expenditure g	18.12*	20.50	18.13	18.18
Transfers τ	7.04	7.06	8.35	8.73
Spread	0.94*	4.35	0.94	7.22

Note: * identifies moments used as calibration targets. See Appendix A-2 for details on sources, definitions, and sample periods for data moments. Since GDP was normalized to 1, all variables in levels are also GDP ratios.

Table 4 shows that the model does well at matching several key features of the data. The averages of total debt, the ratio of domestic to total debt, and spreads were calibration targets, so these moments in the model are close to the data by construction. The rest of the model averages (domestic and external debt, tax revenue, transfers, and government expenditures) approximate well the data averages. Taxes and transfers do not match more accurately because, with the Conesa-Kehoe labor tax rate of $\tau^y = 0.35$ and with GDP net of investment at $Y = 0.76$, the model generates 26.6 percent of GDP in taxes, which is 140 basis points more than in the data and results in average transfers exceeding the data average by the same amount. The model is within a 10-percent margin at matching the crisis peaks of total debt, domestic debt, and the ratio of domestic to total debt.

Table A.8 compares an additional set of model and data moments, including standard deviations (relative to the standard deviation of income), income correlations, and correlations with government expenditures.

Table A.8: Cyclical Moments: Data versus Model

	Standard Deviation		Correl($x, hhdi$)		Correl($x, g/GDP$)	
Variable x	Data	Model	Data	Model	Data	Model
Consumption	0.85	0.84	0.43	0.97	-0.32	-0.76
Trade Balance/GDP	0.63	0.55	-0.31	-0.82	0.15	0.08
Spreads	1.04	2.46	-0.44	-0.004	-0.22	-0.23
Gov. Debt / GDP	1.58	1.23	-0.18	-0.07	0.06	-0.07
Dom. Debt / GDP	1.68	0.32	-0.32	-0.34	-0.10	-0.22

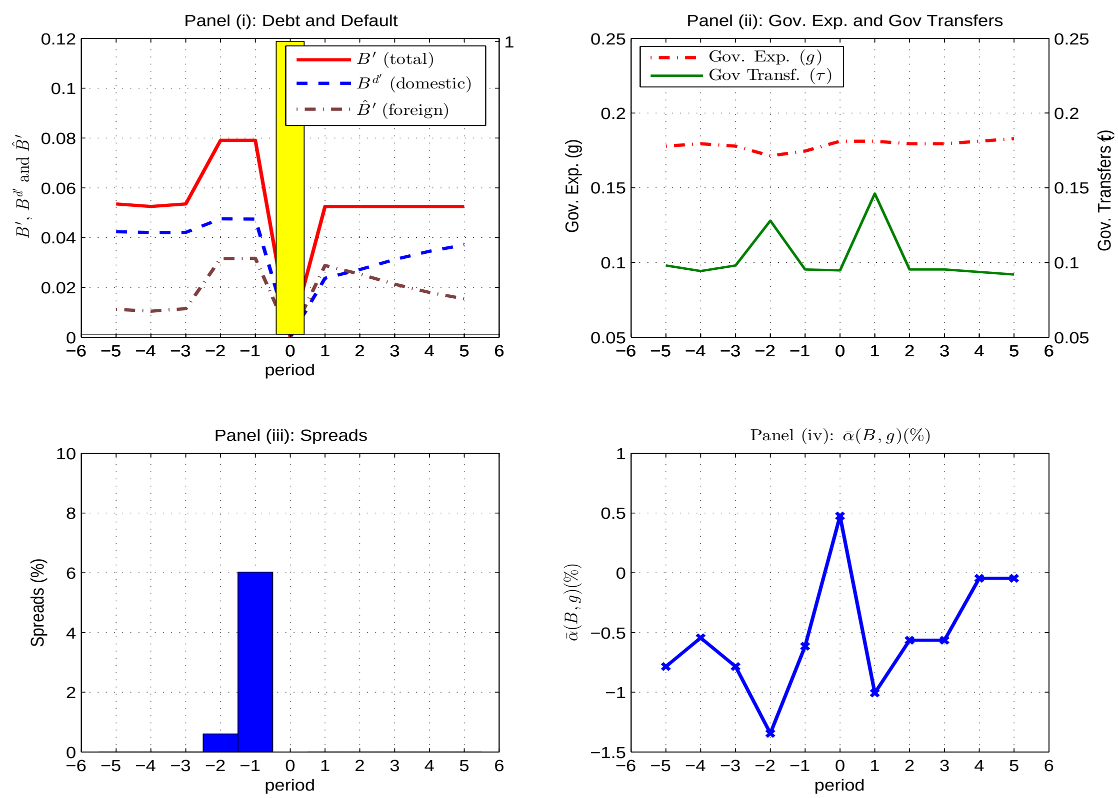
Note: $hhdi$ denotes household disposable income. In the model, $hhdi = (1 - \tau^y)Y + \tau$ and $TB = Y - C - g$. $hhdi$ and C are logged and HP filtered with the smoothing parameter set to 6.25 (annual data). GDP ratios are also HP filtered with the same smoothing parameter. Standard deviations are ratios to the standard deviations of $hhdi$, which are 1.37 and 1.16 in data and model, respectively. Since the data sample for spreads is short (2002-2012) and for a period characterized by a sustained rise in spreads since 2008, we generate comparable model data by isolating events spanning 10 years before spikes in spreads, defining spikes as observations in the 95 percentile. The standard deviation of spreads is demeaned to provide a comparable variability ratio. See Appendix A-2 for details on data sources.

Given the parsimonious structure of the model, it is noteworthy that it can approximate well several key moments of the data, including most co-movements. The model does a good job at approximating the standard deviation of disposable income, as well as the relative standard deviations of consumption, the trade balance, and total debt. On the other hand, the model overestimates the variability of spreads and underestimates that of domestic debt. The correlations with government expenditures produced by the model line up very well with those found in the data. The correlations with debt, domestic debt and spreads are of particular importance for the mechanism driving the model. As we document later in this section, the model predicts that periods with relatively low g weaken default incentives and thus enhance the government's borrowing capacity. Accordingly, the model yields a negative correlation of government expenditures with spreads (-0.23 versus -0.22 in the data) and with domestic debt (-0.22 versus -0.1 in the data), and nearly uncorrelated debt and government expenditures. The model is also very close to matching the correlation between the trade balance and spreads (0.15 in the data versus 0.09 in the model, respectively), which is driven by the same mechanism, since trade deficits are financed with the share of the public debt sold abroad.

The model also approximates well the income correlations of total and domestic debt, and relatively well that of the trade balance. The correlation of consumption with disposable income is close to 1 in the model v. 0.43 in the data, and the model yields uncorrelated spreads and disposable income while in the data the correlation is -0.44.

We study next dynamics around default events. Figure A.9 shows a set of event analysis charts based on the simulated data set with its 73 defaults. The plots show 11-year event windows centered on the year of default at $t = 0$ starting from the median debt level of all default events at $t = -5$. Panel (i) shows total public debt (B) and domestic and foreign debt holdings (B^d and $\hat{B}$, respectively). Panel (ii) shows g and τ. Panel (iii) shows bond spreads. Panel (iv) shows the social welfare gain of default denoted $\overline{\alpha}$.

Figure A.9: Default Event Analysis



The event analysis plots show that a debt crisis in the model appears to emerge suddenly, after seemingly uneventful times. Up to three years before the default, debt is barely moving, spreads are zero, and government expenditures, transfers, and the social welfare gain of default are also relatively stable. In the two years before the default everything changes dramatically. Debt rises sharply by nearly 300 basis points, with both foreign and domestic holdings rising but the former rising faster. Spreads rise very sharply to 100 and 600 basis points in the second and first year before the default, respectively. This follows from a slight drop in g coupled with a larger rise in τ and a sharp drop in $\overline{\alpha}$ at $t = -2$, and then a modest increase in g, and reversals in τ and $\overline{\alpha}$ at $t = -1$.

The reason for the rapid, large changes at $t = -2$ is that the decline in g weakens the government's incentives to default, because the exogenous default cost rises as g falls. The resulting higher borrowing capacity enables the government to redistribute more resources and provide more liquidity to credit-constrained agents by issuing more debt and paying higher transfers. The sharp drop in $\overline{\alpha}$ shows that using the newly gained borrowing capacity in this way is indeed socially optimal. Foreign debt holdings rise more than domestic holdings because domestic agents already have sizable debt holdings for self-insurance, although higher spreads still attract agents with sufficiently high (b, y) to buy more debt.

At $t = -1$, g rises only slightly while debt, and hence transfers, remain unchanged. The higher debt, together with the positive autocorrelation of the g process, strengthen default incentives ($\overline{\alpha}$ rises) and cause an increase in the probability that a default may occur in the following period, causing the sharp increase in spreads to 600 basis points. Then at $t = 0$, g rises slightly again but, at the higher debt, this is enough to cause a large change in $\overline{\alpha}$ by about 100 basis points from -0.5 to 0.5 percent, causing a "sudden" default on a debt ratio practically unchanged from two years prior. In addition, default occurs with relatively low external debt, which is roughly 46 percent of total debt. The surge in spreads at $t = -1$ and the default that followed, both occurring with an unchanged debt, could be viewed as suggesting that equilibrium multiplicity or self-fulfilling expectations were the culprit, but in this simulation this is not the case.

In the early years after a default, g hardly changes but, since the agents' precautionary savings were wiped out, domestic debt holdings rise steadily from 0 to 4 percent of GDP by $t = 5$. This reflects the optimal (gradual) buildup of precautionary savings by agents that draw relatively high income realizations. Total debt and transfers rise sharply in the first year, as the social value of debt starting from zero debt is very high and debt that is not sold at home is sold abroad at zero spread, because repayment incentives are strong ($\overline{\alpha}$ is around -1 percent). By $t = 5$, debt and its foreign and domestic component are approaching the levels they had at $t = -5$. Repayment incentives are weak but still enough to issue debt at zero spread.

A-9 Algorithm Endogenous Partial Default

This Appendix describes the algorithm we constructed to solve for the model's CRME and RME when there is endogenous partial default. The algorithm performs a global solution using value function iteration. We approximate the solution of the infinite horizon economy by solving for the equilibrium of a finite-horizon version of the model for which the finite number of periods (T) is set to a number large enough such that the distance between value functions, government policies and bond prices in the first and second periods are the same up to a convergence criterion. The corresponding first-period functions are then treated as representative of the solution of the infinite-horizon economy.

1. Problem in iteration T, for each $\{b, y\}$ and $\{B', B, g\}$:

   - Government Debt choice: $B'_T(B, g) = 0$.
   - Price Debt: $q_T(B', g) = 0$.
   - Tax no default:
$$\tau_T^{d=0}(B', B, g) = B + g - \tau^y Y$$
     - Define negative consumption flag:
$$\text{flag}_t^{c<0}(B', B, g) = I_{\{(1-\tau^y)y_1 + b_1 - \tau_T(B', B, g) \leq 0\}}$$
   - Define household value (note that at T it does not depend on $\tilde{B}$ since $q_T(B', g) = 0$)
$$\tilde{V}_T^{d=0}(\tilde{B}, y, b, B, g) = u((1-\tau^y)y + b - g - B + \tau^y Y)$$
     - If $\text{flag}_t^{c<0}(\tilde{B}, B, g) = 1$ set $\tilde{V}_T^{d=0}(\tilde{B}, :, :, B, g) = -\infty$
   - In period T, government debt choice $B' = 0$.
   - Household value in $d = 0$: $V_T^{d=0}(y, b, B, g) = \tilde{V}_T^{d=0}(0, y, b, B, g)$
   - Tax under default:
$$\tau_T^{d=1}(B, g, \varphi) = (1-\varphi)B + g - \tau^y Y$$
   - Value in default
$$\tilde{V}_T^{d=1}(y, b, B, g, \varphi) = u((1-\tau^y)y(1-\phi(g)) - g + [1-\varphi](b - B) + \tau^y Y)$$
   - In period T, government $\varphi = 1$, $V_T^{d=1}(y, b, B, g) = \tilde{V}_T^{d=1}(y, b, B, g, 1)$

- Welfare values for default decision

$$W_T^{d=0}(B,g) = \int_{Y\times B} V_T^{d=0}(y,b,B,g)d\omega(b,y)$$

$$W_T^{d=1}(B,g) = \int_{Y\times B} V_T^{d=1}(y,b,B,g)d\omega(b,y)$$

- Default decision (note that $d=1$ implies a given φ)

$$d_T(B,g) = \arg\max_{d=\{0,1\}} \{W_T^{d=0}(B,g), W_T^{d=1}(g)\}$$

- Let

$$V_T(y,b,B,g) = (1-d_T)V_T^{d=0}(y,b,B,g) + d_T V_T^{d=1}(y,b,B,g)$$

2. Problem in iteration $t = T-1,\dots,1$

   - Solve for price function $q_t(B',g)$.
     - Define fraction defaulted (default probability together with fraction defaulted)

$$p_t(B',g) = \sum_{g'} d_{t+1}(B',g')\varphi_{t+1}(B',g')F(g',g)$$

     - price is

$$q_t(B',g) = \frac{1-p_t(B',g)}{1+r}$$

   - Define Tax Function in $d=0$ state

$$\tau_t^{d=0}(B',B,g) = B + g - q_t(B',g)B' - \tau^y Y$$

     - Create flag for negative consumption: Combinations of B', B and g that imply negative consumption for y_1, b_1 when choosing b_1'

$$\text{flag}_t^{c<0}(B',B,g) = I_{\{(1-\tau^y)y_1+b_1-q_t(B',g)B'-\tau_t(B',B,g)\leq 0\}}$$

   - Solve problem household for b,y and $\{\tilde{B},B,g\}$

$$\tilde{V}_t^{d=0}(\tilde{B},y,b,B,g) = \max_{b'} u(c) + \beta E_{g'}[V_{t+1}(b',y',\tilde{B},g')]$$

s.t.

$$c = (1-\tau^y)y + b - q_t(\tilde{B},g)b' - \tau_t(\tilde{B},B,g)$$

- Optimal Debt choice

$$B_t'(B,g) = \arg\max_{\tilde{B}} \int \tilde{V}_t^{d=0}(\tilde{B},y,b,B,g)d\omega(b,y)$$

- Define continuation value for the households under no default

$$V_t^{d=0}(y,b,B,g) = \tilde{V}_t^{d=0}(B_t'(B,g),y,b,B,g)$$

- Tax under default:

$$\tau_t^{d=1}(B,g,\varphi) = (1-\varphi)B + g - \tau^y Y$$

- Value in default

$$\tilde{V}_t^{d=1}(y,b,B,g,\varphi) = u((1-\tau^y)y(1-\phi(g)) - \tau_t^{d=1}(B,g,\varphi)) + \beta E_{g'}[V_{t+1}(0,y',0,g')]$$

- Optimal Fraction of Default Choice

$$\varphi_t(B,g) = \arg\max_{\varphi} \int \tilde{V}_t^{d=1}(y,b,B,g,\varphi)d\omega(b,y)$$

- Define value in default

$$V_t^{d=1}(y,b,B,g) = \tilde{V}_t^{d=1}(y,b,B,g,\varphi_t(B,g))$$

- Auxiliary functions:

$$\hat{p}_t(B,g) = \sum_{g'} d_{t+1}(B_{t+1}'(B,g),g')\varphi_{t+1}(B_{t+1}'(B,g),g'))F(g',g)$$

$$\hat{q}_t(B,g) = \frac{1-\hat{p}_t(B,g)}{1+r}$$

$$\hat{\tau}_t(B,g) = B + g - \hat{q}_t(B,g)B_{t+1}'(B,g) - \tau^y Y$$

$$\hat{\text{flag}}_t^{c<0}(B',B,g) = I_{\{(1-\tau^y)y_1 + b_1 - \hat{q}_t(B,g)B_{t+1}'(B,g) - \hat{\tau}_t(B,g) \leq 0\}}$$

- Government values

$$W_t^{d=0}(B,g) = \int_{Y\times B} V_t^{d=0}(y,b,B,g)d\omega(b,y)$$

$$W_t^{d=1}(B,g) = \int_{Y\times B} V_t^{d=1}(y,b,B,g)d\omega(b,y)$$

- Government default decision

$$d_t(B,g) = \arg\max_{d=\{0,1\}} \{W_t^{d=0}(B,g), W_t^{d=1}(B,g)\}$$

3. After done with solution to periods $t = T-1,\dots,1$ check whether value functions, government policies and bond prices in periods 1 and 2 are sufficiently close. If they are, you are done. If not, increase T and restart.