A Supplementary material: Proofs

A.1 Portfolio problems

Proof of Lemma 1. Notice that (3) implies

$$W_t^D(\boldsymbol{a}_t, k_t) = \boldsymbol{\phi}_t \boldsymbol{a}_t + k_t + \bar{W}_t^D,$$

where

$$\bar{W}_t^D \equiv \max_{\tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2} \left[-\boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1} + \beta \mathbb{E}_t V_{t+1}^D \left(\tilde{a}_{t+1}^m, \delta \tilde{a}_{t+1}^s \right) \right], \tag{27}$$

so (2) implies

$$\begin{aligned} \hat{W}_t^D(\boldsymbol{a}_t, k_t) &= k_t + \bar{W}_t^D + \max_{\hat{\boldsymbol{a}}_t \in \mathbb{R}_+^2} \boldsymbol{\phi}_t \hat{\boldsymbol{a}}_t \\ \text{s.t. } & \hat{a}_t^m + p_t \hat{a}_t^s \le a_t^m + p_t a_t^s. \end{aligned}$$

Hence,

$$\begin{aligned} \hat{a}_t^m(\boldsymbol{a}_t) &\begin{cases} = a_t^m + p_t a_t^s & \text{if } 0 < \varepsilon_t^* \\ \in [0, a_t^m + p_t a_t^s] & \text{if } 0 = \varepsilon_t^* \\ = 0 & \text{if } \varepsilon_t^* < 0, \end{cases} \\ \hat{a}_t^s(\boldsymbol{a}_t) &= (1/p_t)\left[a_t^m + p_t a_t^s - \hat{a}_t^m(\boldsymbol{a}_t) \right], \end{aligned}$$

and

$$\hat{W}_t^D(\boldsymbol{a}_t, k_t) = \max\left(\phi_t^m, \phi_t^s / p_t\right)\left(a_t^m + p_t a_t^s\right) + k_t + \bar{W}_t^D. \tag{28}$$

Also, notice that (4) implies

$$W_t^I(\boldsymbol{a}_t, k_t) = \boldsymbol{\phi}_t \boldsymbol{a}_t - k_t + \bar{W}_t^I, \tag{29}$$

where

$$\bar{W}_t^I \equiv T_t + \max_{\tilde{\boldsymbol{a}}_{t+1} \in \mathbb{R}_+^2} \left[-\boldsymbol{\phi}_t \tilde{\boldsymbol{a}}_{t+1} + \beta \mathbb{E}_t \int V_{t+1}^I \left[\tilde{a}_{t+1}^m, \delta \tilde{a}_{t+1}^s + (1-\delta) A^s, \varepsilon \right] dG(\varepsilon) \right]. \tag{30}$$

With (28) and (29), (1) can be written as

$$\begin{aligned} &\max_{\overline{\boldsymbol{a}}_t^m, k_t} \left[\left(\varepsilon_t^* - \varepsilon \right) \left(\overline{a}_t^m - a_{it}^m \right) \frac{1}{p_t} y_t - k_t \right]^\theta k_t^{1-\theta} \\ &\text{s.t. } 0 \le k_t \le \left(\varepsilon_t^* - \varepsilon \right) \left(\overline{a}_t^m - a_{it}^m \right) \frac{1}{p_t} y_t \end{aligned}$$

with $\overline{a}_t^s = a_{it}^s + (1/p_t)(a_{it}^m - \overline{a}_t^m)$. Hence,

$$\overline{a}_t^m(\boldsymbol{a}_{it}, \varepsilon) \begin{cases} = a_{it}^m + p_t a_{it}^s & \text{if } \varepsilon < \varepsilon_t^* \\ \in [0, a_{it}^m + p_t a_{it}^s] & \text{if } \varepsilon = \varepsilon_t^* \\ = 0 & \text{if } \varepsilon_t^* < \varepsilon, \end{cases}$$

$$\overline{a}_t^s(\boldsymbol{a}_{it}, \varepsilon) = a_{it}^s + (1/p_t)\left[a_{it}^m - \overline{a}_t^m(\boldsymbol{a}_{it}, \varepsilon)\right],$$

and

$$k_t(\boldsymbol{a}_{it}, \varepsilon) = (1-\theta)(\varepsilon - \varepsilon_t^*)\left[\mathbb{I}_{\{\varepsilon_t^* < \varepsilon\}} \frac{1}{p_t} a_{it}^m - \mathbb{I}_{\{\varepsilon < \varepsilon_t^*\}} a_{it}^s\right] y_t.$$

This concludes the proof. ■

Lemma 2 *Let $(\tilde{a}_{dt+1}^m, \tilde{a}_{dt+1}^s)$ and $(\tilde{a}_{it+1}^m, \tilde{a}_{it+1}^s)$ denote the portfolios chosen by a dealer and an investor, respectively, in the second subperiod of period t. These portfolios must satisfy the following first-order necessary and sufficient conditions:*

$$\phi_t^m \geq \beta \mathbb{E}_t \max\left(\phi_{t+1}^m, \phi_{t+1}^s / p_{t+1}\right), \text{ with "} = \text{" if } \tilde{a}_{dt+1}^m > 0 \quad (31)$$

$$\phi_t^s \geq \beta\delta \mathbb{E}_t \max\left(p_{t+1}\phi_{t+1}^m, \phi_{t+1}^s\right), \text{ with "} = \text{" if } \tilde{a}_{dt+1}^s > 0 \quad (32)$$

$$\phi_t^m \geq \beta \mathbb{E}_t \left[\phi_{t+1}^m + \alpha\theta \int_{\varepsilon_{t+1}^*}^{\varepsilon_H} \left(\varepsilon - \varepsilon_{t+1}^*\right) y_{t+1} dG(\varepsilon) \frac{1}{p_{t+1}}\right], \text{ with "} = \text{" if } \tilde{a}_{it+1}^m > 0 \quad (33)$$

$$\phi_t^s \geq \beta\delta \mathbb{E}_t \left[\bar{\varepsilon} y_{t+1} + \phi_{t+1}^s + \alpha\theta \int_{\varepsilon_L}^{\varepsilon_{t+1}^*} \left(\varepsilon_{t+1}^* - \varepsilon\right) y_{t+1} dG(\varepsilon)\right], \text{ with "} = \text{" if } \tilde{a}_{it+1}^s > 0. \quad (34)$$

Proof. With Lemma 1, we can write $V_t^I(\boldsymbol{a}_t, \varepsilon)$ as

$$V_t^I(\boldsymbol{a}_t, \varepsilon) = \left[\alpha\theta(\varepsilon - \varepsilon_t^*)\mathbb{I}_{\{\varepsilon_t^* < \varepsilon\}} \frac{1}{p_t} y_t + \phi_t^m\right] a_t^m + \left\{\left[\varepsilon + \alpha\theta(\varepsilon_t^* - \varepsilon)\mathbb{I}_{\{\varepsilon < \varepsilon_t^*\}}\right] y_t + \phi_t^s\right\} a_t^s + \bar{W}_t^I \quad (35)$$

and $V_t^D(\boldsymbol{a}_t)$ as

$$V_t^D(\boldsymbol{a}_t) = \alpha \int k_t(\boldsymbol{a}_{it}, \varepsilon)\, dH_{It}(\boldsymbol{a}_{it}, \varepsilon) + \max(\phi_t^m, \phi_t^s/p_t)(a_t^m + p_t a_t^s) + \bar{W}_t^D.$$

Since ε is i.i.d. over time, the portfolio that each investor chooses to carry into period $t+1$ is independent of ε. Therefore, we can write $dH_{It}(\boldsymbol{a}_t, \varepsilon) = dF_{It}(\boldsymbol{a}_t)\, dG(\varepsilon)$, where F_{It} is the joint cumulative distribution function of investors' money and equity holdings at the beginning of the OTC round of period t. Thus,

$$V_t^D(\boldsymbol{a}_t) = \max(\phi_t^m, \phi_t^s/p_t)(a_t^m + p_t a_t^s) + V_t^D(\mathbf{0}), \quad (36)$$

where

$$V_t^D(\mathbf{0}) = \alpha(1-\theta)\int(\varepsilon-\varepsilon_t^*)\left[\mathbb{I}_{\{\varepsilon_t^*<\varepsilon\}}\frac{1}{p_t}A_{It}^m - \mathbb{I}_{\{\varepsilon<\varepsilon_t^*\}}A_{It}^s\right]dG(\varepsilon)\,y_t + \bar{W}_t^D.$$

From (36) we have

$$V_{t+1}^D\left(\tilde{a}_{t+1}^m, \delta\tilde{a}_{t+1}^s\right) = \max\left(\phi_{t+1}^m, \phi_{t+1}^s/p_{t+1}\right)\left(\tilde{a}_{t+1}^m + p_{t+1}\delta\tilde{a}_{t+1}^s\right) + V_{t+1}^D(\mathbf{0}),$$

and from (35) we have

$$\begin{aligned}
&\int V_{t+1}^I\left[\tilde{a}_{t+1}^m, \delta\tilde{a}_{t+1}^s + (1-\delta)A^s, \varepsilon\right]dG(\varepsilon) \\
&= \left[\alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon-\varepsilon_{t+1}^*\right)dG(\varepsilon)\frac{1}{p_{t+1}}y_{t+1} + \phi_{t+1}^m\right]\tilde{a}_{t+1}^m \\
&+ \delta\left\{\left[\bar{\varepsilon} + \int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\alpha\theta\left(\varepsilon_{t+1}^*-\varepsilon\right)dG(\varepsilon)\right]y_{t+1} + \phi_{t+1}^s\right\}\tilde{a}_{t+1}^s + \zeta_{t+1},
\end{aligned}$$

where $\zeta_{t+1} \equiv \left\{\left[\bar{\varepsilon} + \alpha\theta\int\left(\varepsilon_{t+1}^*-\varepsilon\right)\mathbb{I}_{\left\{\varepsilon<\varepsilon_{t+1}^*\right\}}dG(\varepsilon)\right]y_{t+1} + \phi_{t+1}^s\right\}(1-\delta)A^s + \bar{W}_{t+1}^I$. Thus, the necessary and sufficient first-order conditions corresponding to the maximization problems in (27) and (30) are as in the statement of the lemma. ■

A.2 Market clearing in the OTC market

Lemma 3 *In period t, the interdealer market-clearing condition for equity is*

$$\left\{\alpha\left[1-G(\varepsilon_t^*)\right]A_{It}^m + \chi(\varepsilon_t^*,0)A_{Dt}^m\right\}\frac{1}{p_t} = \alpha G(\varepsilon_t^*)A_{It}^s + \left[1-\chi(\varepsilon_t^*,0)\right]A_{Dt}^s. \tag{37}$$

Proof. Recall $\bar{A}_{Dt}^s = \int\hat{a}_t^s(\boldsymbol{a}_t)\,dF_{Dt}(\boldsymbol{a}_t)$, so from Lemma 1, we have

$$\bar{A}_{Dt}^s = \chi(\varepsilon_t^*,0)\left(A_{Dt}^s + A_{Dt}^m/p_t\right).$$

Similarly, $\bar{A}_{It}^s = \alpha\int\overline{a}_t^s(\boldsymbol{a}_t,\varepsilon)dH_{It}(\boldsymbol{a}_t,\varepsilon)$, so from Lemma 1, we have

$$\bar{A}_{It}^s = \alpha\left[1-G(\varepsilon_t^*)\right]\left(A_{It}^s + A_{It}^m/p_t\right).$$

With these expressions, the market-clearing condition for equity in the interdealer market of period t, i.e., $\bar{A}_{Dt}^s + \bar{A}_{It}^s = A_{Dt}^s + \alpha A_{It}^s$, can be written as in the statement of the lemma. ■

A.3 Equilibrium characterization

Corollary 2 *A sequence of prices, $\{1/p_t, \phi_t^m, \phi_t^s\}_{t=0}^{\infty}$, together with bilateral terms of trade in the OTC market, $\{\bar{\boldsymbol{a}}_t, k_t\}_{t=0}^{\infty}$, dealer portfolios, $\{\langle \hat{\boldsymbol{a}}_{dt}, \tilde{\boldsymbol{a}}_{dt+1}, \boldsymbol{a}_{dt+1}\rangle_{d\in\mathcal{D}}\}_{t=0}^{\infty}$, and investor portfolios, $\{\langle \tilde{\boldsymbol{a}}_{it+1}, \boldsymbol{a}_{it+1}\rangle_{i\in\mathcal{I}}\}_{t=0}^{\infty}$, constitute an equilibrium if and only if they satisfy the following conditions for all t:*

(i) Intermediation fee and optimal post-trade portfolios in OTC market

$$\begin{aligned}
k_t(\boldsymbol{a}_t, \varepsilon) &= (1-\theta)(\varepsilon - \varepsilon_t^*)\left[\chi(\varepsilon_t^*, \varepsilon)\frac{1}{p_t}a_t^m - [1-\chi(\varepsilon_t^*, \varepsilon)]\,a_t^s\right] y_t \\
\bar{a}_t^m(\boldsymbol{a}_t, \varepsilon) &= [1-\chi(\varepsilon_t^*, \varepsilon)](a_t^m + p_t a_t^s) \\
\bar{a}_t^s(\boldsymbol{a}_t, \varepsilon) &= \chi(\varepsilon_t^*, \varepsilon)(1/p_t)(a_t^m + p_t a_t^s) \\
\hat{\boldsymbol{a}}_t(\boldsymbol{a}_t) &= \bar{\boldsymbol{a}}_t(\boldsymbol{a}_t, 0).
\end{aligned}$$

(ii) Interdealer market clearing

$$\{\alpha[1-G(\varepsilon^*)]\,A_{It}^m + \chi(\varepsilon_t^*, 0)\,A_{Dt}^m\}\frac{1}{p_t} = \alpha G(\varepsilon^*)\,A_{It}^s + [1-\chi(\varepsilon_t^*, 0)]\,A_{Dt}^s,$$

where $A_{jt}^m \equiv \int a_t^m dF_{jt}(\boldsymbol{a}_t)$ and $A_{jt}^s \equiv \int a_t^s dF_{jt}(\boldsymbol{a}_t)$ for $j \in \{D, I\}$.

(iii) Optimal end-of-period portfolios:

$$\begin{aligned}
\phi_t^m &\geq \beta\mathbb{E}_t \max\left(\phi_{t+1}^m, \phi_{t+1}^s/p_{t+1}\right) \\
\phi_t^s &\geq \beta\delta\mathbb{E}_t \max\left(p_{t+1}\phi_{t+1}^m, \phi_{t+1}^s\right) \\
\phi_t^m &\geq \beta\mathbb{E}_t\left[\phi_{t+1}^m + \alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right)dG(\varepsilon)\frac{1}{p_{t+1}}y_{t+1}\right] \\
\phi_t^s &\geq \beta\delta\mathbb{E}_t\left[\bar{\varepsilon}y_{t+1} + \phi_{t+1}^s + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right)y_{t+1}dG(\varepsilon)\right]
\end{aligned}$$

with

$$\begin{aligned}
\left[\phi_t^m - \beta\mathbb{E}_t \max\left(\phi_{t+1}^m, \phi_{t+1}^s/p_{t+1}\right)\right]\tilde{a}_{dt+1}^m &= 0 \\
\left[\phi_t^s - \beta\delta\mathbb{E}_t \max\left(p_{t+1}\phi_{t+1}^m, \phi_{t+1}^s\right)\right]\tilde{a}_{dt+1}^s &= 0 \\
\left\{\phi_t^m - \beta\mathbb{E}_t\left[\phi_{t+1}^m + \alpha\theta\int_{\varepsilon_{t+1}^*}^{\varepsilon_H}\left(\varepsilon - \varepsilon_{t+1}^*\right)dG(\varepsilon)\frac{1}{p_{t+1}}y_{t+1}\right]\right\}\tilde{a}_{it+1}^m &= 0 \\
\left\{\phi_t^s - \beta\delta\mathbb{E}_t\left[\bar{\varepsilon}y_{t+1} + \phi_{t+1}^s + \alpha\theta\int_{\varepsilon_L}^{\varepsilon_{t+1}^*}\left(\varepsilon_{t+1}^* - \varepsilon\right)y_{t+1}dG(\varepsilon)\right]\right\}\tilde{a}_{it+1}^s &= 0
\end{aligned}$$

for all $d \in \mathcal{D}$ and all $i \in \mathcal{I}$, and

$$
\begin{aligned}
a^m_{jt+1} &= \tilde{a}^m_{jt+1} \\
a^s_{jt+1} &= \delta \tilde{a}^s_{jt+1} + \mathbb{I}_{\{j \in \mathcal{I}\}} (1-\delta) A^s \\
\tilde{a}^k_{jt+1} &\in \mathbb{R}_+ \text{ for } k \in \{s, m\}
\end{aligned}
$$

for all $j \in \mathcal{D} \cup \mathcal{I}$.

(iv) End-of-period market clearing

$$
\begin{aligned}
\tilde{A}^s_{Dt+1} + \tilde{A}^s_{It+1} &= A^s \\
\tilde{A}^m_{Dt+1} + \tilde{A}^m_{It+1} &= A^m_{t+1},
\end{aligned}
$$

where $\tilde{A}^k_{Dt+1} \equiv \int_{\mathcal{D}} \tilde{a}^k_{xt+1} dx$ and $\tilde{A}^k_{It+1} \equiv \int_{\mathcal{I}} \tilde{a}^k_{xt+1} dx$ for $k \in \{s, m\}$.

Proof. *Follows immediately from Definition 1 together with Lemma 1, Lemma 2, and Lemma 3.* ■

Lemma 4 *Consider $\hat{\mu}$ and $\bar{\mu}$ as defined in (5). Then $\hat{\mu} < \bar{\mu}$.*

Proof of Lemma 4. Define $\Upsilon(\zeta) : \mathbb{R} \to \mathbb{R}$ by $\Upsilon(\zeta) \equiv \bar{\beta}\left[1 + \alpha\theta(1 - \bar{\beta}\delta)\zeta\right]$. Let $\hat{\zeta} \equiv \frac{(1-\alpha\theta)(\hat{\varepsilon}-\bar{\varepsilon})}{\alpha\theta\hat{\varepsilon}}$ and $\bar{\zeta} \equiv \frac{\bar{\varepsilon}-\varepsilon_L}{\beta\delta\bar{\varepsilon}+(1-\beta\delta)\varepsilon_L}$, so that $\hat{\mu} = \Upsilon(\hat{\zeta})$ and $\bar{\mu} = \Upsilon(\bar{\zeta})$. Since Υ is strictly increasing, $\hat{\mu} < \bar{\mu}$ if and only if $\hat{\zeta} < \bar{\zeta}$. With (6) and the fact that $\bar{\varepsilon} \equiv \int_{\varepsilon_L}^{\varepsilon_H} \varepsilon dG(\varepsilon) = \varepsilon_H - \int_{\varepsilon_L}^{\varepsilon_H} G(\varepsilon)\, d\varepsilon$,

$$
\hat{\zeta} = \frac{\int_{\hat{\varepsilon}}^{\varepsilon_H} [1 - G(\varepsilon)]\, d\varepsilon}{\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\hat{\varepsilon}} G(\varepsilon)\, d\varepsilon},
$$

so clearly,

$$
\hat{\zeta} < \frac{\int_{\varepsilon_L}^{\varepsilon_H} [1 - G(\varepsilon)]\, d\varepsilon}{\bar{\varepsilon}} = \frac{\bar{\varepsilon} - \varepsilon_L}{\bar{\varepsilon}} < \bar{\zeta}.
$$

Hence, $\hat{\mu} < \bar{\mu}$. ■

Proof of Proposition 1. In an equilibrium with no money (or no valued money), there is no trade in the OTC market. From Lemma 2, the first-order conditions for a dealer $d \in \mathcal{D}$ and an investor $i \in \mathcal{I}$ in the time t Walrasian market are

$$
\begin{aligned}
&\phi^s_t \geq \beta\delta \mathbb{E}_t \phi^s_{t+1}, \text{ “ = ” if } \tilde{a}^s_{t+1d} > 0 \\
&\phi^s_t \geq \beta\delta \mathbb{E}_t \left(\bar{\varepsilon} y_{t+1} + \phi^s_{t+1}\right), \text{ “ = ” if } \tilde{a}^s_{t+1i} > 0.
\end{aligned}
$$

In a recursive equilibrium, $\mathbb{E}_t(\phi^s_{t+1}/\phi^s_t) = \bar{\gamma}$, and $\beta\delta\bar{\gamma} < 1$ is a maintained assumption, so no dealer holds equity. The Walrasian market for equity can only clear if $\phi^s = \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\bar{\varepsilon}$. This establishes parts (i) and (iii) in the statement of the proposition.

Next, we turn to monetary equilibria. In a recursive equilibrium, the Euler equations (31)-(34) become

$$\mu \geq \bar{\beta}, \text{ “ = ” if } \tilde{a}^m_{dt+1} > 0 \tag{38}$$

$$\phi^s \geq \bar{\beta}\delta\bar{\phi}^s, \text{ “ = ” if } \tilde{a}^s_{dt+1} > 0 \tag{39}$$

$$1 \geq \frac{\bar{\beta}}{\mu}\left[1 + \frac{\alpha\theta}{\varepsilon^* + \phi^s}\int_{\varepsilon^*}^{\varepsilon_H}(\varepsilon - \varepsilon^*)\,dG(\varepsilon)\right], \text{ “ = ” if } \tilde{a}^m_{it+1} > 0 \tag{40}$$

$$\phi^s \geq \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^* - \varepsilon)\,dG(\varepsilon)\right], \text{ “ = ” if } \tilde{a}^s_{it+1} > 0. \tag{41}$$

(We have used the fact that, as will become clear below, $\bar{\phi}^s \equiv \varepsilon^* + \phi^s \geq \varepsilon_L + \phi^s > \phi^s$ in any equilibrium.) Under our maintained assumption $\bar{\beta} < \mu$, (38) implies $\tilde{a}^m_{dt+1} = Z_D = 0$, so (40) must hold with equality for some investor in a monetary equilibrium. Thus, in order to find a monetary equilibrium, there are three possible equilibrium configurations to consider depending on the binding patterns of the complementary slackness conditions associated with (39) and (41). The interdealer market-clearing condition, $\bar{A}^s_{Dt} + \bar{A}^s_{It} = A^s_{Dt} + \alpha A^s_{It}$, must hold for all three configurations. Lemma 3 shows that this condition is equivalent to (37) and in a recursive equilibrium (37) reduces to

$$Z = \frac{\varepsilon^* + \phi^s}{\alpha\left[1 - G(\varepsilon^*)\right]}\left\{\alpha G(\varepsilon^*)\,A^s_I + \left[1 - \chi(\varepsilon^*, 0)\right]A^s_D\right\}.$$

This condition in turn reduces to (12) if, as shown below, the equilibrium has $0 < \varepsilon^*$. The rest of the proof proceeds in three steps.

Step 1: Try to construct a recursive monetary equilibrium with $\tilde{a}^s_{dt+1} = 0$ for all $d \in \mathcal{D}$ and $\tilde{a}^s_{it+1} > 0$ for some $i \in \mathcal{I}$. The equilibrium conditions for this case are (12) together with

$$\phi^s > \bar{\beta}\delta\bar{\phi}^s \tag{42}$$

$$1 = \frac{\bar{\beta}}{\mu}\left[1 + \frac{\alpha\theta}{\varepsilon^* + \phi^s}\int_{\varepsilon^*}^{\varepsilon_H}(\varepsilon - \varepsilon^*)\,dG(\varepsilon)\right] \tag{43}$$

$$\phi^s = \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^* - \varepsilon)\,dG(\varepsilon)\right] \tag{44}$$

and

$$\tilde{a}^m_{dt+1} = 0 \text{ for all } d \in \mathcal{D} \tag{45}$$

$$\tilde{a}^m_{it+1} \geq 0, \text{ with `` > '' for some } i \in \mathcal{I} \tag{46}$$

$$\tilde{a}^s_{dt+1} = 0 \text{ for all } d \in \mathcal{D} \tag{47}$$

$$\tilde{a}^s_{it+1} \geq 0, \text{ with `` > '' for some } i \in \mathcal{I}. \tag{48}$$

Conditions (43) and (44) are to be solved for the two unknowns ε^* and ϕ^s. Substitute (44) into (43) to obtain

$$1 = \frac{\bar{\beta}}{\mu}\left[1 + \alpha\theta \frac{\int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right]}\right], \tag{49}$$

which is a single equation in ε^*. Define

$$T(x) \equiv \frac{\int_x^{\varepsilon_H} (\varepsilon - x)\, dG(\varepsilon)}{\frac{1}{1-\bar{\beta}\delta}x + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\hat{T}(x)} - \frac{\mu - \bar{\beta}}{\bar{\beta}\alpha\theta} \tag{50}$$

with

$$\hat{T}(x) \equiv \bar{\varepsilon} - x + \alpha\theta \int_{\varepsilon_L}^{x} (x - \varepsilon)\, dG(\varepsilon), \tag{51}$$

and notice that ε^* solves (49) if and only if it satisfies $T(\varepsilon^*) = 0$. T is a continuous real-valued function on $[\varepsilon_L, \varepsilon_H]$, with

$$T(\varepsilon_L) = \frac{\bar{\varepsilon} - \varepsilon_L}{\varepsilon_L + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\bar{\varepsilon}} - \frac{\mu - \bar{\beta}}{\bar{\beta}\alpha\theta},$$

$$T(\varepsilon_H) = -\frac{\mu - \bar{\beta}}{\bar{\beta}\alpha\theta} < 0,$$

and

$$T'(x) = -\frac{[1-G(x)]\left\{x + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{x} G(\varepsilon) d\varepsilon\right]\right\} + \left[\int_x^{\varepsilon_H} [1-G(\varepsilon)] d\varepsilon\right]\left\{1 + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\alpha\theta G(x)\right\}}{\left\{x + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{x} G(\varepsilon) d\varepsilon\right]\right\}^2} < 0.$$

Hence, if $T(\varepsilon_L) > 0$, or equivalently, if $\mu < \bar{\mu}$ (with $\bar{\mu}$ is as defined in (5)), then there exists a unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that satisfies $T(\varepsilon^*) = 0$ (and $\varepsilon^* \downarrow \varepsilon_L$ as $\mu \uparrow \bar{\mu}$). Once we know ε^*, ϕ^s is given by (44). Given ε^* and ϕ^s, the values of Z, $\bar{\phi}^s$, ϕ^m_t, and p_t are obtained using (12) (with $A^s_I = A^s$ and $A^s_D = 0$), (9), (10), and (11). To conclude this step, notice that for this case to be an equilibrium, (42) must hold, or equivalently, using $\bar{\phi}^s = \varepsilon^* + \phi^s$ and (44), it must be that $\hat{T}(\varepsilon^*) > 0$, where $\hat{T}$ is the continuous function on $[\varepsilon_L, \varepsilon_H]$ defined in (51). Notice that

$\hat{T}'(x) = -[1-\alpha\theta G(x)] < 0$, and $\hat{T}(\varepsilon_H) = -(1-\alpha\theta)(\varepsilon_H - \bar{\varepsilon}) < 0 < \bar{\varepsilon} - \varepsilon_L = \hat{T}(\varepsilon_L)$, so there exists a unique $\hat{\varepsilon} \in (\varepsilon_L, \varepsilon_H)$ such that $\hat{T}(\hat{\varepsilon}) = 0$. (Since $\hat{T}(\bar{\varepsilon}) > 0$, and $\hat{T}' < 0$, it follows that $\bar{\varepsilon} < \hat{\varepsilon}$.) Then $\hat{T}'(x) < 0$ implies $\hat{T}(\varepsilon^*) \geq 0$ if and only if $\varepsilon^* \leq \hat{\varepsilon}$, with "=" for $\varepsilon^* = \hat{\varepsilon}$. With (50), we know that $\varepsilon^* < \hat{\varepsilon}$ if and only if $T(\hat{\varepsilon}) < 0 = T(\varepsilon^*)$, i.e., if and only if

$$\bar{\beta}\left[1 + \frac{\left(1-\bar{\beta}\delta\right)\alpha\theta \int_{\hat{\varepsilon}}^{\varepsilon_H} (\varepsilon - \hat{\varepsilon})\, dG(\varepsilon)}{\hat{\varepsilon}}\right] < \mu.$$

Since $\hat{T}(\hat{\varepsilon}) = -(1-\alpha\theta)(\hat{\varepsilon} - \bar{\varepsilon}) + \alpha\theta \int_{\hat{\varepsilon}}^{\varepsilon_H} (\varepsilon - \hat{\varepsilon})\, dG(\varepsilon) = 0$, this last condition is equivalent to $\hat{\mu} < \mu$, where $\hat{\mu}$ is as defined in (5). The allocations and asset prices described in this step correspond to those in the statement of the proposition for $\mu \in (\hat{\mu}, \bar{\mu})$.

Step 2: Try to construct a recursive monetary equilibrium with $a^s_{dt+1} > 0$ for some $d \in \mathcal{D}$ and $a^s_{it+1} = 0$ for all $i \in \mathcal{I}$. The equilibrium conditions are (12), (43), (45), and (46), together with

$$\phi^s = \bar{\beta}\delta\bar{\phi}^s \tag{52}$$

$$\phi^s > \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right],\ \text{“ = ” if } \tilde{a}^s_{it+1} > 0. \tag{53}$$

$$\tilde{a}^s_{dt+1} \geq 0, \text{ with “ > ” for some } d \in \mathcal{D} \tag{54}$$

$$\tilde{a}^s_{it+1} = 0, \text{ for all } i \in \mathcal{I}. \tag{55}$$

The conditions (43) and (52) are to be solved for ε^* and ϕ^s. First use $\bar{\phi}^s = \varepsilon^* + \phi^s$ in (52) to obtain

$$\phi^s = \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\varepsilon^*. \tag{56}$$

Substitute (56) in (43) to obtain

$$1 = \frac{\bar{\beta}}{\mu}\left[1 + \frac{\alpha\theta\left(1-\bar{\beta}\delta\right)\int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^*}\right], \tag{57}$$

which is a single equation in ε^*. Define

$$R(x) \equiv \frac{\left(1-\bar{\beta}\delta\right)\int_{x}^{\varepsilon_H} (\varepsilon - x)\, dG(\varepsilon)}{x} - \frac{\mu - \bar{\beta}}{\bar{\beta}\alpha\theta} \tag{58}$$

and notice that ε^* solves (57) if and only if it satisfies $R(\varepsilon^*) = 0$. R is a continuous real-valued function on $[\varepsilon_L, \varepsilon_H]$, with

$$R(\varepsilon_L) = \frac{\left(1-\bar{\beta}\delta\right)(\bar{\varepsilon}-\varepsilon_L)}{\varepsilon_L} - \frac{\mu-\bar{\beta}}{\bar{\beta}\alpha\theta}$$
$$R(\varepsilon_H) = -\frac{\mu-\bar{\beta}}{\bar{\beta}\alpha\theta}$$

and

$$R'(x) = -\frac{[1-G(x)]\,x + \int_x^{\varepsilon_H}[1-G(\varepsilon)]\,d\varepsilon}{\frac{1}{1-\bar{\beta}\delta}x^2} < 0.$$

Hence, if $R(\varepsilon_L) > 0$, or equivalently, if

$$\mu < \bar{\beta}\left[1 + \frac{\alpha\theta\left(1-\bar{\beta}\delta\right)(\bar{\varepsilon}-\varepsilon_L)}{\varepsilon_L}\right] \equiv \mu^o,$$

then there exists a unique $\varepsilon^* \in (\varepsilon_L, \varepsilon_H)$ that satisfies $R(\varepsilon^*) = 0$ (and $\varepsilon^* \downarrow \varepsilon_L$ as $\mu \uparrow \mu^o$). Having solved for ε^*, ϕ^s is obtained from (56). Given ε^* and ϕ^s, the values of Z, $\bar{\phi}^s$, ϕ_t^m, and p_t are obtained using (12) (with $A_D^s = A^s - A_I^s = \delta A^s$), (9), (10), and (11). Notice that for this case to be an equilibrium (53) must hold, or equivalently, using (56), it must be that $\hat{T}(\varepsilon^*) < 0$, which in turn is equivalent to $\hat{\varepsilon} < \varepsilon^*$. With (58), we know that $\hat{\varepsilon} < \varepsilon^*$ if and only if $R(\varepsilon^*) = 0 < R(\hat{\varepsilon})$, i.e., if and only if

$$\mu < \bar{\beta}\left[1 + \frac{\alpha\theta\left(1-\bar{\beta}\delta\right)\int_{\hat{\varepsilon}}^{\varepsilon_H}(\varepsilon-\hat{\varepsilon})\,dG(\varepsilon)}{\hat{\varepsilon}}\right],$$

which using $\hat{T}(\hat{\varepsilon}) = 0$ can be written as $\mu < \hat{\mu}$. To summarize, the prices and allocations constructed in this step constitute a recursive monetary equilibrium provided $\mu \in (\bar{\beta}, \min(\hat{\mu}, \mu^o))$. To conclude this step, we show that $\hat{\mu} < \bar{\mu} < \mu^o$, which together with the previous step will mean that there is no recursive monetary equilibrium for $\mu \geq \bar{\mu}$ (thus establishing part (*ii*) in the statement of the proposition). It is clear that $\bar{\mu} < \mu^o$, and we know that $\hat{\mu} < \bar{\mu}$ from Lemma 4. Therefore, the allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\mu \in (\bar{\beta}, \min(\hat{\mu}, \mu^o)) = (\bar{\beta}, \hat{\mu})$.

Step 3: Try to construct a recursive monetary equilibrium with $\tilde{a}_{dt+1}^s > 0$ for some $d \in \mathcal{D}$ and $\tilde{a}_{it+1}^s > 0$ for some $i \in \mathcal{I}$. The equilibrium conditions are (12), (43), (44), (45), (46), and (52) with

$$\tilde{a}_{it+1}^s \geq 0 \text{ and } \tilde{a}_{dt+1}^s \geq 0, \text{ with " > " for some } i \in \mathcal{I} \text{ or some } d \in \mathcal{I}.$$

Notice that ε^* and ϕ^s are obtained as in Step 2. Now, however, (44) must also hold, which together with (56) implies we must have $\hat{T}(\varepsilon^*) = 0$, or equivalently, $\varepsilon^* = \hat{\varepsilon}$. In other words, this condition requires $R(\hat{\varepsilon}) = \hat{T}(\hat{\varepsilon})$, or equivalently, we must have $\mu = \hat{\mu}$. As before, the market-clearing condition (12) is used to obtain Z, while (9), (10), and (11) imply $\bar{\phi}^s$, ϕ_t^m, and p_t, respectively. The allocations and asset prices described in this step correspond to those in the statement of the proposition for the case with $\mu = \hat{\mu}$.

Combined, Steps 1, 2, and 3 prove part (iv) in the statement of the proposition. Part $(v)(a)$ is immediate from (44) and (50), and part $(v)(b)$ from (56) and (58). ■

Corollary 3 *The marginal valuation, ε^*, characterized in Proposition 1 is strictly decreasing in the rate of inflation, i.e., $\frac{\partial \varepsilon^*}{\partial \mu} < 0$ both for $\mu \in (\bar{\beta}, \hat{\mu})$ and for $\mu \in (\hat{\mu}, \bar{\mu})$.*

Proof. For $\mu \in (\bar{\beta}, \hat{\mu})$, implicitly differentiate $R(\varepsilon^*) = 0$ (with R given by (58)), and for $\mu \in (\hat{\mu}, \bar{\mu})$, implicitly differentiate $T(\varepsilon^*) = 0$ (with T given by (50)) to obtain

$$\frac{\partial \varepsilon^*}{\partial \mu} = \begin{cases} -\frac{\varepsilon^*}{\bar{\beta}\alpha\theta\left(1-\bar{\beta}\delta\right)[1-G(\varepsilon^*)]+\mu-\bar{\beta}} & \text{if } \bar{\beta} < \mu < \hat{\mu} \\ -\frac{\bar{\beta}\alpha\theta\int_{\varepsilon^*}^{\varepsilon_H}[1-G(\varepsilon)]d\varepsilon}{\left\{1+\bar{\beta}\alpha\theta\left[\frac{\delta G(\varepsilon^*)}{1-\bar{\beta}\delta}+\frac{1-G(\varepsilon^*)}{\mu-\bar{\beta}}\right]\right\}\left(\mu-\bar{\beta}\right)^2} & \text{if } \hat{\mu} < \mu < \bar{\mu}. \end{cases}$$

Clearly, $\partial \varepsilon^*/\partial \mu < 0$ for $\mu \in (\bar{\beta}, \hat{\mu})$ and for $\mu \in (\hat{\mu}, \bar{\mu})$. ■

A.4 Positive implications: asset prices

A.4.1 Monetary policy

Proof of Proposition 2. Recall that $\partial \varepsilon^*/\partial \mu < 0$ (Corollary 3). (i) From (8),

$$\frac{\partial \phi^s}{\partial \mu} = \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\mathbb{I}_{\{\bar{\beta}<\mu\leq\hat{\mu}\}} + \mathbb{I}_{\{\hat{\mu}<\mu<\bar{\mu}\}}\alpha\theta G(\varepsilon^*)\right]\frac{\partial \varepsilon^*}{\partial \mu} < 0.$$

(ii) Condition (9) implies $\partial \bar{\phi}^s/\partial \mu = \partial \varepsilon^*/\partial \mu + \partial \phi^s/\partial \mu < 0$. (iii) From (12) it is clear that $\partial Z/\partial \varepsilon^* > 0$, so $\partial Z/\partial \mu = (\partial Z/\partial \varepsilon^*)(\partial \varepsilon^*/\partial \mu) < 0$. From (10), $\partial \phi_t^m/\partial \mu = (y_t/A_t^m)\,\partial Z/\partial \mu < 0$. ■

For Proposition 3 we consider a formulation of the model where the length of the time period becomes arbitrarily short. This limiting economy can be interpreted as an approximation to a continuous-time version of our discrete-time economy.[49] To this end, generalize the baseline discrete-time model by allowing the period length to be an arbitrary constant, and then take

[49]This formulation is useful since the model period in our quantitative implementation of the theory is very short (a trading day).

the limit as this constant becomes arbitrarily small. Let Δ denote the length of the model period, and define the discount rate, r, the expected dividend growth rate, g, the depreciation rate, d, and the money growth rate, m, as $\beta \equiv (1+r\Delta)^{-1}$, $\bar{\gamma} \equiv 1+g\Delta$, $\delta \equiv 1-d\Delta$, $\mu \equiv 1+m\Delta$. Over a time period of length Δ, the dividend is $y_t\Delta$, and consumption of the dividend good is $\varepsilon y_t \Delta$. In this context we focus on recursive equilibria where, as $\Delta \to 0$, real asset prices are time-invariant linear functions of the *dividend rate*, y_t. Specifically, let $\Phi_t^s(\Delta)$ and $\Phi_t^m(\Delta) A_t^m$ denote the real equity price and the real aggregate money balance, respectively, in the discrete-time economy with time periods of length Δ. We look for recursive equilibria of this discrete-time economy such that $\Phi_t^s(\Delta) = \Phi^s(\Delta) y_t \Delta$, $\bar{\Phi}_t^s(\Delta) = \bar{\Phi}^s(\Delta) y_t \Delta$, and $\Phi_t^m(\Delta) A_t^m = \mathrm{Z}(\Delta) y_t \Delta$, where $\Phi^s(\Delta)$, $\bar{\Phi}^s(\Delta)$, and $\mathrm{Z}(\Delta)$ are time-invariant functions with the property that $\lim_{\Delta\to 0} \Phi^s(\Delta)\Delta = \phi^s$, $\lim_{\Delta\to 0} \bar{\Phi}^s(\Delta)\Delta = \bar{\phi}^s$, and $\lim_{\Delta\to 0} \mathrm{Z}(\Delta)\Delta = Z$, with $\phi^s, Z \in \mathbb{R}$.[50] The thresholds in (5) now generalize to $\hat{\mu} = 1+\hat{m}\Delta$ and $\bar{\mu} = 1+\bar{m}\Delta$, where

$$\hat{m} \equiv \frac{(1-\alpha\theta)(\hat{\varepsilon}-\bar{\varepsilon})(r-g+d)}{\hat{\varepsilon}} - (r-g)$$
$$\bar{m} \equiv \frac{\alpha\theta(\bar{\varepsilon}-\varepsilon_L)(r-g+d)}{\bar{\varepsilon}} - (r-g).$$

Also, if we relabel $\underline{\mu} \equiv \bar{\beta}$, we now have $\underline{\mu} = 1+\underline{m}\Delta$, where $\underline{m} \equiv g-r$. Then from part (*ii*) of Proposition 1, we know that a stationary monetary equilibrium exists if $m \in [\underline{m}, \bar{m})$. Notice that $\bar{m} < d$, so $m < d$ in any monetary equilibrium. The monetary equilibrium, i.e., (8), (9), (10), (12), and (13), generalize to

$$\Phi_t^s(\Delta) = \Phi^s(\Delta) y_t \Delta$$
$$\Phi^s(\Delta) = \begin{cases} \frac{(1+g\Delta)(1-d\Delta)}{1+r\Delta-(1+g\Delta)(1-d\Delta)}\varepsilon^* & \text{if } \underline{m} < m \leq \hat{m} \\ \frac{(1+g\Delta)(1-d\Delta)}{1+r\Delta-(1+g\Delta)(1-d\Delta)}\left[\bar{\varepsilon} + \alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^*-\varepsilon)\,dG(\varepsilon)\right] & \text{if } \hat{m} < m < \bar{m} \end{cases}$$
$$\bar{\Phi}_t^s(\Delta) = \bar{\Phi}^s(\Delta) y_t \Delta, \text{ with } \bar{\Phi}^s(\Delta) = \varepsilon^* + \Phi^s(\Delta)$$
$$\Phi_t^m(\Delta) = \frac{\mathrm{Z}(\Delta) y_t \Delta}{A_t^m}$$
$$\mathrm{Z}(\Delta) y_t \Delta = \frac{\alpha G(\varepsilon^*) A_I^s + A_D^s}{\alpha[1-G(\varepsilon^*)]}\left[\varepsilon^* y_t \Delta + \Phi_t^s(\Delta)\right]$$

and

$$\frac{\int_{\varepsilon^*}^{\varepsilon_H}(\varepsilon-\varepsilon^*)\,dG(\varepsilon)}{\varepsilon^* + \frac{(1+g\Delta)(1-d\Delta)}{1+r\Delta}\left[\bar{\varepsilon}-\varepsilon^*+\alpha\theta\int_{\varepsilon_L}^{\varepsilon^*}(\varepsilon^*-\varepsilon)\,dG(\varepsilon)\right]\mathbb{I}_{\{\hat{m}<m\}}} - \frac{[(1+m\Delta)(1+r\Delta)-(1+g\Delta)](1+r\Delta)}{[1+r\Delta-(1+g\Delta)(1-d\Delta)](1+g\Delta)}\frac{1}{\alpha\theta} = 0.$$

[50]The discrete-time formulation we laid out previously, corresponds to a special case of this formulation with $\Delta = 1$, $\Phi_t^s(1) \equiv \phi_t^s$, $\Phi_t^m(1) \equiv \phi_t^m$, $\Phi^s(1) \equiv \phi^s$, and $\mathrm{Z}(1) \equiv Z$.

As $\Delta \to 0$, these conditions become

$$\phi_t^s = \phi^s y_t, \text{ with } \phi^s = \begin{cases} \frac{1}{r-g+d}\varepsilon^* & \text{if } \underline{m} < m \le \hat{m} \\ \frac{1}{r-g+d}\left[\bar{\varepsilon} + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right] & \text{if } \hat{m} < m < \bar{m} \end{cases} \tag{59}$$

$$\bar{\phi}_t^s = \phi_t^s \tag{60}$$

$$\phi_t^m = \frac{Z}{A_t^m} y_t \tag{61}$$

$$Z = \frac{\alpha G(\varepsilon^*) A_I^s + A_D^s}{\alpha [1 - G(\varepsilon^*)]}\phi^s \tag{62}$$

$$0 = \frac{\alpha\theta \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*)\, dG(\varepsilon)}{\varepsilon^* + \left[\bar{\varepsilon} - \varepsilon^* + \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon)\, dG(\varepsilon)\right] \mathbb{I}_{\{\hat{m}<m\}}} - \frac{r-g+m}{r-g+d}, \tag{63}$$

where $\phi_t^s \equiv \lim_{\Delta\to 0} \Phi_t^s(\Delta)$, $\bar{\phi}_t^s \equiv \lim_{\Delta\to 0} \bar{\Phi}_t^s(\Delta)$, and $\phi_t^m \equiv \lim_{\Delta\to 0} \Phi_t^m(\Delta)$.

Corollary 4 *In any monetary equilibrium of the continuous-time economy, $\partial\varepsilon^*/\partial r < 0$.*

Proof. From (63), it is clear that $\partial\varepsilon^*/\partial m < 0$, and

$$\frac{\partial\varepsilon^*}{\partial r} = \frac{d-m}{r-g+d}\frac{\partial\varepsilon^*}{\partial m}.$$

In any monetary equilibrium, $m < \bar{m} < d$, so $\partial\varepsilon^*/\partial r < 0$ in any monetary equilibrium. ■

Proof of Proposition 3. (*i*) From (59),

$$\frac{\partial\phi^s}{\partial r} = \frac{1}{r-g+d}\left\{-\phi^s + \left[\mathbb{I}_{\{\underline{m}<m\le\hat{m}\}} + \mathbb{I}_{\{\hat{m}<m<\bar{m}\}}\alpha\theta G(\varepsilon^*)\right]\frac{\partial\varepsilon^*}{\partial r}\right\} < 0,$$

where the inequality follows from Corollary 4. (*ii*) From (62) it is clear that $\partial Z/\partial\varepsilon^* > 0$, so $\partial Z/\partial r = (\partial Z/\partial\varepsilon^*)(\partial\varepsilon^*/\partial r) < 0$. From (61), $\partial\phi_t^m/\partial r = (y_t/A_t^m)\,\partial Z/\partial r < 0$. ■

A.4.2 OTC frictions: trading delays and market power

Proof of Proposition 4. From condition (13),

$$\frac{\partial\varepsilon^*}{\partial(\alpha\theta)} = \frac{\frac{\mu-\bar{\beta}}{\alpha\theta}[\varepsilon^* + \bar{\beta}\delta(\bar{\varepsilon} - \varepsilon^*)\mathbb{I}_{\{\hat{\mu}<\mu\}}]}{\bar{\beta}\alpha\theta(1-\bar{\beta}\delta)[1-G(\varepsilon^*)] + (\mu-\bar{\beta})\left\{1 + \bar{\beta}\delta[\alpha\theta G(\varepsilon^*) - 1]\mathbb{I}_{\{\hat{\mu}<\mu\}}\right\}} > 0. \tag{64}$$

(*i*) From (8),

$$\frac{\partial\phi^s}{\partial(\alpha\theta)} = \begin{cases} \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\frac{\partial\varepsilon^*}{\partial(\alpha\theta)} > 0 & \text{if } \bar{\beta} < \mu \le \hat{\mu} \\ \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon + \alpha\theta G(\varepsilon^*)\frac{\partial\varepsilon^*}{\partial(\alpha\theta)}\right] > 0 & \text{if } \hat{\mu} < \mu < \bar{\mu}. \end{cases}$$

(*ii*) From (9), $\partial\bar{\phi}^s/\partial(\alpha\theta) = \partial\varepsilon^*/\partial(\alpha\theta) + \partial\phi^s/\partial(\alpha\theta) > 0$. (*iii*) For $\mu \in (\hat{\mu}, \bar{\mu})$, (12) implies $\partial Z/\partial\alpha = (\partial Z/\partial\varepsilon^*)(\partial\varepsilon^*/\partial\alpha) > 0$ and therefore $\partial\phi_t^m/\partial\alpha = (\partial Z/\partial\alpha)(y_t/A_t^m) > 0$. ■

A.5 Positive implications: financial liquidity

In this section, we use the theory to study the determinants of standard measures of market liquidity: liquidity provision by dealers, trade volume, and bid-ask spreads.

A.5.1 Liquidity provision by dealers

Broker-dealers in financial markets provide liquidity (*immediacy*) to investors by finding them counterparties for trade, or by trading with them out of their own account, effectively becoming their counterparty. The following result characterizes the effect of inflation on dealers' provision of liquidity by accumulating assets.

Proposition 7 *In the recursive monetary equilibrium: (i) dealers' provision of liquidity by accumulating assets, i.e.,* A_D^s*, is nonincreasing in the inflation rate. (ii) For any* μ *close to* $\bar{\beta}$*, dealers' provision of liquidity by accumulating assets is nonmonotonic in* $\alpha\theta$*, i.e.,* $A_D^s = 0$ *for* $\alpha\theta$ *close to* 0 *and close to* 1*, but* $A_D^s > 0$ *for intermediate values of* $\alpha\theta$*.*

Proof. (i) The result is immediate from the expression for A_D^s in Proposition 1. (ii) From (5) and (6),

$$\frac{\partial \hat{\mu}}{\partial (\alpha\theta)} = \bar{\beta}\left(1 - \bar{\beta}\delta\right)\left\{\frac{(1-\alpha\theta)\,\bar{\varepsilon}}{\left[1-\alpha\theta G\left(\hat{\varepsilon}\right)\right]\hat{\varepsilon}^2}\int_{\varepsilon_L}^{\hat{\varepsilon}} G\left(\varepsilon\right) d\varepsilon - \frac{\hat{\varepsilon}-\bar{\varepsilon}}{\hat{\varepsilon}}\right\}.$$

Notice that $\partial\hat{\mu}/\partial\left(\alpha\theta\right)$ approaches a positive value as $\alpha\theta \to 0$ and a negative value as $\alpha\theta \to 1$. Also, $\hat{\mu} \to \bar{\beta}$ both when $\alpha\theta \to 0$ and when $\alpha\theta \to 1$. Hence, $\mu > \bar{\beta} = \lim_{\alpha\theta\to 0}\hat{\mu} = \lim_{\alpha\theta\to 1}\hat{\mu}$ for a range of values of $\alpha\theta$ close to 0 and a range of values of $\alpha\theta$ close to 1. For those ranges of values of $\alpha\theta$, $A_D^s = 0$. In between those ranges there must exist values of $\alpha\theta$ such that $\mu < \hat{\mu}$, which implies $A_D^s > 0$. ■

Part (i) of Proposition 7 is related to the discussion that followed Proposition 1. The expected return from holding equity is larger for investors than for dealers with high inflation ($\mu > \hat{\mu}$) because in that case the expected resale value of equity in the OTC market is relatively low and dealers only buy equity to resell in the OTC market, while investors also buy it with the expectation of getting utility from the dividend flow. For low inflation ($\mu < \hat{\mu}$), dealers value equity more than investors because the OTC resale value is high and they have a higher probability of making capital gains from reselling than investors, and this trading advantage more than compensates for the fact that investors enjoy the additional utility from the dividend flow. Part (*ii*) of Proposition 7 states that given a low enough rate of inflation, dealers' incentive to hold equity inventories overnight is nonmonotonic in the degree of OTC frictions as measured by $\alpha\theta$. In particular, dealers will not hold inventories if $\alpha\theta$ is either very small or very large. If $\alpha\theta$ is close to zero, few investors contact the interdealer market, and this makes the equity price in the OTC market very low, which in turn implies too small a capital gain to induce dealers to hold equity overnight. Conversely, if $\alpha\theta$ is close to one, a dealer has no trading advantage over an investor in the OTC market and since the investor gets utility from the dividend while the dealer does not, the willingness to pay for the asset in the centralized market is higher for the investor than for the dealer, and therefore it is investors and not dealers who carry the asset overnight into the OTC market.

A.5.2 Trade volume

Proof of Proposition 5. (*i*) Differentiate (15) to get

$$\frac{\partial \mathcal{V}}{\partial \mu} = 2\alpha G'(\varepsilon^*)(A^s - \delta \tilde{A}_D^s)\frac{\partial \varepsilon^*}{\partial \mu} < 0,$$

where the inequality follows from Corollary 3. (*ii*) Differentiate (15) to get

$$\frac{\partial \mathcal{V}}{\partial r} = 2\alpha G'(\varepsilon^*)(A^s - \delta \tilde{A}_D^s)\frac{\partial \varepsilon^*}{\partial r} < 0,$$

where the inequality follows from Corollary 4. (*iii*) From (15),

$$\begin{aligned}
\frac{\partial \mathcal{V}}{\partial \theta} &= 2\alpha G'(\varepsilon^*)(A^s - \delta \tilde{A}_D^s)\frac{\partial \varepsilon^*}{\partial \theta} \\
\frac{\partial \mathcal{V}}{\partial \alpha} &= 2\left[G(\varepsilon^*) + \alpha G'(\varepsilon^*)\frac{\partial \varepsilon^*}{\partial \alpha}\right](A^s - \delta \tilde{A}_D^s),
\end{aligned}$$

and both are positive since $\partial\varepsilon^*/\partial(\alpha\theta) > 0$ (see (64)). ■

A.5.3 Bid-ask spreads

Bid-ask spreads and intermediation fees are a popular measure of market liquidity as they constitute the main out-of-pocket transaction cost that investors bear in OTC markets. Lemma 1 shows that when dealers execute trades on behalf of their investors, they charge a fee $k_t(\boldsymbol{a}_t, \varepsilon)$ that is linear in the trade size. This means that when an investor with $\varepsilon > \varepsilon_t^*$ wants to buy equity, the dealer charges him an *ask price*, $p_t^a(\varepsilon) = p_t\phi_t^m + (1-\theta)(\varepsilon - \varepsilon_t^*)y_t$ per share. When an investor with $\varepsilon < \varepsilon_t^*$ wants to sell, the dealer pays him a *bid price*, $p_t^b(\varepsilon) = p_t\phi_t^m -$

$(1-\theta)\left(\varepsilon_t^*-\varepsilon\right)y_t$ per share. Define $\mathcal{S}_t^a\left(\varepsilon\right)=\frac{p_t^a(\varepsilon)-p_t\phi_t^m}{p_t\phi_t^m}$ and $\mathcal{S}_t^b\left(\varepsilon\right)=\frac{p_t\phi_t^m-p_t^b(\varepsilon)}{p_t\phi_t^m}$, i.e., the ask spread and bid spread, respectively, expressed as fractions of the price of the asset in the interdealer market. Then in a recursive equilibrium, the ask spread earned by a dealer when trading with an investor with $\varepsilon>\varepsilon^*$ is $\mathcal{S}^a\left(\varepsilon\right)=\frac{(1-\theta)(\varepsilon-\varepsilon^*)}{\varepsilon^*+\phi^s}$ and the bid spread earned by a dealer when trading with an investor with $\varepsilon<\varepsilon^*$ is $\mathcal{S}^b\left(\varepsilon\right)=\frac{(1-\theta)(\varepsilon^*-\varepsilon)}{\varepsilon^*+\phi^s}$. The average real spread earned by dealers is $\bar{\mathcal{S}}=\int\left[\mathcal{S}^a\left(\varepsilon\right)\mathbb{I}_{\{\varepsilon^*<\varepsilon\}}+\mathcal{S}^b\left(\varepsilon\right)\mathbb{I}_{\{\varepsilon<\varepsilon^*\}}\right]dG\left(\varepsilon\right)$. The change $\bar{\mathcal{S}}$ in response to changes in μ or α is ambiguous in general.[51]

A.6 Testable implications

Proof of Proposition 6. We start with the general model described in Section 6.1, which nests the model of Section 2 provided dealers do not hold asset inventories (the model of Section 6.1 considers an environment where dealers cannot hold inventories). The equilibrium conditions for the model of Section 6.1 for the case of no policy uncertainty (as is the case in the model of Section 2) are reported in Proposition 13 (Appendix C, Section C.3). Even without policy uncertainty, the model of Section 6.1 is more general than the model of Section 2 in that the former has multiple assets indexed by $s\in\{1,2,...,N\}$ that differ in terms of the trading probability, α^s. Consider the equilibrium conditions (120) and (121), that determine ϕ^s and ε^{s*} for asset of type s. By following steps similar to those that preceeded the proof of Proposition 3 in Section A.4, in the continuous-time limit of the discrete-time economy, conditions (120) and (121) become

$$\phi^s=\frac{1}{r-g+d}\left[\bar{\varepsilon}+\alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}}(\varepsilon^{s*}-\varepsilon)dG(\varepsilon)\right] \tag{65}$$

$$0=\frac{\alpha^s\theta\int_{\varepsilon^{s*}}^{\varepsilon_H}\left(\varepsilon-\varepsilon^{s*}\right)dG(\varepsilon)}{\bar{\varepsilon}+\alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}}(\varepsilon^{s*}-\varepsilon)dG(\varepsilon)}-\frac{r-g+m}{r-g+d}. \tag{66}$$

Notice that (65) and (66), i.e., the equilibrium conditions that determine ϕ^s and ε^{s*} for each asset s in the multiple-asset model are identical to the corresponding conditions (59) and (63) for the continuous-time approximation to the single-asset model of Section 2 (again, provided dealers are assumed to hold no assets, e.g., as would be the case if they either cannot, or if they can, but $\hat{m}<m$).

[51]The reason is that the spread $\mathcal{S}^a\left(\varepsilon\right)$ charged to buyers is decreasing in ε^* while the spread $\mathcal{S}^b\left(\varepsilon\right)$ charged to sellers may be increasing in ε^*. For example, if $\mu\in\left(\bar{\beta},\hat{\mu}\right)$, it is easy to show $\partial\mathcal{S}^a\left(\varepsilon\right)/\partial\varepsilon^*=-\partial\mathcal{S}^b\left(\varepsilon\right)/\partial\varepsilon^*<0$.

For any $x \in [\varepsilon_L, \varepsilon_H]$, define

$$I_B(x) \equiv \int_x^{\varepsilon_H} (\varepsilon - x)\, dG(\varepsilon)$$
$$I_S(x) \equiv \int_{\varepsilon_L}^{x} (x - \varepsilon) dG(\varepsilon)$$

and notice that $I_B'(x) = -[1 - G(x)]$ and $I_S'(x) = G(x)$, so a first-order Taylor approximation around $x_0 \in (\varepsilon_L, \varepsilon_H)$ gives

$$I_B(x) \approx I_B(x_0) - [1 - G(x_0)](x - x_0)$$
$$I_S(x) \approx I_S(x_0) + G(x_0)(x - x_0).$$

Let $\varepsilon^{s*} \equiv \varepsilon^{s*}(r, m, \alpha^s) \in (\varepsilon_L, \varepsilon_H)$ denote the value of ε^{s*} that solves (66) for given (r, m, α^s), and let $\varepsilon_0^{s*} \equiv \varepsilon^{s*}(r_0, m_0, \alpha_0^s) \in (\varepsilon_L, \varepsilon_H)$ for some (r_0, m_0, α_0^s). Use the Taylor approximation to get

$$(r - g + d)\, \phi^s \approx \bar{\varepsilon} + \alpha^s \theta \left[I_S(\varepsilon_0^{s*}) + G(\varepsilon_0^{s*})(\varepsilon^{s*} - \varepsilon_0^{s*})\right] \tag{67}$$

$$\frac{r - g + m}{r - g + d} \approx \frac{I_B(\varepsilon_0^{s*}) - [1 - G(\varepsilon_0^{s*})](\varepsilon^{s*} - \varepsilon_0^{s*})}{\frac{\bar{\varepsilon}}{\alpha^s \theta} + I_S(\varepsilon_0^{s*}) + G(\varepsilon_0^{s*})(\varepsilon^{s*} - \varepsilon_0^{s*})}. \tag{68}$$

Condition (68) implies

$$\varepsilon^{s*}(r, m, \alpha^s) \approx \frac{I_B(\varepsilon_0^{s*}) + [1 - G(\varepsilon_0^{s*})]\varepsilon_0^{s*} - \left(\frac{r-g+m}{r-g+d}\right)\left[\frac{\bar{\varepsilon}}{\alpha^s\theta} + I_S(\varepsilon_0^{s*}) - G(\varepsilon_0^{s*})\varepsilon_0^{s*}\right]}{\left(\frac{r-g+m}{r-g+d}\right) G(\varepsilon_0^{s*}) + [1 - G(\varepsilon_0^{s*})]}$$

and a first-order Taylor approximation gives

$$\begin{aligned}\varepsilon^{s*}(r, m, \alpha^s) \approx\ & \varepsilon_0^{s*} + \frac{\partial \varepsilon^{s*}(r_0, m_0, \alpha_0^s)}{\partial r}(r - r_0) + \frac{\partial \varepsilon^{s*}(r_0, m_0, \alpha_0^s)}{\partial m}(m - m_0) \\ & + \frac{\partial \varepsilon^{s*}(r_0, m_0, \alpha_0^s)}{\partial \alpha^s}(\alpha^s - \alpha_0^s),\end{aligned} \tag{69}$$

where

$$\frac{\partial \varepsilon^{s*}(r, m, \alpha^s)}{\partial r} = -\frac{\left[\frac{\bar{\varepsilon}}{\alpha^s\theta} + I_S(\varepsilon_0^{s*})\right][1 - G(\varepsilon_0^{s*})] + I_B(\varepsilon_0^{s*})\, G(\varepsilon_0^{s*})}{\left\{\left(\frac{r-g+m}{r-g+d}\right) G(\varepsilon_0^{s*}) + [1 - G(\varepsilon_0^{s*})]\right\}^2} \frac{d - m}{(r - g + d)^2} \tag{70}$$

$$\frac{\partial \varepsilon^{s*}(r, m, \alpha^s)}{\partial m} = -\frac{G(\varepsilon_0^{s*})\, I_B(\varepsilon_0^{s*}) + [1 - G(\varepsilon_0^{s*})]\left[\frac{\bar{\varepsilon}}{\alpha^s\theta} + I_S(\varepsilon_0^{s*})\right]}{(r - g + d)\left\{\left(\frac{r-g+m}{r-g+d}\right) G(\varepsilon_0^{s*}) + [1 - G(\varepsilon_0^{s*})]\right\}^2} \tag{71}$$

$$\frac{\partial \varepsilon^{s*}(r, m, \alpha^s)}{\partial \alpha^s} = \frac{\left(\frac{r-g+m}{r-g+d}\right)\frac{\bar{\varepsilon}}{(\alpha^s)^2\theta}}{\left(\frac{r-g+m}{r-g+d}\right) G(\varepsilon_0^{s*}) + [1 - G(\varepsilon_0^{s*})]}. \tag{72}$$

Conditions (67) and (69) imply

$$\begin{aligned}\log\phi^s \approx & -\log(r-g+d)\\ & +\log\Big\{\bar{\varepsilon}+\alpha^s\theta\Big[I_S(\varepsilon_0^{s*})+G(\varepsilon_0^{s*})\Big(\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial r}(r-r_0)+\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial m}(m-m_0)\\ & +\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}(\alpha^s-\alpha_0^s)\Big)\Big]\Big\}.\end{aligned}$$

Without loss of generality, let $m=m_0$; then

$$\frac{\partial\log\phi^s}{\partial r}=-\frac{1}{r-g+d}+\frac{\alpha^s\theta G(\varepsilon_0^{s*})\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial r}}{\bar{\varepsilon}+\alpha^s\theta\Big[I_S(\varepsilon_0^{s*})+G(\varepsilon_0^{s*})\Big(\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial r}(r-r_0)+\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}(\alpha^s-\alpha_0^s)\Big)\Big]} \tag{73}$$

and

$$\frac{\partial^2\log\phi^s}{\partial\alpha^s\partial r}=\frac{\theta G(\varepsilon_0^{s*})\Big[\bar{\varepsilon}-(\alpha^s)^2\theta G(\varepsilon_0^{s*})\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}\Big]\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial r}}{\Big\{\bar{\varepsilon}+\alpha^s\theta\Big[I_S(\varepsilon_0^{s*})+G(\varepsilon_0^{s*})\Big(\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial r}(r-r_0)+\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}(\alpha^s-\alpha_0^s)\Big)\Big]\Big\}^2}. \tag{74}$$

From (70), we see that in any monetary equilibrium, $\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)/\partial r<0$ (since $m<\bar{m}<d$, in any monetary equilibrium), so it is clear from (73) that $\partial\log\phi^s/\partial r<0$. This establishes part (*ii*) in the statement of the proposition. By using (72), we know that

$$\bar{\varepsilon}-(\alpha^s)^2\theta G(\varepsilon_0^{s*})\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}=\frac{1-G(\varepsilon_0^{s*})}{\left(\frac{r-g+m}{r-g+d}\right)G(\varepsilon_0^{s*})+[1-G(\varepsilon_0^{s*})]}\bar{\varepsilon}>0. \tag{75}$$

With this inequality and $\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)/\partial r<0$, it is clear from (74) that $\partial^2\log\phi^s/(\partial\alpha^s\partial r)<0$. This establishes part (*iv*) in the statement of the proposition.

Without loss of generality, let $r=r_0$; then

$$\frac{\partial\log\phi^s}{\partial m}=\frac{\alpha^s\theta G(\varepsilon_0^{s*})\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial m}}{\bar{\varepsilon}+\alpha^s\theta\Big[I_S(\varepsilon_0^{s*})+G(\varepsilon_0^{s*})\Big(\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial m}(m-m_0)+\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}(\alpha^s-\alpha_0^s)\Big)\Big]} \tag{76}$$

and

$$\frac{\partial^2\log\phi^s}{\partial\alpha^s\partial m}=\frac{\theta G(\varepsilon_0^{s*})\Big[\bar{\varepsilon}-(\alpha^s)^2\theta G(\varepsilon_0^{s*})\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}\Big]\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial m}}{\Big\{\bar{\varepsilon}+\alpha^s\theta\Big[I_S(\varepsilon_0^{s*})+G(\varepsilon_0^{s*})\Big(\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial m}(m-m_0)+\frac{\partial\varepsilon^{s*}(r_0,m_0,\alpha_0^s)}{\partial\alpha^s}(\alpha^s-\alpha_0^s)\Big)\Big]\Big\}^2}. \tag{77}$$

In Section 3, we defined $1+\pi\equiv\mu/\bar{\gamma}$ for our baseline model. In the generalized discrete-time formulation with period of length Δ, this definition generalizes to $1+\pi\Delta\equiv\frac{1+m\Delta}{1+g\Delta}$, which for Δ small implies $\pi \approx m - g$. Hence it is clear that keeping g constant, $\partial \log \phi^s / \partial m$ and $\partial^2 \log \phi^s / (\partial \alpha^s \partial m)$ have the same sign as $\partial \log \phi^s / \partial \pi$ and $\partial^2 \log \phi^s / (\partial \alpha^s \partial \pi)$, respectively. From (71), we see that in any monetary equilibrium, $\partial \varepsilon^{s*} (r_0, m_0, \alpha_0^s) / \partial m < 0$, so it is clear from (76) that $\partial \log \phi^s / \partial m < 0$. This establishes part (i) in the statement of the proposition. With (75) and $\partial \varepsilon^{s*} (r_0, m_0, \alpha_0^s) / \partial m < 0$, it is clear from (77) that $\partial^2 \log \phi^s / (\partial \alpha^s \partial m) < 0$. This establishes part (iii) in the statement of the proposition. ■

A.7 Speculative premium

According to Proposition 1, in a monetary equilibrium the equity price, ϕ^s, is larger than the expected present discounted value that any agent assigns to the dividend stream, i.e., $\hat{\phi}_t^s \equiv \left[\bar{\beta}\delta/(1-\bar{\beta}\delta)\right] \bar{\varepsilon} y_t$. We follow Harrison and Kreps (1978) and call the equilibrium value of the asset in excess of the expected present discounted value of the dividend, i.e., $\phi_t^s - \hat{\phi}_t^s$, the *speculative premium* that investors are willing to pay in anticipation of the capital gains they will reap when reselling the asset to investors with higher valuations in the future.[52] Thus, we say investors exhibit *speculative behavior* if the prospect of reselling a stock makes them willing to pay more for it than they would if they were obliged to hold it forever. Investors exhibit speculative behavior in the sense that they buy with the expectation to resell, and naturally the asset price incorporates the value of this option to resell.

The speculative premium in a monetary equilibrium is $\mathcal{P}_t = \mathcal{P} y_t$, where

$$\mathcal{P} = \begin{cases} \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta} \left(\varepsilon^* - \bar{\varepsilon}\right) & \text{if } \bar{\beta} < \mu \leq \hat{\mu} \\ \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta} \alpha\theta \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\, d\varepsilon & \text{if } \hat{\mu} < \mu < \bar{\mu}. \end{cases}$$

The speculative premium is nonnegative in any monetary equilibrium, i.e., $\mathcal{P}_t \geq 0$, with "=" only if $\mu = \bar{\mu}$. Since $\partial \varepsilon^* / \partial \mu < 0$ (see Corollary 3), it is immediate that the speculative premium is decreasing in the rate of inflation. Intuitively, anticipated inflation reduces the

[52] It is commonplace to define the *fundamental value* of the asset as the expected present discounted value of the dividend stream and to call any transaction value in excess of this benchmark a *bubble*. In fact, our notion of *speculative premium* corresponds to the notion of *speculative bubble* that is used in the modern literature on bubbles. See, e.g., Barlevy (2007), Brunnermeier (2008), Scheinkman and Xiong (2003a, 2003b), Scheinkman (2013), and Xiong (2013), who discuss Harrison and Kreps (1978) in the context of what is generally known as the *resale option theory of bubbles*. One could argue, of course, that the relevant notion of "fundamental value" should be calculated through market aggregation of diverse investor valuations and taking into account the monetary policy stance as well as all the details of the market structure in which the asset is traded (such as the frequency of trading opportunities and the degree of market power of financial intermediaries), which ultimately also factor into the asset price in equilibrium. We adopt the terminology used by Harrison and Kreps (1978) to avoid semantic controversies.

real money balances used to finance asset trading, which limits the ability of high-valuation traders to purchase the asset from low-valuation traders. As a result, the speculative premium is decreasing in μ. Since $\partial\varepsilon^*/\partial(\alpha\theta) > 0$ (see the proof of Proposition 4), the speculative premium is increasing in α and θ. Intuitively, the speculative premium is the value of the option to resell the equity to a higher valuation investor in the future, and the value of this resale option to the investor increases with the probability α that the investor gets a trading opportunity in an OTC trading round and with the probability θ that he can capture the gains from trade in those trades. So in low-inflation regimes, the model predicts large trade volume and a large speculative premium. The following proposition summarizes these results.

Proposition 8 *In the recursive monetary equilibrium: (i) $\partial\mathcal{P}/\partial\mu < 0$, (ii) $\partial\mathcal{P}/\partial(\alpha\theta) > 0$.*

Proof. (i) For $\bar{\beta} < \mu \leq \hat{\mu}$, $\partial\mathcal{P}/\partial\mu = \left[\bar{\beta}\delta/(1-\bar{\beta}\delta)\right](\partial\varepsilon^*/\partial\mu) < 0$, and for $\hat{\mu} < \mu < \bar{\mu}$, $\partial\mathcal{P}/\partial\mu = \left[\bar{\beta}\delta/(1-\bar{\beta}\delta)\right]\alpha\theta G(\varepsilon^*)(\partial\varepsilon^*/\partial\mu) < 0$. (ii) For $\bar{\beta} < \mu \leq \hat{\mu}$, $\partial\mathcal{P}/\partial(\alpha\theta) = \left[\bar{\beta}\delta/(1-\bar{\beta}\delta)\right](\partial\varepsilon^*/\partial(\alpha\theta)) > 0$, and for $\hat{\mu} < \mu < \bar{\mu}$, $\partial\mathcal{P}/\partial\mu = \left[\bar{\beta}\delta/(1-\bar{\beta}\delta)\right]\{\alpha\theta G(\varepsilon^*)\left[\partial\varepsilon^*/\partial(\alpha\theta)\right] + \int_{\varepsilon_L}^{\varepsilon^*} G(\varepsilon)\,d\varepsilon\} > 0$. ■

Together, Proposition 5 and Proposition 8 imply that changes in the trading probability will generate a positive correlation between trade volume and the size of the speculative premium. The same is true of changes in the bargaining power.[53]

Proposition 9 *Assume $G(\varepsilon;\sigma)$ is a differentiable function of the parameter σ that indexes a family of mean-preserving spreads, so that for any $\sigma < \sigma'$, $G(\cdot;\sigma')$ is a mean-preserving spread of $G(\cdot;\sigma)$. Then in the recursive monetary equilibrium, $\partial\phi^s/\partial\sigma > 0$ and $\partial\bar{\phi}^s/\partial\sigma > 0$.*

Proof of Proposition 9. From the definition of the mean-preserving spread, for any $\Delta > 0$,

$$\int_{\varepsilon_L}^{x}\left[G(\varepsilon;\sigma+\Delta) - G(\varepsilon;\sigma)\right]d\varepsilon \geq 0 \text{ for all } x \in (\varepsilon_L, \varepsilon_H),$$

with "=" if $x \in \{\varepsilon_L, \varepsilon_H\}$, and therefore

$$\lim_{\Delta\to 0}\int_{\varepsilon_L}^{x}\frac{\left[G(\varepsilon;\sigma+\Delta) - G(\varepsilon;\sigma)\right]}{\Delta}d\varepsilon = \int_{\varepsilon_L}^{x} G_\sigma(\varepsilon;\sigma)\,d\varepsilon \geq 0 \text{ for all } x \in (\varepsilon_L, \varepsilon_H),$$

[53] The positive correlation between trade volume and the size of the speculative premium is a feature of historical episodes that are usually regarded as "bubbles"—a point emphasized by Scheinkman and Xiong (2003a, 2003b) and Scheinkman (2013).

with "=" if $x \in \{\varepsilon_L, \varepsilon_H\}$, where $G_\sigma(\varepsilon;\sigma) \equiv \partial G_\sigma(\varepsilon;\sigma)/\partial\sigma$. With this notation, the equilibrium mapping (13) is

$$T(x;\sigma) = \frac{\frac{1-\bar{\beta}\delta}{1-\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}}\int_x^{\varepsilon_H}[1-G(\varepsilon;\sigma)]\,d\varepsilon}{x+\frac{\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}}{1-\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}}\left[\bar{\varepsilon}+\alpha\theta\int_{\varepsilon_L}^{x}G(\varepsilon;\sigma)\,d\varepsilon\right]} - \frac{\mu-\bar{\beta}}{\bar{\beta}\alpha\theta},$$

and the equilibrium ε^* satisfies $T(\varepsilon^*;\sigma)=0$. By implicitly differentiating this condition, we get

$$\frac{\partial\varepsilon^*}{\partial\sigma} = -\frac{\frac{\alpha\theta\bar{\beta}}{1-\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}}\left(\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}-\frac{1-\bar{\beta}\delta}{\mu-\bar{\beta}}\right)\int_{\varepsilon_L}^{\varepsilon^*}G_\sigma(\varepsilon;\sigma)\,d\varepsilon}{1+\frac{\alpha\theta\bar{\beta}}{1-\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}}\left[G(\varepsilon^*;\sigma)\,\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}+[1-G(\varepsilon^*;\sigma)]\frac{1-\bar{\beta}\delta}{\mu-\bar{\beta}}\right]}.$$

If $\mu \in (\bar{\beta},\hat{\mu})$, then $\partial\varepsilon^*/\partial\sigma > 0$ since $(1-\bar{\beta}\delta)/(\mu-\bar{\beta}) - \delta\mathbb{I}_{\{\hat{\mu}<\mu\}} = (1-\delta\bar{\beta})/(\mu-\bar{\beta}) > 0$. If $\mu \in (\hat{\mu},\bar{\mu})$, then $\partial\varepsilon^*/\partial\sigma > 0$ since

$$\delta\mu < \delta\bar{\mu} = 1-(1-\bar{\beta}\delta)\frac{\bar{\beta}\delta(1-\alpha\theta)\bar{\varepsilon}+\left[1-\bar{\beta}\delta(1-\alpha\theta)\right]\varepsilon_L}{\bar{\beta}\delta\bar{\varepsilon}+(1-\bar{\beta}\delta)\varepsilon_L} < 1$$

implies $-\left[\delta-(1-\bar{\beta}\delta)/(\mu-\bar{\beta})\right] = (1-\delta\mu)/(\mu-\bar{\beta}) > 0$. Given that $\partial\varepsilon^*/\partial\sigma > 0$ for all $\mu \in (\bar{\beta},\bar{\mu})$, (8) and (9) imply $\partial\phi^s/\partial\sigma > 0$ and $\partial\bar{\phi}^s/\partial\sigma > 0$, respectively. ■

The following proposition shows there is a certain equivalence between α and G as fundamental determinants of trading activity.

Proposition 10 *Consider Economy A with contact probability α and distribution of valuations G on $[\varepsilon_L,\varepsilon_H]$ and Economy B with contact probability $\tilde{\alpha}$ and distribution of valuations $\tilde{G}$ on $[\tilde{\varepsilon}_L,\tilde{\varepsilon}_H]$ (and all other primitives of Economy B are as in Economy A). Let ε^* and $\tilde{\varepsilon}^*$ denote the equilibrium marginal valuation for Economy A and Economy B, respectively. Then for any $\tilde{\alpha} > \alpha$, there exists a $\tilde{G}$ such that*

$$\tilde{\varepsilon}^* = \frac{\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}\left(1-\frac{\alpha}{\tilde{\alpha}}\right)}{1-\bar{\beta}\delta\left(1-\mathbb{I}_{\{\hat{\mu}<\mu\}}\right)}\bar{\varepsilon}+\left[1-\frac{\bar{\beta}\delta\mathbb{I}_{\{\hat{\mu}<\mu\}}\left(1-\frac{\alpha}{\tilde{\alpha}}\right)}{1-\bar{\beta}\delta\left(1-\mathbb{I}_{\{\hat{\mu}<\mu\}}\right)}\right]\varepsilon^*,$$

and moreover, trade volume in Economy B is the same as in Economy A.

Proof of Proposition 10. In Economy A the marginal investor valuation, ε^*, is characterized by (13), while in Economy B the marginal investor valuation is the $\tilde{\varepsilon}^*$ that solves

$$\frac{(1-\bar{\beta}\delta)\,\tilde{\alpha}\theta\int_{\tilde{\varepsilon}^*}^{\tilde{\varepsilon}_H}[1-\tilde{G}(\varepsilon)]d\varepsilon}{(1-\bar{\beta}\delta)\,\tilde{\varepsilon}^*+\bar{\beta}\delta\left[\int_{\tilde{\varepsilon}_L}^{\tilde{\varepsilon}_H}\varepsilon d\tilde{G}(\varepsilon)+\tilde{\alpha}\theta\int_{\tilde{\varepsilon}_L}^{\tilde{\varepsilon}^*}\tilde{G}(\varepsilon)\,d\varepsilon\right]\mathbb{I}_{\{\hat{\mu}<\mu\}}} - \frac{\mu-\bar{\beta}}{\bar{\beta}} = 0.$$

Define

$$\tilde{G}(\varepsilon) = \begin{cases} 0 & \text{for } \varepsilon \leq \tilde{\varepsilon}_L \\ \frac{\alpha}{\tilde{\alpha}} G(\varepsilon - c) + \left(1 - \frac{\alpha}{\tilde{\alpha}}\right) \mathbb{I}_{\{\varepsilon^* < \varepsilon - c\}} & \text{for } \tilde{\varepsilon}_L \leq \varepsilon \leq \tilde{\varepsilon}_H \\ 1 & \text{for } \tilde{\varepsilon}_H < \varepsilon \end{cases} \tag{78}$$

with $\tilde{\varepsilon}_L \equiv \varepsilon_L + c$, $\tilde{\varepsilon}_H \equiv \varepsilon_H + c$ and

$$c \equiv \frac{\bar{\beta}\delta \mathbb{I}_{\{\hat{\mu} < \mu\}}}{1 - \bar{\beta}\delta \left(1 - \mathbb{I}_{\{\hat{\mu} < \mu\}}\right)} \left(1 - \frac{\alpha}{\tilde{\alpha}}\right) (\bar{\varepsilon} - \varepsilon^*). \tag{79}$$

With (78) and (79), the equilibrium mapping for Economy B becomes

$$\frac{\left(1 - \bar{\beta}\delta\right) \tilde{\alpha}\theta \int_{\tilde{\varepsilon}^* - c}^{\varepsilon_H} \left[1 - \frac{\alpha}{\tilde{\alpha}} G(z) - \left(1 - \frac{\alpha}{\tilde{\alpha}}\right) \mathbb{I}_{\{\varepsilon^* < z\}}\right] dz}{\left(1 - \bar{\beta}\delta\right) (\tilde{\varepsilon}^* - c) + \bar{\beta}\delta \left[\bar{\varepsilon} + \tilde{\alpha}\theta \int_{\varepsilon_L}^{\tilde{\varepsilon}^* - c} \left\{\frac{\alpha}{\tilde{\alpha}} G(z) + \left(1 - \frac{\alpha}{\tilde{\alpha}}\right) \mathbb{I}_{\{\varepsilon^* < z\}}\right\} dz\right] \mathbb{I}_{\{\hat{\mu} < \mu\}}} - \frac{\mu - \bar{\beta}}{\bar{\beta}} = 0.$$

If we replace $\tilde{\varepsilon}^* = \varepsilon^* + c$ in this last expression, it reduces to (13), a condition that holds because ε^* is the equilibrium marginal valuation for Economy A. Hence, $\tilde{\varepsilon}^* = \varepsilon^* + c$ with c given by (79) is the equilibrium marginal valuation for Economy B. Notice that $\tilde{\alpha}\tilde{G}(\tilde{\varepsilon}^*) = \tilde{\alpha}\tilde{G}(\varepsilon^* + c) = \alpha G(\varepsilon^*)$, so (15) implies that trade volume in Economy B is the same as in Economy A. ■

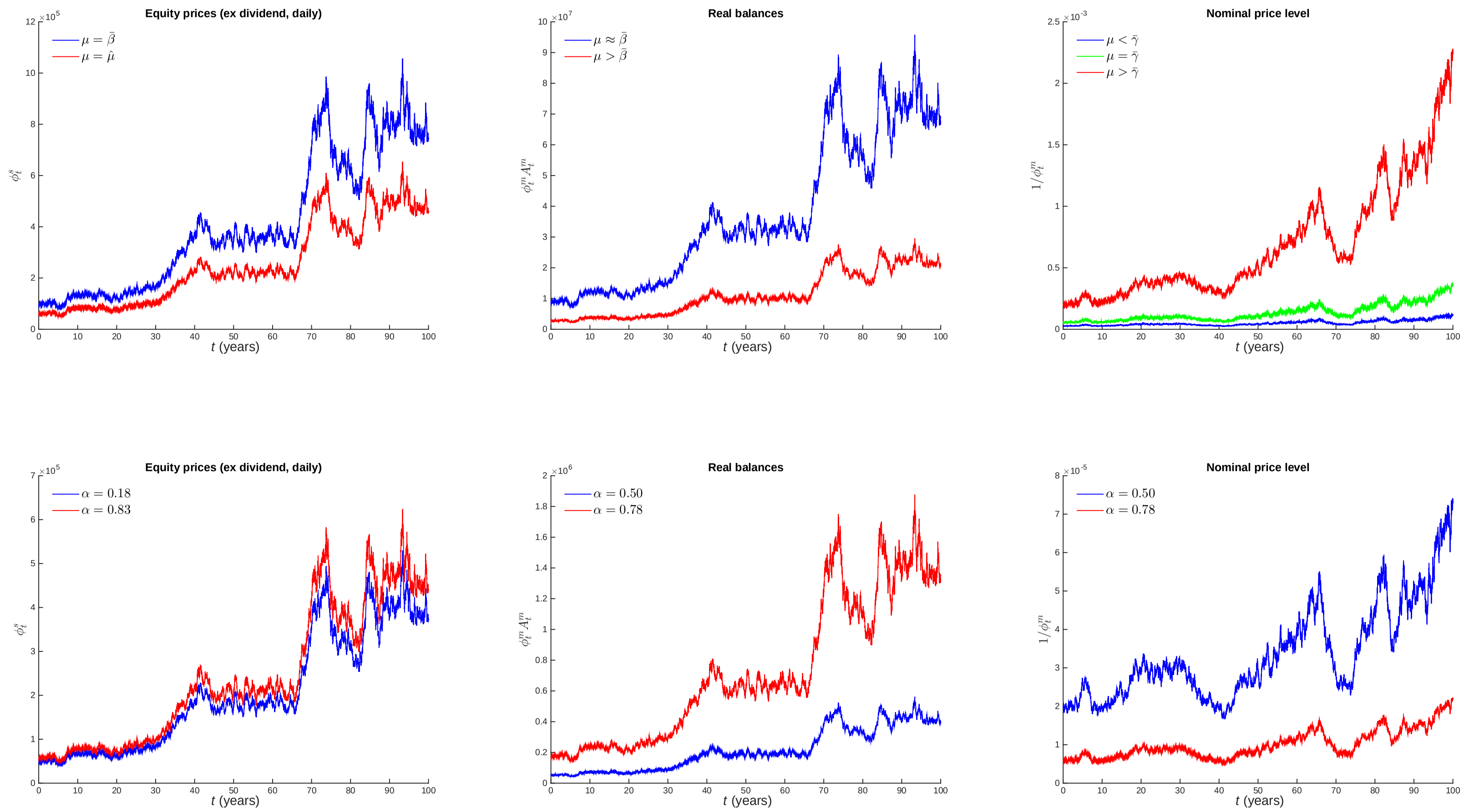

Figure 8: Effects of monetary policy and OTC frictions on asset prices.

B Supplementary material: Data, estimation, and simulation

B.1 Heteroskedasticity-based estimator

In this section we explain the H-based estimator used in Section 5.2. Rigobon and Sack (2004) show that the response of asset prices to changes in monetary policy can be identified based on the increase in the variance of policy shocks that occurs on days of FOMC announcements. They argue that this approach tends to be more reliable than the event-study approach based on daily data because identification relies on a weaker set of conditions.

The idea behind the heteroskedasticity-based estimator of Rigobon and Sack (2004) is as follows. Suppose the change in the policy rate, Δi_t, and Y_t (where Y_t could be the stock market return, $\mathcal{R}_t^I$, or the turnover rate, $\mathcal{T}_t^I$) are jointly determined by

$$\Delta i_t = \kappa Y_t + \varpi x_t + \epsilon_t \tag{80}$$

$$Y_t = \rho \Delta i_t + x_t + \eta_t, \tag{81}$$

where ϵ_t is a monetary policy shock and η_t is a shock to the asset price. To fix ideas, suppose $Y_t = \mathcal{R}_t^I$. Then equation (80) represents the monetary policy reaction to asset returns and possibly other variables represented by x_t. Equation (81) represents the reaction of asset prices to the policy rate and x_t. The disturbances ϵ_t and η_t are assumed to have no serial correlation and to be uncorrelated with each other and with x_t. We are interested in estimating the parameter ρ. Let Σ_v denote the variance of some variable v. If (80) and (81) were the true model and one were to run an OLS regression on an equation like (19), there would be a simultaneity bias if $\kappa \neq 0$ and $\Sigma_\eta > 0$, and an omitted variable bias if $\varpi \neq 0$ and $\Sigma_x > 0$. Conditions (80) and (81) can be solved for $\Delta i_t = \frac{1}{1-\rho\kappa}\left[\epsilon_t + \kappa \eta_t + (\kappa + \varpi) x_t\right]$ and $Y_t = \frac{1}{1-\rho\kappa}\left[\rho \epsilon_t + \eta_t + (1 + \rho\varpi) x_t\right]$. Divide the data sample into two subsamples: one consisting of FOMC policy announcement days and another consisting of the trading days immediately before the policy announcement days. In what follows we refer to these subsamples as S_1 and S_0, respectively. Let Ω^k denote the covariance matrix of Δi_t and $\mathcal{R}_t^I$ for $t \in S_k$, for $k \in \{0, 1\}$. Then

$$\Omega^k = \frac{1}{(1-\rho\kappa)^2} \begin{bmatrix} \Omega_{11}^k & \Omega_{12}^k \\ \Omega_{21}^k & \Omega_{22}^k \end{bmatrix},$$

where $\Omega_{11}^k \equiv \Sigma_\epsilon^k + \kappa^2 \Sigma_\eta^k + (\kappa + \varpi)^2 \Sigma_x^k$, $\Omega_{12}^k = \Omega_{21}^k \equiv \rho \Sigma_\epsilon^k + \kappa \Sigma_\eta^k + (\kappa + \varpi)(1 + \rho\varpi) \Sigma_x^k$, $\Omega_{22}^k \equiv \rho^2 \Sigma_\epsilon^k + \Sigma_\eta^k + (1 + \rho\varpi)^2 \Sigma_x^k$, and Σ_x^k denotes the variance of variable x in subsample S_k, for

$k \in \{0, 1\}$. Provided $\Sigma_x^1 = \Sigma_x^0$ and $\Sigma_\eta^1 = \Sigma_\eta^0$,

$$\Omega^1 - \Omega^0 = \frac{\Sigma_\epsilon^1 - \Sigma_\epsilon^0}{(1-\rho\kappa)^2} \begin{bmatrix} 1 & \rho \\ \rho & \rho^2 \end{bmatrix}.$$

Hence, if $\Sigma_\epsilon^1 - \Sigma_\epsilon^0 > 0$, then ρ can be identified from the difference in the covariance matrices of the two subsamples. This suggests a natural way to estimate ρ. Replace Ω^1 and Ω^0 with their sample estimates, denoted $\hat{\Omega}^1$ and $\hat{\Omega}^0$. Define $\hat{\Omega} \equiv \hat{\Omega}^1 - \hat{\Omega}^0$ and use $\hat{\Omega}_{ij}$ to denote the (i, j) element of $\hat{\Omega}$. Then ρ can be estimated by $\hat{\Omega}_{12}/\hat{\Omega}_{11} \equiv \hat{\rho}$. Rigobon and Sack (2004) show that this estimate can be obtained by regressing $\mathcal{R}_t^I$ on Δi_t over the combined sample $S_0 \cup S_1$ using a standard instrumental variables regression.

The standard deviation of Δi_t is 4.37 basis points (bps) in subsample S_0 and 6.29 bps in subsample S_1. The standard deviation of $\mathcal{R}_t^I$ is 86.10 bps in subsample S_0 and 92.96 bps in subsample S_1. The correlation between Δi_t and $\mathcal{R}_t^I$ is 0.057 in subsample S_0 and -0.37 in subsample S_1. Stock returns are more volatile on the days of monetary policy announcements than on other days, which is consistent with policy actions inducing some reaction in the stock market. The relatively large negative correlation between the policy rate and stock returns for announcement days contrasts with the much smaller and positive correlation for non-announcement days, suggesting that the negative effect of surprise increases in the nominal rate on stock prices that has been documented in the empirical literature (e.g., Bernanke and Kuttner, 2005, Rigobon and Sack, 2004).

B.2 High-frequency IV estimator

In this section we consider a version of the event-study estimator that, instead of *daily* changes in the interest rate, uses intraday high-frequency tick-by-tick interest rate data to isolate the change in the interest rate that takes place over a narrow window around each policy announcement. We refer to this as the *high-frequency instrumental variable estimator* (or "HFIV" estimator, for short).

Specifically, the HFIV estimator is obtained by estimating (19), where instead of directly using the daily change in the 3-month Eurodollar future rate, we instrument for it using the daily *imputed change* in the 30-day federal funds futures rate from the level it has 20 minutes after the FOMC announcement and the level it has 10 minutes before the FOMC announcement.[54] By

[54] By "daily imputed" we mean that in order to interpret the change in the federal funds futures rate as the surprise component of the change in the daily policy rate, it is adjusted for the fact that the federal funds futures

focusing on changes in a proxy for the policy rate in a very narrow 30-minute window around the time of the policy announcement, the resulting HFIV estimator addresses the omitted variable bias and the concern that the Eurodollar futures rate may itself respond to market conditions on policy announcement days.

The data for the high-frequency interest change are constructed as follows. For each announcement day $t \in S_1$, we define $z_t \equiv i_{t,m_t^*+20} - i_{t,m_t^*-10}$, where $i_{t,m}$ denotes the (daily imputed) 30-day federal funds futures rate on minute m of day t, and for any $t \in S_1$, m_t^* denotes the time of day (measured in minutes) when the FOMC announcement was made.[55] We then estimate b in (19) using the following two-stage least squares (2SLS) procedure. Define $\Delta i_t^{ed} \equiv i_t^{ed} - i_{t-1}^{ed}$, where i_t^{ed} denotes the rate implied (for day t) by the 3-month Eurodollar futures contract with closest expiration date at or after day t. First, run the regression $\Delta i_t^{ed} = \kappa_0 + \kappa z_t + \eta_t$ on sample S_1 (where η_t is an error term) to obtain the OLS estimates of κ_0 and κ, namely $\hat{\kappa}_0$ and $\hat{\kappa}$. Second, construct the fitted values $\hat{z}_t \equiv \hat{\kappa}_0 + \hat{\kappa} z_t$ and run the event-study regression (19) setting $\Delta i_t = \hat{z}_t$.

B.3 More on disaggregative announcement-day effects

In Section 5.3 and Section B.2, we sorted stocks into 20 portfolios according to the level of turnover of each individual stock and found that changes in the nominal rate affect stocks with different turnover liquidity differently, with more liquid stocks responding more than less liquid stocks. In this section, we complement that analysis by using an alternative procedure to sort stocks into portfolios. Specifically, in this section we sort stocks according to the sensitivity of their individual return to changes in an aggregate (marketwide) measure of turnover. This alternative criterion is useful for two reasons. First, it will allow us to control for some differences across stocks, such as the conventional risk factors used in empirical asset-pricing models. Second, this sorting criterion emphasizes the responsiveness of the individual stock return to changes in an aggregate measure of turnover, which is another manifestation of the transmission mechanism that operates in the theory. To construct the portfolios, we proceed as follows.

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contracts settle on the effective federal funds rate *averaged over the month covered by the contract.* See Section B.4.3 for details.

[55]We use the data set constructed by Gorodnichenko and Weber (2016) with tick-by-tick data of the federal funds futures trading on the CME Globex electronic trading platform (as opposed to the open-outcry market). The variable we denote as z_t is the same variable that Gorodnichenko and Weber denote as v_t. The data are available at http://faculty.chicagobooth.edu/michael.weber/research/data/replication_dataset_gw.xlsx.

For each individual stock s in our sample, we use daily time-series data to run

$$\mathcal{R}_t^s = \alpha^s + \beta_0^s \mathcal{T}_t^I + \sum_{j=1}^{K} \beta_j^s f_{j,t} + \varepsilon_t^s, \tag{82}$$

where ε_t^s is an error term, $\mathcal{R}_t^s$ is the daily stock return (between day t and day $t-1$), $\mathcal{T}_t^I$ is the aggregate (marketwide) turnover rate on day t, and $\{f_{j,t}\}_{j=1}^K$ are K pricing factors. We set $K = 3$, with $f_{1,t} = MKT_t$, $f_{2,t} = HML_t$, and $f_{3,t} = SMB_t$, where MKT_t is a broad measure of the market excess return, HML_t is the return of a portfolio of stocks with high book-to-market value minus the return of a portfolio of stocks with low book-to-market value, and SMB_t is the return of a portfolio of small-cap stocks minus the return of a portfolio of large-cap stocks. That is, MKT_t is the typical CAPM factor, while HML_t and SMB_t are the long-short spreads constructed by sorting stocks according to book-to-market value and market capitalization, respectively, as in the Fama and French (1993) three-factor model.[56] Let t_k denote the day of the k^{th} policy announcement (we use 133 policy announcement days from our sample period 1994-2007). For each stock s, regression (82) is run 133 times, once for each policy announcement day, each time using the sample of all trading days between day t_{k-1} and day t_k. Thus, for each stock s we obtain 532 estimates, $\{\{\beta_j^s(k)\}_{j=0}^3\}_{k=1}^{133}$, where $\beta_j^s(k)$ denotes the estimate for the beta corresponding to factor j for stock s, estimated on the sample consisting of all trading days between the policy announcement days t_{k-1} and t_k. For each policy announcement day, t_k, we sort all NYSE stocks into 20 portfolios by assigning stocks with $\beta_0^s(k)$ ranked between the $[5(i-1)]^{\text{th}}$ percentile and $(5i)^{\text{th}}$ percentile to the i^{th} portfolio, for $i = 1, ..., 20$. For each portfolio $i \in \{1, ..., 20\}$ constructed in this manner, we compute the daily return, $\mathcal{R}_t^i$, and the daily change in the turnover rate, $\mathcal{T}_t^i - \mathcal{T}_{t-1}^i$, and run the event-study regression (19) portfolio-by-portfolio, first with $Y_t^i = \mathcal{R}_t^i$ and then with $Y_t^i = \mathcal{T}_t^i - \mathcal{T}_{t-1}^i$, as in

[56] In order to construct the Fama-French factors HML_t and SMB_t, stocks are sorted into six portfolios obtained from the intersections of two portfolios formed on size (as measured by market capitalization and labeled "Small" and "Big") and three portfolios formed on the ratio of book value to market value (labeled "Value," "Neutral," and "Growth"). Then $SMB_t = (1/3)\left(\mathcal{R}_t^{SG} + \mathcal{R}_t^{SN} + \mathcal{R}_t^{SV}\right) - (1/3)\left(\mathcal{R}_t^{BG} + \mathcal{R}_t^{BN} + \mathcal{R}_t^{BV}\right)$ and $HML_t = (1/2)\left(\mathcal{R}_t^{SV} + \mathcal{R}_t^{BV}\right) - (1/2)\left(\mathcal{R}_t^{SG} + \mathcal{R}_t^{BG}\right)$, where $\mathcal{R}_t^{BG}$ denotes the return on portfolio "Big-Growth," "$\mathcal{R}_t^{SV}$" denotes the return on portfolio "Small-Value," and so on. For a detailed description of the breakpoints used to define the six portfolios, see Kenneth French's website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/six_portfolios. The CAPM factor, MKT_t, is a broad measure of the market excess return, specifically, the value-weighted return of all CRSP firms incorporated in the United States and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares and price data at the beginning of t, and good return data for t minus the one-month Treasury bill rate (from Ibbotson Associates). The data for the three Fama-French factors were obtained from Wharton Research Data Services (WRDS).

Section 5.3.

For each of the 20 portfolios, Table 4 reports estimates of the responses (on the day of the policy announcement) of the return of the portfolio to a 1 pp increase in the policy rate. Estimates are negative, as predicted by the theory. Also as predicted by the theory, the magnitude of the estimates tends to be larger for portfolios with higher indices. From these estimates we learn that stocks whose returns are more sensitive to aggregate measures of aggregate market turnover tend to experience larger declines in returns in response to unexpected increases in the nominal rate. This finding is in line with the turnover-liquidity channel of monetary policy.

Notice that by sorting portfolios on the β_0's estimated from (82), we are controlling for the three standard Fama-French factors. To explore how the portfolios sorted in this manner vary in terms of the three standard Fama-French factors, we construct the series of monthly return for each of the 20 portfolios for the period 1994-2007, $\{(\mathcal{R}_t^i)_{i=1}^{20}\}$, and run (82) to estimate the vector of betas, $\{\{\beta_j^i\}_{i=1}^{20}\}_{j=0}^{3}$. The estimated betas corresponding to each portfolio are displayed in Figure 9.[57] Notice that there is no correlation between the turnover-liquidity betas, $\{\beta_0^i\}_{i=1}^{20}$, and the CAPM betas, $\{\beta_1^i\}_{i=1}^{20}$. To get a sense of whether the different cross-portfolio responses of returns to policy shocks documented in Table 4 can be accounted for by the standard CAPM, consider the following back-of-the-envelope calculation. Let b denote the effect of a 1 bp increase in the policy rate on the marketwide stock return on the day of the policy announcement (e.g., the E-based estimate obtained from running (19) with $Y_t^I = \mathcal{R}_t^I$). Then according to the basic CAPM model, the effect on portfolio $i \in \{1, ..., 20\}$ would be $\tilde{b}^i \equiv \beta_1^i \times b$, where $\{\beta_1^i\}_{i=1}^{20}$ is the vector of betas estimated on monthly data for each of the 20 portfolios sorted on β_0^i (plotted in Figure 9). Figure 10 plots $\{(i, \tilde{b}^i)\}_{i=1}^{20}$ and $\{(i, b^i)\}_{i=1}^{20}$, where $\{b^i\}_{i=1}^{20}$ corresponds to the E-based estimates for the effect of monetary policy on returns reported in Table 4.

B.4 VAR estimation

B.4.1 Identification

We conjecture that the data, $\{Y_t\}$ with $Y_t \in \mathbb{R}^n$, correspond to an equilibrium that can be approximated by a structural vector autoregression (SVAR),

$$KY_t = \sum_{j=1}^{J} C_j Y_{t-j} + \varepsilon_t, \tag{83}$$

[57] The vector $\{\beta_0^i\}_{i=1}^{20}$ shown in the figure has been normalized by dividing it by $|\beta_0^1|$.

where K and C_j are $n \times n$ matrices, $J \geq 1$ is an integer that denotes the maximum number of lags, and $\varepsilon_t \in \mathbb{R}^n$ is a vector of structural shocks, with $\mathbb{E}(\varepsilon_t) = 0$, $\mathbb{E}(\varepsilon_t \varepsilon_t') = I$, and $\mathbb{E}(\varepsilon_t \varepsilon_s') = 0$ for $s \neq t$, where 0 is a conformable matrix of zeroes and I denotes the n-dimensional identity. If K is invertible, (83) can be represented by the reduced-form VAR

$$Y_t = \sum_{j=1}^{J} B_j Y_{t-j} + u_t, \tag{84}$$

where $B_j = K^{-1} C_j$ and

$$u_t = K^{-1} \varepsilon_t \tag{85}$$

is an error term with

$$\Xi \equiv \mathbb{E}\left(u_t u_t'\right) = K^{-1} K^{-1\prime}. \tag{86}$$

The reduced-form VAR (84) can be estimated to obtain the matrices $\{B_j\}_{j=1}^{J}$, and the residuals $\{u_t\}$ from the estimation can be used to calculate Ξ. From (85), we know that the disturbances of the reduced-form VAR (84) are linear combinations of the structural shocks, ε_t, so in order to use (84) and the estimates $\{B_j\}_{j=1}^{J}$ to compute the impulse responses to the structural shocks, it is necessary to find the n^2 elements of the matrix K^{-1}. However, given the known covariance matrix Ξ, (86) only provides $n(n+1)/2$ independent equations involving the elements of K^{-1}, so $n(n-1)/2$ additional independent conditions would be necessary to find all elements of K^{-1}. This is the well-known identification problem of the SVAR (83). Only three specific elements of K^{-1} are relevant for our analysis. To find them, we use an identification scheme that relies on external instruments.[58]

The VAR we estimate consists of three variables, i.e., $Y_t = \left(i_t, \mathcal{R}_t^I, \mathcal{T}_t^I\right)'$, where i_t, $\mathcal{R}_t^I$, and $\mathcal{T}_t^I$ are the measures of the policy rate, the stock return, and turnover described in Sections 5.1 and 5.2. Denote $\varepsilon_t = \left(\varepsilon_t^i, \varepsilon_t^{\mathcal{R}}, \varepsilon_t^{\mathcal{T}}\right)'$, $u_t = \left(u_t^i, u_t^{\mathcal{R}}, u_t^{\mathcal{T}}\right)'$, and

$$K^{-1} = \begin{bmatrix} k_i^i & k_i^{\mathcal{R}} & k_i^{\mathcal{T}} \\ k_{\mathcal{R}}^i & k_{\mathcal{R}}^{\mathcal{R}} & k_{\mathcal{R}}^{\mathcal{T}} \\ k_{\mathcal{T}}^i & k_{\mathcal{T}}^{\mathcal{R}} & k_{\mathcal{T}}^{\mathcal{T}} \end{bmatrix}.$$

Then $u_t = K^{-1} \varepsilon_t$ can be written as

$$\begin{bmatrix} u_t^i \\ u_t^{\mathcal{R}} \\ u_t^{\mathcal{T}} \end{bmatrix} = \begin{bmatrix} k_i^i \\ k_{\mathcal{R}}^i \\ k_{\mathcal{T}}^i \end{bmatrix} \varepsilon_t^i + \begin{bmatrix} k_i^{\mathcal{R}} \\ k_{\mathcal{R}}^{\mathcal{R}} \\ k_{\mathcal{T}}^{\mathcal{R}} \end{bmatrix} \varepsilon_t^{\mathcal{R}} + \begin{bmatrix} k_i^{\mathcal{T}} \\ k_{\mathcal{R}}^{\mathcal{T}} \\ k_{\mathcal{T}}^{\mathcal{T}} \end{bmatrix} \varepsilon_t^{\mathcal{T}}. \tag{87}$$

[58] The identification methodology has been used by Mertens and Ravn (2013), Stock and Watson (2012), Gertler and Karadi (2015), Hamilton (2003), and Kilian (2008a, 2008b), among others.

Since we are only interested in the impulse responses for the monetary shock, ε_t^i, it suffices to find the first column of K^{-1}. The identification problem we face, of course, stems from the fact that the structural shocks, $\left(\varepsilon_t^i, \varepsilon_t^{\mathcal{R}}, \varepsilon_t^{\mathcal{T}}\right)$, are unobservable and some of the elements of K^{-1} are unknown (three elements are unknown in this 3×3 case). Suppose we had data on $\left\{\varepsilon_t^i\right\}$. Then we could run the regression $u_t^i = \kappa_i^i \varepsilon_t^i + \eta_t$ to estimate κ_i^i, where η_t is an error term. From (87) we have $\eta_t = k_i^{\mathcal{R}} \varepsilon_t^{\mathcal{R}} + k_i^{\mathcal{T}} \varepsilon_t^{\mathcal{T}}$, so $\mathbb{E}\left(\varepsilon_t^i \eta_t\right) = \mathbb{E}\left[\varepsilon_t^i \left(k_i^{\mathcal{R}} \varepsilon_t^{\mathcal{R}} + k_i^{\mathcal{T}} \varepsilon_t^{\mathcal{T}}\right)\right] = 0$ (since we are assuming $\mathbb{E}(\varepsilon_t \varepsilon_t') = I$), and thus the estimate of κ_i^i could be used to identify k_i^i (up to a constant) via the population regression of u_t^i onto ε_t^i. Since ε_t^i is unobservable, one natural alternative is to find a proxy (instrumental) variable for it. Suppose there is a variable z_t such that

$$\mathbb{E}\left(z_t \varepsilon_t^{\mathcal{R}}\right) = \mathbb{E}\left(z_t \varepsilon_t^{\mathcal{T}}\right) = 0 < \mathbb{E}\left(z_t \varepsilon_t^i\right) \equiv v \text{ for all } t.$$

Then

$$\Lambda \equiv \mathbb{E}(z_t u_t) = K^{-1} \mathbb{E}(z_t \varepsilon_t) = \left(k_i^i, k_{\mathcal{R}}^i, k_{\mathcal{T}}^i\right)' v. \tag{88}$$

Since $\Lambda = (\Lambda_1, \Lambda_2, \Lambda_3)'$ is a known (3×1) vector, we can identify the coefficients of interest, $\left(k_i^i, k_{\mathcal{R}}^i, k_{\mathcal{T}}^i\right)$ up to the sign of the scalar v. To see this, notice (88) implies

$$v k_i^i = \Lambda_1 \tag{89}$$

$$v k_{\mathcal{R}}^i = \Lambda_2 \tag{90}$$

$$v k_{\mathcal{T}}^i = \Lambda_3 \tag{91}$$

with

$$v^2 = \mathbb{E}(z_t u_t)' \Xi^{-1} \mathbb{E}(z_t u_t). \tag{92}$$

Since the sign of v is unknown, we could look for restrictions that do not involve v, and in this case these conditions only provide two additional restrictions on $\left(k_i^i, k_{\mathcal{R}}^i, k_{\mathcal{T}}^i\right)$, i.e., combining (89) with (90), and (89) with (91), yields

$$\frac{k_{\mathcal{R}}^i}{k_i^i} = \frac{\Lambda_2}{\Lambda_1} \tag{93}$$

$$\frac{k_{\mathcal{T}}^i}{k_i^i} = \frac{\Lambda_3}{\Lambda_1}. \tag{94}$$

Thus, $k_{\mathcal{R}}^i$ and $k_{\mathcal{T}}^i$ are identified. From (89), k_i^i is also identified but up to the sign of v.

Notice that if we run a 2SLS regression of $u_t^{\mathcal{R}}$ on u_t^i using z_t as an instrument for u_t^i, then the estimate of the slope coefficient on this regression is Λ_2/Λ_1. Similarly, Λ_3/Λ_1 corresponds to the instrumental variable estimate of the slope coefficient of a regression of $u_t^{\mathcal{T}}$ on u_t^i using z_t as an instrument for u_t^i.

In our application, as an instrument for the structural monetary policy shock, ε_t^i, we use the (daily imputed) change in the 30-day federal funds futures from the level it has 10 minutes before the FOMC announcement and the level it has 20 minutes after the FOMC announcement.[59] That is, we restrict our sample to $t \in S_1$ and set $\{z_t\} = \left\{i_{t,m_t^*+20} - i_{t,m_t^*-10}\right\}$, where $i_{t,m}$ denotes the (daily imputed) 30-day federal funds futures rate on minute m of day t, and for any $t \in S_1$, m_t^* denotes the time of day (measured in minutes) when the FOMC announcement was made.[60] All this leads to the following procedure, used by Mertens and Ravn (2013), Stock and Watson (2012), and Gertler and Karadi (2015), to identify the coefficients needed to estimate the empirical impulse responses to a monetary policy shock:

Step 1: Estimate the reduced-form VAR by least squares over the whole sample of all trading days to obtain the coefficients $\{B_j\}_{j=1}^J$ and the residuals $\{u_t\}$.

Step 2: Run the regression $u_t^i = \kappa_0 + \kappa^i z_t + \eta_t$ on sample S_1 to obtain the OLS estimates of κ_0 and κ^i, namely $\hat{\kappa}_0$ and $\hat{\kappa}^i$, and construct the fitted values $\hat{u}_t^i = \hat{\kappa}_0 + \hat{\kappa}^i z_t$.

Step 3: Run the regressions $u_t^{\mathcal{R}} = \kappa_0 + \kappa^{\mathcal{R}} \hat{u}_t^i + \eta_t$ and $u_t^{\mathcal{T}} = \kappa_0 + \kappa^{\mathcal{T}} \hat{u}_t^i + \eta_t$ on sample S_1 to obtain the OLS estimates of $\kappa^{\mathcal{R}}$ and $\kappa^{\mathcal{T}}$, namely $\hat{\kappa}^{\mathcal{R}}$ and $\hat{\kappa}^{\mathcal{T}}$. Since $\hat{\kappa}^{\mathcal{R}} = \Lambda_2/\Lambda_1$ and $\hat{\kappa}^{\mathcal{T}} = \Lambda_3/\Lambda_1$, (93) and (94) imply $\hat{\kappa}^{\mathcal{R}} = k_{\mathcal{R}}^i/k_i^i$ and $\hat{\kappa}^{\mathcal{T}} = k_{\mathcal{T}}^i/k_i^i$.

For the purpose of getting impulse responses with respect to the shock ε_t^i, the scale and sign of k_i^i are irrelevant since the shock ε_t^i is typically normalized to have any desired impact on a given variable.[61] For example, in our impulse responses we normalize the shock ε_t^i so that it

[59] By "daily imputed" we mean that in order to interpret the change in the federal funds futures rate as the surprise component of the change in the daily policy rate, it is adjusted for the fact that the federal funds futures contracts settle on the effective federal funds rate *averaged over the month covered by the contract*. See Section B.4.3 for details.

[60] We use the data set constructed by Gorodnichenko and Weber (2016) with tick-by-tick data of the federal funds futures trading on the CME Globex electronic trading platform (as opposed to the open-outcry market). The variable we call z_t is the same variable that Gorodnichenko and Weber denote as v_t. Their data are available at http://faculty.chicagobooth.edu/michael.weber/research/data/replication_dataset_gw.xlsx. We have also performed the estimations using a different instrument for the high-frequency external identification scheme, namely the 3-month Eurodollar rate (on the nearest futures contract to expire after the FOMC announcement) from the level it has 10 minutes before the FOMC announcement and the level it has 20 minutes after the FOMC announcement. That is, we restrict our sample to $t \in S_1$ and set $\{z_t\} = \{i_{t,m_t^*+20}^{ed} - i_{t,m_t^*-10}^{ed}\}$, where $i_{t,m}^{ed}$ denotes the 3-month Eurodollar futures rate on minute m of day t, and for any $t \in S_1$, m_t^* denotes the time of day (measured in minutes) when the FOMC announcement was made. The results were essentially the same.

[61] Alternatively, (89) and (92) can be combined to get $k_i^i = \Lambda_1/v$, which is then identified up to the sign of v.

induces a 1 pp increase in the level of the policy rate i_t on impact. To see this, consider (87) with $\varepsilon_t^{\mathcal{R}} = \varepsilon_t^{\mathcal{T}} = 0$. Then for any k_i^i, the shock that induces an x pp increase in the level of the policy rate on impact (e.g., at $t = 0$) is $\varepsilon_0^i = (x/100)/k_i^i = (x/100)/(\Lambda_1/v)$.

B.4.2 Confidence intervals for impulse responses

The 95 percent confidence intervals for the impulse response coefficients estimated from the data are computed using a recursive wild bootstrap using 10,000 replications, as in Gonçalves and Kilian (2004) and Mertens and Ravn (2013). The procedure is as follows. Given the estimates of the reduced-form VAR, $\{\hat{B}_j\}_{j=1}^J$, and the residual, $\{\hat{u}_t\}$, we generate bootstrap draws, $\{Y_t^b\}$, recursively, by $Y_t^b = \sum_{j=1}^J \hat{B}_j Y_{t-j} + e_t^b \hat{u}_t$, where e_t^b is the realization of a scalar random variable taking values of -1 or 1, each with probability 1/2. Our identification procedure also requires us to generate bootstrap draws for the proxy variable, $\{z_t^b\}$, so following Mertens and Ravn (2013), we generate random draws for the proxy variable via $z_t^b = e_t^b z_t$. We then use the bootstrap samples $\{Y_t^b\}$ and $\{z_t^b\}$ to reestimate the VAR coefficients and compute the associated impulse responses (applying the covariance restrictions implied by the bootstrapped instrument z_t^b). This gives one bootstrap estimate of the impulse response coefficients. The confidence intervals are the percentile intervals of the distribution of 10,000 bootstrap estimates for the impulse response coefficients.

B.4.3 Changes in federal funds future rate and unexpected policy rate changes

Fix a month, s, and let the intervals $\{[t, t+1]\}_{t=1}^T$ denote the T days of the month. Let $\{f_{s,t}^0\}_{t=1}^T$ denote the market prices of the federal funds futures contract *at the end* of day t of month s. The superscript "0" indicates that the contract corresponds to the current month, s.[62] Let $\{r_t\}_{t=1}^T$ be the (average) daily fededral funds rate calculated *at the end* of day t. Finally, for $j = 1, ..., T - t$, let $E_t r_{t+j}$ denote the expectation of the spot federal funds rate on day $t + j$ conditional on the information available at the end of day t. Then, since federal funds futures contracts settle on the average daily rate of the month, we have

$$f_{s,t}^0 = \frac{1}{T}\left[\sum\nolimits_{i=1}^t r_i + \sum\nolimits_{i=t+1}^T E_t r_i\right], \text{ for } t = 1, ..., T.$$

[62] Contracts can range from 1 to 5 months. For example, $f_{s,t}^5$ would be the price of the 5-month forward on day t of month s.

Hence, for $t = 1, ..., T$,

$$f^0_{s,t} - f^0_{s,t-1} = \frac{1}{T} r_t - \frac{1}{T} E_{t-1} r_t + \frac{1}{T} \sum\nolimits_{i=t+1}^{T} E_t r_i - \frac{1}{T} \sum\nolimits_{i=t+1}^{T} E_{t-1} r_i,$$

where $f^0_{s,0} \equiv f^1_{s-1,T}$. Assume the federal funds rate changes at most once during the month, and suppose it is known that the announcement takes place at the beginning of day $t \geq 1$.[63] Then

$$E_t r_i = r_t \text{ for } i = t, ..., T$$
$$E_{t-1} r_i = E_{t-1} r_t \text{ for } i = t+1, ...T.$$

Thus, the change in the forward rate at the time of the announcement, i.e., $t = 1, ..., T$, is

$$f^0_{s,t} - f^0_{s,t-1} = \frac{T+1-t}{T} \left(r_t - E_{t-1} r_t \right), \tag{95}$$

where $r_t - E_{t-1} r_t$ is the surprise change in the federal funds rate on day t (the day of the policy announcement). If we know the daily change in the forward rate at the time of the announcement, $f^0_{s,t} - f^0_{s,t-1}$, then from (95) we can recover the unexpected change in the federal funds rate on the day of the FOMC announcement, t, as follows:

$$r_{t+1} - E_t r_{t+1} = \frac{T}{T-t} \left(f^0_{s,t+1} - f^0_{s,t} \right) \text{ for } t = 0, ..., T-1. \tag{96}$$

This condition is the same as condition (7) in Kuttner (2001), which is the convention used by the event-study literature to map the change in the 30-day federal funds futures rate on the day of the FOMC policy announcement into the surprise change in the daily policy rate on the day of the announcement. In terms of the notation for our high-frequency instrument introduced in Section B.4.1, we set $z_t = \frac{T}{T-t} \left(f^0_{s,t+1} - f^0_{s,t} \right) \equiv i_{t,m^*_t+20} - i_{t,m^*_t-10}$, where $f^0_{s,t+1} - f^0_{s,t}$ is measured (using high-frequency data) as the change in the 30-day federal funds futures rate over a 30-minute window around the FOMC announcement that takes place on day t.

[63] If r_t were the actual target federal funds rate, then the assumption that it changes at most once in the month would be exactly true for most of our sample; see, e.g., footnote 16 in Gorodnichenko and Weber (2016). In general this has to be regarded as an approximation, since on any given day the effective federal funds rate, r_t, can and does deviate somewhat from the announced federal funds rate target rate (see Afonso and Lagos, 2014).

		E-based		H-based		HFIV	
Portfolio	Turnover beta	Estimate	Std. dev.	Estimate	Std. dev.	Estimate	Std. dev.
1	-22.58	-1.90	2.36	-4.54	5.11	-5.01	3.60
2	-13.04	-4.54^{**}	1.97	-7.25^{*}	4.56	-6.49^{**}	3.39
3	-9.83	-2.64^{**}	1.46	-6.25^{*}	3.98	-3.78^{*}	2.62
4	-7.68	-5.07^{***}	1.59	-9.31^{**}	4.69	-6.43^{***}	2.62
5	-5.99	-4.79^{***}	1.67	-8.88^{**}	4.61	-6.68^{***}	2.83
6	-4.62	-4.39^{***}	1.21	-8.29^{**}	3.90	-6.68^{***}	2.52
7	-3.42	-3.92^{***}	1.51	-7.23^{**}	3.94	-6.33^{**}	2.82
8	-2.33	-3.80^{***}	1.24	-8.43^{**}	4.30	-5.71^{***}	2.32
9	-1.31	-4.64^{***}	1.21	-9.59^{***}	4.12	-6.70^{***}	2.34
10	-0.36	-4.16^{***}	1.13	-7.73^{**}	3.49	-6.87^{***}	2.41
11	0.52	-4.26^{***}	1.46	-7.72^{**}	3.67	-7.79^{***}	2.36
12	1.44	-5.15^{***}	1.12	-10.27^{***}	4.08	-7.86^{***}	2.39
13	2.44	-5.60^{***}	1.31	-10.79^{***}	4.04	-9.32^{***}	2.53
14	3.56	-6.52^{***}	1.41	-12.09^{***}	4.64	-10.52^{***}	2.85
15	4.79	-7.09^{***}	1.63	-11.90^{***}	4.22	-11.89^{***}	3.09
16	6.20	-7.74^{***}	1.81	-13.23^{***}	4.67	-11.78^{***}	3.10
17	7.88	-6.24^{***}	2.20	-10.57^{**}	4.65	-11.88^{***}	3.73
18	10.03	-6.75^{***}	2.09	-11.75^{***}	4.44	-13.87^{***}	3.48
19	13.25	-10.33^{***}	2.68	-17.40^{***}	6.03	-18.80^{***}	5.11
20	25.04	-11.51^{***}	3.35	-18.75^{***}	6.21	-23.19^{***}	5.39

Table 4: Empirical responses of NYSE stock returns to monetary policy across portfolios sorted on return sensitivity to aggregate turnover. *** denotes significance at the 1 percent level, ** significance at the 5 percent level, * significance at the 10 percent level.

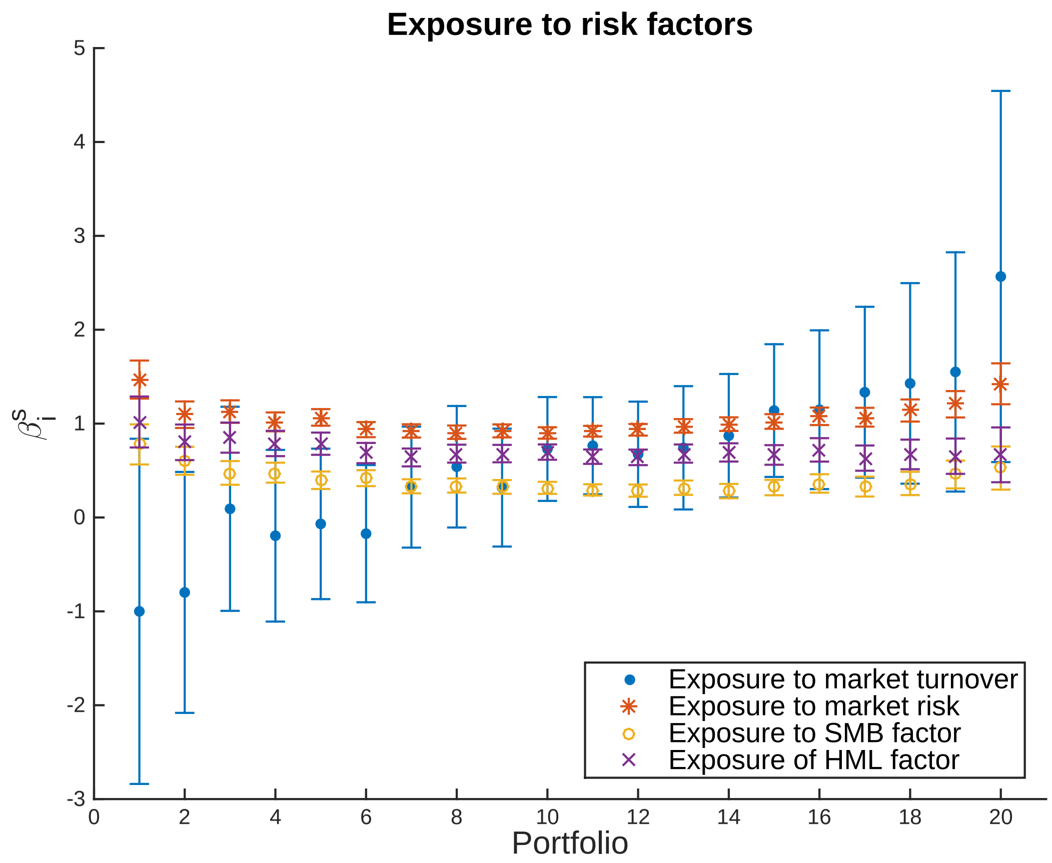

Figure 9: Betas for the 20 portfolios sorted on sensitivity of return to aggregate turnover.

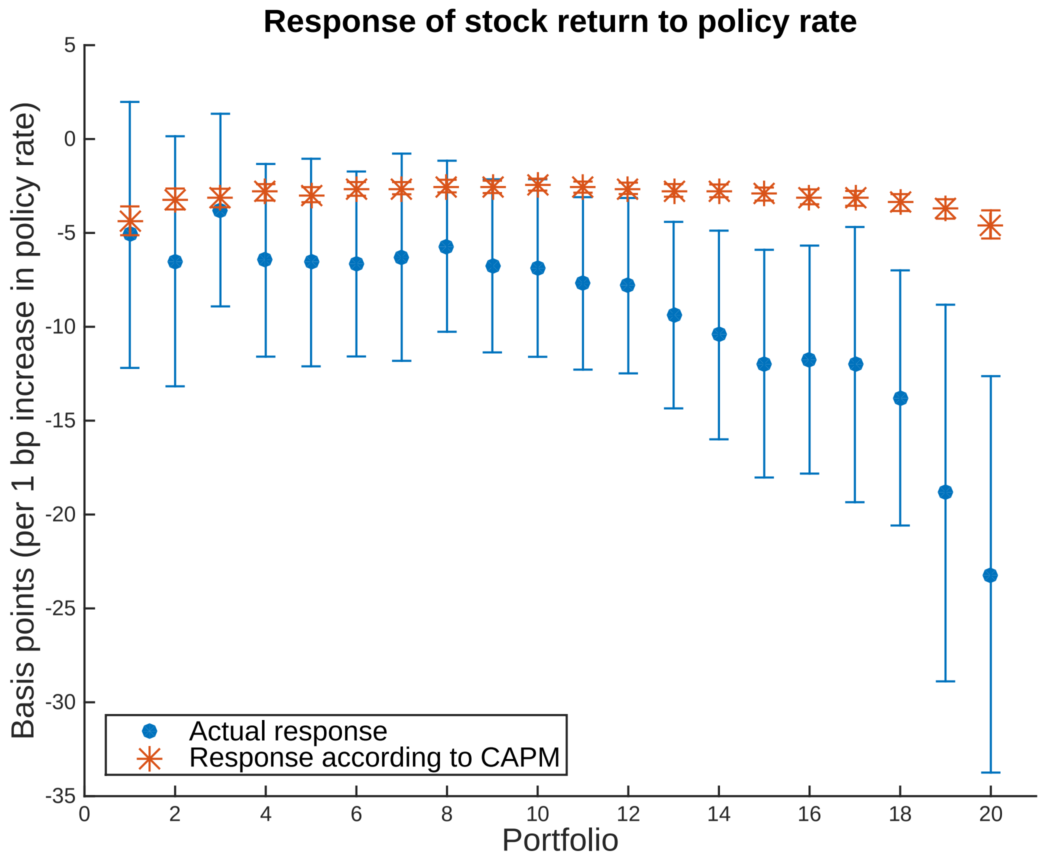

Figure 10: E-based estimates of responses of announcement-day stock returns to a 1 basis point surprise increase in the policy rate: CAPM vs. response based on portfolio analysis (for portfolios sorted on sensitivity of return to aggregate turnover).

C Supplementary material: Theory

C.1 Efficiency

Consider a social planner who wishes to maximize the sum of all agents' expected discounted utilities subject to the same meeting frictions that agents face in the decentralized formulation. Specifically, in the first subperiod of every period, the planner can only reallocate assets among all dealers and the measure α of investors who contact dealers at random. We restrict attention to symmetric allocations (identical agents receive equal treatment). Let c_{Dt} and h_{Dt} denote a dealer's consumption and production of the homogeneous consumption good in the second subperiod of period t. Let c_{It} and h_{It} denote an investor's consumption and production of the homogeneous consumption good in the second subperiod of period t. Let $\tilde{a}_{Dt}$ denote the beginning-of-period t (before depreciation) equity holding of a dealer, and let a'_{Dt} denote the equity holding of a dealer at the end of the first subperiod of period t (after OTC trade). Let $\tilde{a}_{It}$ denote the beginning-of-period t (before depreciation and endowment) asset holding of an investor. Finally, let a'_{It} denote a measure on $\mathcal{F}([\varepsilon_L, \varepsilon_H])$, the Borel σ-field defined on $[\varepsilon_L, \varepsilon_H]$. The measure a'_{It} is interpreted as the distribution of post-OTC-trade asset holdings among investors with different valuations who contacted a dealer in the first subperiod of period t. With this notation, the planner's problem consists of choosing a nonnegative allocation,

$$\left\{ \left[\tilde{a}_{jt}, a'_{jt}, c_{jt}, h_{jt} \right]_{j \in \{D,I\}} \right\}_{t=0}^{\infty},$$

to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha \int_{\varepsilon_L}^{\varepsilon_H} \varepsilon y_t a'_{It}(d\varepsilon) + (1-\alpha) \int_{\varepsilon_L}^{\varepsilon_H} \varepsilon y_t a_{It} dG(\varepsilon) + c_{Dt} + c_{It} - h_{Dt} - h_{It} \right]$$

(the expectation operator $\mathbb{E}_0$ is with respect to the probability measure induced by the dividend process) subject to

$$\tilde{a}_{Dt} + \tilde{a}_{It} \le A^s \tag{97}$$

$$a'_{Dt} + \alpha \int_{\varepsilon_L}^{\varepsilon_H} a'_{It}(d\varepsilon) \le a_{Dt} + \alpha a_{It} \tag{98}$$

$$c_{Dt} + c_{It} \le h_{Dt} + h_{It} \tag{99}$$

$$a_{Dt} = \delta \tilde{a}_{Dt} \tag{100}$$

$$a_{It} = \delta \tilde{a}_{It} + (1-\delta) A^s. \tag{101}$$

According to Proposition 11, the efficient allocation is characterized by the following two properties: (a) only dealers carry equity between periods, and (b) among those investors who have a trading opportunity with a dealer, only those with the highest valuation hold equity shares at the end of the first subperiod.

Proposition 11 *The efficient allocation satisfies the following two conditions for every t: (a) $\tilde{a}_{Dt} = A^s - \tilde{a}_{It} = A^s$ and (b) $a'_{It}(E) = \mathbb{I}_{\{\varepsilon_H \in E\}} [\delta + \alpha(1-\delta)] A^s/\alpha$, where $\mathbb{I}_{\{\varepsilon_H \in E\}}$ is an indicator function that takes the value 1 if $\varepsilon_H \in E$, and 0 otherwise, for any $E \in \mathcal{F}([\varepsilon_L, \varepsilon_H])$.*

Proof of Proposition 11. The choice variable a'_{Dt} does not appear in the planner's objective function, so $a'_{Dt} = 0$ at an optimum. Also, (99) must bind for every t at an optimum, so the planner's problem is equivalent to

$$\max_{\{\tilde{a}_{Dt}, \tilde{a}_{It}, a'_{It}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha \int_{\varepsilon_L}^{\varepsilon_H} \varepsilon a'_{It}(d\varepsilon) + (1-\alpha)\bar{\varepsilon} a_{It}\right] y_t$$

$$\text{s.t. (97), (100), (101), and } \alpha \int_{\varepsilon_L}^{\varepsilon_H} a'_{It}(d\varepsilon) \le a_{Dt} + \alpha a_{It}.$$

Let W^* denote the maximum value of this problem. Then clearly, $W^* \le \bar{W}^*$, where

$$\bar{W}^* = \max_{\{\tilde{a}_{Dt}, \tilde{a}_{It}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_H (\tilde{a}_{Dt} + \alpha \tilde{a}_{It}) + (1-\alpha)\bar{\varepsilon}\tilde{a}_{It}\right] \delta y_t + w$$

s.t. (97), where $w \equiv [\alpha\varepsilon_H + (1-\alpha)\bar{\varepsilon}](1-\delta) A^s \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t y_t$. Rearrange the expression for $\bar{W}^*$ and substitute (97) (at equality) to obtain

$$\begin{aligned}\bar{W}^* &= \max_{\{\tilde{a}_{It}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{\varepsilon_H A^s - (1-\alpha)(\varepsilon_H - \bar{\varepsilon})\tilde{a}_{It}\right\} \delta y_t + w \\ &= \left\{\delta\varepsilon_H + (1-\delta)[\alpha\varepsilon_H + (1-\alpha)\bar{\varepsilon}]\right\} A^s \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t y_t.\end{aligned}$$

The allocation consisting of $\tilde{a}_{Dt} = A^s$, $\tilde{a}_{It} = 0$, and the Dirac measure defined in the statement of the proposition achieve $\bar{W}^*$ and therefore solve the planner's problem. ■

C.2 Examples

In this section we present two examples for which the basic model of Section 2 can be solved in closed form.

Example 1 *Suppose that the probability distribution over investor valuations is concentrated on two points: ε_L with probability π_L and ε_H with probability π_H, with $\bar{\varepsilon} = \pi_H \varepsilon_H + \pi_L \varepsilon_L$. Then (13) implies*

$$\varepsilon^* = \begin{cases} \frac{\varepsilon_H}{1+\frac{\mu-\bar{\beta}}{\alpha\theta\bar{\beta}(1-\bar{\beta}\delta)\pi_H}} & \text{if } \bar{\beta} < \mu \leq \hat{\mu} \\ \frac{\bar{\beta}\alpha\theta\left(1-\bar{\beta}\delta\right)\pi_H\varepsilon_H-\left(\mu-\bar{\beta}\right)\bar{\beta}\delta(\bar{\varepsilon}-\alpha\theta\pi_L\varepsilon_L)}{\bar{\beta}\alpha\theta\left(1-\bar{\beta}\delta\right)\pi_H+\left(\mu-\bar{\beta}\right)\left[1-\bar{\beta}\delta(1-\alpha\theta\pi_L)\right]} & \text{if } \hat{\mu} < \mu < \bar{\mu} \end{cases}$$

with

$$\hat{\mu} = \bar{\beta}\left[1 + \frac{\left(1-\bar{\beta}\delta\right)(1-\alpha\theta)\,\alpha\theta\pi_L\,(\bar{\varepsilon}-\varepsilon_L)}{\bar{\varepsilon}-\alpha\theta\pi_L\varepsilon_L}\right] \quad \text{and} \quad \bar{\mu} = \bar{\beta}\left[1 + \frac{\left(1-\bar{\beta}\delta\right)\alpha\theta\,(\bar{\varepsilon}-\varepsilon_L)}{\bar{\beta}\delta\bar{\varepsilon}+\left(1-\bar{\beta}\delta\right)\varepsilon_L}\right].$$

Given ε^*, the closed-form expressions for the equilibrium allocation are given in Proposition 1.

Example 2 *Suppose that the probability distribution over investor valuations is distributed uniformly on $[0,1]$. Then (13) implies*

$$\varepsilon^* = \begin{cases} \frac{\alpha\theta\left(1-\bar{\beta}\delta\right)+\iota-\sqrt{\left[\alpha\theta\left(1-\bar{\beta}\delta\right)+\iota\right]^2-\left[\alpha\theta\left(1-\bar{\beta}\delta\right)\right]^2}}{\alpha\theta\left(1-\bar{\beta}\delta\right)} & \text{if } \bar{\beta} < \mu \leq \hat{\mu} \\ \frac{\left(1-\bar{\beta}\delta\right)(\alpha\theta+\iota)-\sqrt{\left[\left(1-\bar{\beta}\delta\right)(\alpha\theta+\iota)\right]^2-\alpha\theta\bar{\beta}\delta\left[1-\bar{\beta}\delta(1+\iota)\right](\bar{\iota}-\iota)}}{\alpha\theta\left[1-\bar{\beta}\delta(1+\iota)\right]} & \text{if } \hat{\mu} < \mu < \bar{\mu} \end{cases}$$

with

$$\hat{\mu} = \bar{\beta}\left[1 + \frac{\left(1-\bar{\beta}\delta\right)(1-\alpha\theta)\,(\hat{\varepsilon}-1/2)}{\hat{\varepsilon}}\right] \quad \text{and} \quad \bar{\mu} = \bar{\beta}\left[1 + \frac{\alpha\theta\left(1-\bar{\beta}\delta\right)}{\bar{\beta}\delta}\right]$$

and where $\bar{\iota} \equiv \left(\bar{\mu}-\bar{\beta}\right)/\bar{\beta}$ and $\hat{\varepsilon} = (1-\sqrt{1-\alpha\theta})/(\alpha\theta)$. Given ε^*, the closed-form expressions for the equilibrium allocation are given in Proposition 1.

C.3 Equilibrium conditions for the general model

In this section we derive the equilibrium conditions for the general model of Section 6. We specialize the analysis to recursive equilibria in which prices are time-invariant functions of an aggregate state vector that follows a time-invariant law of motion. The state vector is $\boldsymbol{x}_t = (A_t^m, y_t, \boldsymbol{\tau}_t) \in \mathbb{R}_+^5$, with $\boldsymbol{\tau}_t \equiv (\omega_t, \mu_t, r_t)$. Asset prices in a recursive equilibrium will be $\phi_t^s = \phi^s(\boldsymbol{x}_t)$, $\bar{\phi}_t^s = \bar{\phi}^s(\boldsymbol{x}_t)$, $\phi_t^m = \phi^m(\boldsymbol{x}_t)$, $p_t^s = p^s(\boldsymbol{x}_t)$, $q_t = q(\boldsymbol{x}_t)$, and $\varepsilon_t^{s*} = \varepsilon^{s*}(\boldsymbol{x}_t)$. Let A_t^{mk} denote the amount of money that investors have available to trade asset $k \in \bar{\mathbb{N}} \equiv \mathbb{N} \cup \{b\}$ at the beginning of period t (i.e., the bond, if $k = b$, and equity, if $k \in \mathbb{N}$). The laws of motion for the state variables A_t^m, y_t, and $\boldsymbol{\tau}_t$ are exogenous (as described above) while $A_t^{mk} = \Psi^k(\boldsymbol{x}_t)$, where the decision rule Ψ^k, for $k \in \bar{\mathbb{N}}$, is determined in equilibrium.

The investor's value functions are

$$W^I(a_t^{mb}, a_t^b, (a_t^{ms}, a_t^s)_{s\in\mathbb{N}}, k_t; \boldsymbol{x}_t) = \sum_{s\in\mathbb{N}} [\phi^m(\boldsymbol{x}_t) a_t^{ms} + \phi^s(\boldsymbol{x}_t) a_t^s] \\ + \phi^m(\boldsymbol{x}_t)(a_t^{mb} + a_t^b) - k_t + \bar{W}^I(\boldsymbol{x}_t),$$

where a_t^b denotes the quantity of bonds that the investor brings into the second subperiod of period t, with

$$\bar{W}^I(\boldsymbol{x}_t) \equiv T(\boldsymbol{x}_t) + \max_{(\tilde{a}_{t+1}^m, (\tilde{a}_{t+1}^s)_{s\in\mathbb{N}}) \in \mathbb{R}_+^{N+1}} \left\{ -\phi^m(\boldsymbol{x}_t)\tilde{a}_{t+1}^m - \sum_{s\in\mathbb{N}} \phi^s(\boldsymbol{x}_t)\tilde{a}_{t+1}^s \right. \\ \left. + \frac{1}{1+r_t}\mathbb{E}\left[\bar{V}^I(\tilde{a}_{t+1}^m, (a_{t+1}^s)_{s\in\mathbb{N}}; \boldsymbol{x}_{t+1}) \middle| \boldsymbol{x}_t\right] \right\},$$

$$\bar{V}^I(\tilde{a}_{t+1}^m, (a_{t+1}^s)_{s\in\mathbb{N}}; \boldsymbol{x}_{t+1}) = \max_{(a_{t+1}^{mk})_{k\in\bar{\mathbb{N}}} \in \mathbb{R}_+^{N+1}} \int V^I(a_{t+1}^{mb}, (a_{t+1}^{ms}, a_{t+1}^s)_{s\in\mathbb{N}}, \varepsilon; \boldsymbol{x}_{t+1}) dG(\varepsilon) \tag{102}$$

$$\text{s.t. } \sum_{k\in\bar{\mathbb{N}}} a_{t+1}^{mk} \leq \tilde{a}_{t+1}^m,$$

and

$$V^I(a_{t+1}^{mb}, (a_{t+1}^{ms}, a_{t+1}^s)_{s\in\mathbb{N}}, \varepsilon; \boldsymbol{x}_{t+1}) = \phi^m(\boldsymbol{x}_{t+1}) \{a_{t+1}^{mb} + [1 - q(\boldsymbol{x}_{t+1})] a_{t+1}^b(a_{t+1}^{mb}, q(\boldsymbol{x}_{t+1}))\} \\ + \sum_{s\in\mathbb{N}} \{\phi^m(\boldsymbol{x}_{t+1}) a_{t+1}^{ms} + [\varepsilon y_{t+1} + \phi^s(\boldsymbol{x}_{t+1})] a_{t+1}^s\} \\ + \sum_{s\in\mathbb{N}} \left[\alpha^s \theta \frac{\varepsilon - \varepsilon^{s*}(\boldsymbol{x}_{t+1})}{p^s(\boldsymbol{x}_{t+1})} y_{t+1} \mathbb{I}_{\{\varepsilon^{s*}(\boldsymbol{x}_{t+1}) < \varepsilon\}} a_{t+1}^{ms}\right] \\ + \sum_{s\in\mathbb{N}} \{\alpha^s \theta [\varepsilon^{s*}(\boldsymbol{x}_{t+1}) - \varepsilon] y_{t+1} \mathbb{I}_{\{\varepsilon < \varepsilon^{s*}(\boldsymbol{x}_{t+1})\}} a_{t+1}^s\} \\ + \bar{W}^I(\boldsymbol{x}_{t+1}),$$

where $a_t^b(a_t^{mb}, q_t)$ is the bond demand of an agent who carries a_t^{mb} dollars into the bond market in state $\boldsymbol{x}_t$, and $a_{t+1}^s \equiv \delta\tilde{a}_{t+1}^s + (1-\delta)A^s$. In writing $V^I(\cdot)$ we have used the fact that Lemma 1 still characterizes the equilibrium post-trade portfolios in the OTC market. The following lemma characterizes an investor's demand in the bond market.

Lemma 5 *Consider an investor who brings a_t^{mb} dollars to the bond market of period t. The bond demand, $a_t^b(a_t^{mb}, q_t)$ and the post-trade bond-market cash holdings, $\bar{a}_t^{mb}(a_t^{mb}, q_t) = a_t^{mb} -$*

$q_t a_t^b(a_t^{mb}, q_t)$, are given by

$$a_t^b(a_t^{mb}, q_t) = \chi(q_t, 1)\frac{a_t^{mb}}{q_t}$$
$$\bar{a}_t^{mb}(a_t^{mb}, q_t) = [1 - \chi(q_t, 1)]\, a_t^{mb},$$

where $\chi(\cdot,\cdot)$ is the function defined in Lemma 1.

Proof. The investor's problem in the bond market of period t is

$$\max_{(\bar{a}_t^{mb}, a_t^b)\in\mathbb{R}_+^2} W^I(\bar{a}_t^{mb}, a_t^b, (a_t^{ms}, a_t^s)_{s\in\mathbb{N}}, k_t; \boldsymbol{x}_t) \text{ s.t. } \bar{a}_t^{mb} + q_t a_t^b \le a_t^{mb}.$$

This problem can be written as

$$\max_{a_t^b\in[0, a_t^{mb}/q_t]} \phi^m(\boldsymbol{x}_t)\left[(a_t^{mb} + (1-q_t)\, a_t^b\right] + W^I\left((a_t^{ms}, a_t^s)_{s\in\mathbb{N}}, 0, 0, k_t; \boldsymbol{x}_t\right),$$

and the solution is as in the statement of the lemma. ■

The market-clearing condition for bonds is $a_t^b\left(A_t^{mb}, q_t\right) = B_t$, which implies the equilibrium nominal price of a bond is $q_t = \min\left(A_t^{mb}/B_t, 1\right)$, or in the recursive equilibrium,

$$q(\boldsymbol{x}_t) = \min\left\{\frac{\Psi^b(\boldsymbol{x}_t)}{\omega_t A_t^m}, 1\right\}.$$

With Lemma 5, the investor's value function in the first subperiod becomes

$$\begin{aligned} V^I(a_{t+1}^{mb}, (a_{t+1}^{ms}, a_{t+1}^s)_{s\in\mathbb{N}}, \varepsilon; \boldsymbol{x}_{t+1}) = {} & \frac{\phi^m(\boldsymbol{x}_{t+1})}{q(\boldsymbol{x}_{t+1})} a_{t+1}^{mb} + \bar{W}^I(\boldsymbol{x}_{t+1}) \\ & + \sum_{s\in\mathbb{N}}\left\{\phi^m(\boldsymbol{x}_{t+1})\, a_{t+1}^{ms} + [\varepsilon y_{t+1} + \phi^s(\boldsymbol{x}_{t+1})]\, a_{t+1}^s\right\} \\ & + \sum_{s\in\mathbb{N}} \alpha^s\theta\frac{\varepsilon - \varepsilon^{s*}(\boldsymbol{x}_{t+1})}{p^s(\boldsymbol{x}_{t+1})} y_{t+1}\mathbb{I}_{\{\varepsilon^{s*}(\boldsymbol{x}_{t+1})<\varepsilon\}} a_{t+1}^{ms} \\ & + \sum_{s\in\mathbb{N}} \alpha^s\theta\left[\varepsilon^{s*}(\boldsymbol{x}_{t+1}) - \varepsilon\right] y_{t+1}\mathbb{I}_{\{\varepsilon<\varepsilon^{s*}(\boldsymbol{x}_{t+1})\}} a_{t+1}^s. \end{aligned}$$

The following lemma characterizes the optimal partition of money across asset classes chosen by an investor at the beginning of the period.

Lemma 6 *The $(a_{t+1}^{mk})_{k\in\bar{\mathbb{N}}}$ that solves (102) satisfies*

$$\frac{\partial\bar{V}^I(\tilde{a}_{t+1}^m, (a_{t+1}^s)_{s\in\mathbb{N}}; \boldsymbol{x}_{t+1})}{\partial\tilde{a}_{t+1}^m} \ge \phi^m(\boldsymbol{x}_{t+1}) + \alpha^s\theta\int_{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^{s*}(\boldsymbol{x}_{t+1})}{p^s(\boldsymbol{x}_{t+1})} y_{t+1}\, dG(\varepsilon) \tag{103}$$

$$\frac{\partial\bar{V}^I(\tilde{a}_{t+1}^m, (a_{t+1}^s)_{s\in\mathbb{N}}; \boldsymbol{x}_{t+1})}{\partial\tilde{a}_{t+1}^m} \ge \frac{\phi^m(\boldsymbol{x}_{t+1})}{q(\boldsymbol{x}_{t+1})}, \tag{104}$$

where (103) holds with "=" if $a_{t+1}^{ms} > 0$ and (104) holds with "=" if $a_{t+1}^{mb} > 0$.

Proof. The objective function on the right side of (102) can be written as

$$\begin{aligned}
&\int V^I(a^{mb}_{t+1},(a^{ms}_{t+1},a^s_{t+1})_{s\in\mathbb{N}},\varepsilon;\boldsymbol{x}_{t+1})dG(\varepsilon)\\
&=\sum_{s\in\mathbb{N}}\left\{\phi^m\left(\boldsymbol{x}_{t+1}\right)a^{ms}_{t+1}+\left[\bar{\varepsilon}y_{t+1}+\phi^s\left(\boldsymbol{x}_{t+1}\right)\right]a^s_{t+1}\right\}\\
&+\sum_{s\in\mathbb{N}}\alpha^s\theta\int_{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}{p^s\left(\boldsymbol{x}_{t+1}\right)}y_{t+1}dG(\varepsilon)a^{ms}_{t+1}\\
&+\sum_{s\in\mathbb{N}}\alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}\left[\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)-\varepsilon\right]y_{t+1}dG(\varepsilon)a^s_{t+1}\\
&+\frac{\phi^m\left(\boldsymbol{x}_{t+1}\right)}{q\left(\boldsymbol{x}_{t+1}\right)}a^{mb}_{t+1}+\bar{W}^I\left(\boldsymbol{x}_{t+1}\right).
\end{aligned}$$

The Lagrangian for the maximization in (102) is

$$\begin{aligned}
\hat{\mathcal{L}}((a^{ms}_{t+1})_{s\in\bar{\mathbb{N}}};\tilde{a}^m_{t+1},\boldsymbol{x}_{t+1})=&\sum_{s\in\mathbb{N}}\left[\phi^m\left(\boldsymbol{x}_{t+1}\right)+\alpha^s\theta\int_{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}{p^s\left(\boldsymbol{x}_{t+1}\right)}y_{t+1}dG(\varepsilon)\right]a^{ms}_{t+1}\\
&+\frac{\phi^m\left(\boldsymbol{x}_{t+1}\right)}{q\left(\boldsymbol{x}_{t+1}\right)}a^{mb}_{t+1}+\sum_{k\in\bar{\mathbb{N}}}\zeta^{mk}\left(\boldsymbol{x}_{t+1}\right)a^{mk}_{t+1}+\xi\left(\boldsymbol{x}_{t+1}\right)\left(\tilde{a}^m_{t+1}-\sum_{k\in\bar{\mathbb{N}}}a^{mk}_{t+1}\right),
\end{aligned}$$

where $\xi\left(\boldsymbol{x}_{t+1}\right)$ is the multiplier on the feasibility constraint in state $\boldsymbol{x}_{t+1}$ and $(\zeta^{mk}\left(\boldsymbol{x}_{t+1}\right))_{k\in\bar{\mathbb{N}}}$ are the multipliers on the nonnegativity constraints. The first-order conditions are

$$\frac{\phi^m\left(\boldsymbol{x}_{t+1}\right)}{q\left(\boldsymbol{x}_{t+1}\right)}+\zeta^{mb}\left(\boldsymbol{x}_{t+1}\right)-\xi\left(\boldsymbol{x}_{t+1}\right)=0$$

$$\phi^m\left(\boldsymbol{x}_{t+1}\right)+\alpha^s\theta\int_{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}{p^s\left(\boldsymbol{x}_{t+1}\right)}y_{t+1}dG(\varepsilon)+\zeta^{ms}\left(\boldsymbol{x}_{t+1}\right)-\xi\left(\boldsymbol{x}_{t+1}\right)=0,$$

for all $s\in\mathbb{N}$. Finally, notice that $\xi\left(\boldsymbol{x}_{t+1}\right)=\partial\hat{\mathcal{L}}/\partial\tilde{a}^m_{t+1}=\partial\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1})/\partial\tilde{a}^m_{t+1}$. ∎

The following lemma characterizes an investor's optimal portfolio choice in the second subperiod of any period with state $\boldsymbol{x}_t$.

Lemma 7 *The portfolio $(\tilde{a}^m_{t+1},(\tilde{a}^s_{t+1})_{s\in\mathbb{N}})$ chosen by an investor in the second subperiod of period t with state $\boldsymbol{x}_t$ of a recursive equilibrium, satisfies*

$$\begin{aligned}
\phi^s\left(\boldsymbol{x}_t\right)&\geq\frac{\delta}{1+r_t}\mathbb{E}\left[\bar{\varepsilon}y_{t+1}+\phi^s\left(\boldsymbol{x}_{t+1}\right)+\alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}\left[\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)-\varepsilon\right]y_{t+1}dG(\varepsilon)\,\middle|\,\boldsymbol{x}_t\right]\\
\phi^m\left(\boldsymbol{x}_t\right)&\geq\frac{1}{1+r_t}\mathbb{E}\left[\phi^m\left(\boldsymbol{x}_{t+1}\right)+\alpha^s\theta\int_{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}^{\varepsilon_H}\frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}{p^s\left(\boldsymbol{x}_{t+1}\right)}y_{t+1}dG(\varepsilon)\,\middle|\,\boldsymbol{x}_t\right]\\
\phi^m\left(\boldsymbol{x}_t\right)&\geq\frac{1}{1+r_t}\mathbb{E}\left[\frac{\phi^m\left(\boldsymbol{x}_{t+1}\right)}{q\left(\boldsymbol{x}_{t+1}\right)}\,\middle|\,\boldsymbol{x}_t\right],
\end{aligned}$$

where the first condition holds with "=" if $\tilde{a}^s_{t+1} > 0$, the second condition holds with "=" if $a^{ms}_{t+1} > 0$, and the third condition holds with "=" if $a^{mb}_{t+1} > 0$.

Proof. The investor's maximization problem in the second subperiod is

$$\max_{(\tilde{a}^m_{t+1},(\tilde{a}^s_{t+1})_{s\in\mathbb{N}})\in\mathbb{R}^{N+1}_+} \left\{ -\phi^m(\boldsymbol{x}_t)\,\tilde{a}^m_{t+1} - \sum_{s\in\mathbb{N}} \phi^s(\boldsymbol{x}_t)\,\tilde{a}^s_{t+1} + \frac{1}{1+r_t}\mathbb{E}\left[\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1})\middle|\,\boldsymbol{x}_t\right]\right\},$$

with

$$\begin{aligned}
&\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1}) \\
&= \bar{W}^I(\boldsymbol{x}_{t+1}) + \max_{\{a^{mk}_{t+1}\}_{k\in\bar{\mathbb{N}}}\in\mathbb{R}^{N+1}_+} \hat{\mathcal{L}}((a^{ms}_{t+1})_{s\in\bar{\mathbb{N}}};\tilde{a}^m_{t+1},\boldsymbol{x}_{t+1}) \\
&+ \sum_{s\in\mathbb{N}}\left[\bar{\varepsilon}y_{t+1} + \phi^s(\boldsymbol{x}_{t+1}) + \alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}\left[\varepsilon^{s*}(\boldsymbol{x}_{t+1})-\varepsilon\right]y_{t+1}dG(\varepsilon)\right]a^s_{t+1},
\end{aligned}$$

where $\hat{\mathcal{L}}\left((a^{ms}_{t+1})_{s\in\mathbb{N}};\tilde{a}^m_{t+1},\boldsymbol{x}_{t+1}\right)$ is defined in the proof of Lemma 6. We then have,

$$\begin{aligned}
\tfrac{\partial\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1})}{\partial a^s_{t+1}} &= \bar{\varepsilon}y_{t+1} + \phi^s(\boldsymbol{x}_{t+1}) + \alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}\left[\varepsilon^{s*}(\boldsymbol{x}_{t+1})-\varepsilon\right]y_{t+1}dG(\varepsilon) \\
\tfrac{\partial\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1})}{\partial\tilde{a}^m_{t+1}} &= \xi(\boldsymbol{x}_{t+1}).
\end{aligned}$$

The first-order conditions for the investor's optimization problem in the second subperiod are

$$\begin{aligned}
-\phi^m(\boldsymbol{x}_t) + \frac{1}{1+r_t}\mathbb{E}\left[\frac{\partial\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1})}{\partial\tilde{a}^m_{t+1}}\middle|\,\boldsymbol{x}_t\right] &\leq 0, \text{ with `` = '' if } \tilde{a}^m_{t+1} > 0 \\
-\phi^s(\boldsymbol{x}_t) + \frac{1}{1+r_t}\mathbb{E}\left[\frac{\partial\bar{V}^I(\tilde{a}^m_{t+1},(a^s_{t+1})_{s\in\mathbb{N}};\boldsymbol{x}_{t+1})}{\partial\tilde{a}^s_{t+1}}\middle|\,\boldsymbol{x}_t\right] &\leq 0, \text{ with `` = '' if } \tilde{a}^s_{t+1} > 0,
\end{aligned}$$

or equivalently,

$$\begin{aligned}
\phi^m(\boldsymbol{x}_t) &\geq \frac{1}{1+r_t}\mathbb{E}\left[\xi(\boldsymbol{x}_{t+1})|\,\boldsymbol{x}_t\right], \text{ with `` = '' if } \tilde{a}^m_{t+1} > 0 \\
\phi^s(\boldsymbol{x}_t) &\geq \frac{\delta}{1+r_t}\mathbb{E}\left[\bar{\varepsilon}y_{t+1} + \phi^s(\boldsymbol{x}_{t+1}) + \alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}(\boldsymbol{x}_{t+1})}\left[\varepsilon^{s*}(\boldsymbol{x}_{t+1})-\varepsilon\right]y_{t+1}dG(\varepsilon)\middle|\,\boldsymbol{x}_t\right],
\end{aligned}$$

with "=" if $\tilde{a}^s_{t+1} > 0$, for $s\in\mathbb{N}$. By Lemma 6, the first condition can be written as

$$\phi^m(\boldsymbol{x}_t) \geq \frac{1}{1+r_t}\mathbb{E}\left[\frac{\phi^m(\boldsymbol{x}_{t+1})}{q(\boldsymbol{x}_{t+1})}\middle|\,\boldsymbol{x}_t\right],$$

with "=" if $a^{mb}_{t+1} > 0$, or as

$$\phi^{m}\left(\boldsymbol{x}_{t}\right) \geq \frac{1}{1+r_{t}} \mathbb{E}\left[\phi^{m}\left(\boldsymbol{x}_{t+1}\right)+\alpha^{s} \theta \int_{\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}^{\varepsilon_{H}} \frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}{p^{s}\left(\boldsymbol{x}_{t+1}\right)} y_{t+1} dG(\varepsilon) \right| \boldsymbol{x}_{t}\Bigg],$$

with "=" if $a^{ms}_{t+1} > 0$, for $s \in \mathbb{N}$. ■

Definition 2 *A recursive monetary equilibrium for the multiple asset economy with open-market operations (in which only investors can hold assets overnight) is a collection of functions,* $\{\phi^{m}(\cdot), q(\cdot), \Psi^{b}(\cdot), \{\phi^{s}(\cdot), p^{s}(\cdot), \varepsilon^{s*}(\cdot), \Psi^{s}(\cdot)\}_{s\in\mathbb{N}}\}$, *that satisfy*

$$\begin{aligned}
\phi^{s}\left(\boldsymbol{x}_{t}\right) &= \frac{\delta}{1+r_{t}} \mathbb{E}\left[\bar{\varepsilon} y_{t+1}+\phi^{s}\left(\boldsymbol{x}_{t+1}\right)+\alpha^{s} \theta \int_{\varepsilon_{L}}^{\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}\left[\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)-\varepsilon\right] y_{t+1} dG(\varepsilon) \right| \boldsymbol{x}_{t}\Bigg] \\
\phi^{m}\left(\boldsymbol{x}_{t}\right) &= \frac{1}{1+r_{t}} \mathbb{E}\left[\phi^{m}\left(\boldsymbol{x}_{t+1}\right)+\alpha^{s} \theta \int_{\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}^{\varepsilon_{H}} \frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t+1}\right)}{p^{s}\left(\boldsymbol{x}_{t+1}\right)} y_{t+1} dG(\varepsilon) \right| \boldsymbol{x}_{t}\Bigg] \\
\frac{\phi^{m}\left(\boldsymbol{x}_{t}\right)}{q\left(\boldsymbol{x}_{t}\right)} &= \phi^{m}\left(\boldsymbol{x}_{t}\right)+\alpha^{s} \theta \int_{\varepsilon^{s*}\left(\boldsymbol{x}_{t}\right)}^{\varepsilon_{H}} \frac{\varepsilon-\varepsilon^{s*}\left(\boldsymbol{x}_{t}\right)}{p^{s}\left(\boldsymbol{x}_{t}\right)} y_{t} dG(\varepsilon) \\
q\left(\boldsymbol{x}_{t}\right) &= \min[A^{mb}_{t}/(\omega_{t} A^{m}_{t}), 1] \\
p^{s}\left(\boldsymbol{x}_{t}\right) &= \frac{\left[1-G\left(\varepsilon^{s*}\left(\boldsymbol{x}_{t}\right)\right)\right] A^{ms}_{t}}{G\left(\varepsilon^{s*}\left(\boldsymbol{x}_{t}\right)\right) A^{s}} \\
\varepsilon^{s*}\left(\boldsymbol{x}_{t}\right) &= \frac{p^{s}\left(\boldsymbol{x}_{t}\right) \phi^{m}\left(\boldsymbol{x}_{t}\right)-\phi^{s}\left(\boldsymbol{x}_{t}\right)}{y_{t}} \\
A^{mk}_{t} &= \Psi^{k}\left(\boldsymbol{x}_{t}\right), \text{ for } k \in \bar{\mathbb{N}} \\
A^{m}_{t} &= \sum_{k\in\bar{\mathbb{N}}} A^{mk}_{t}.
\end{aligned}$$

Suppose $\boldsymbol{x}_t = (A^m_t, y_t, \omega_i, \mu_i, r_i)$ and focus on a recursive equilibrium with the property that real prices are linear functions of the aggregate dividend. Then, under the conjecture

$$\phi^{s}\left(\boldsymbol{x}_{t}\right) = \phi^{s}_{i} y_{t} \tag{105}$$

$$\bar{\phi}^{s}\left(\boldsymbol{x}_{t}\right) = \bar{\phi}^{s}_{i} y_{t} \tag{106}$$

$$\phi^{m}\left(\boldsymbol{x}_{t}\right) A^{m}_{t} = Z_{i} y_{t} \tag{107}$$

$$A^{mk}_{t} = \Psi^{k}\left(\boldsymbol{x}_{t}\right) = \lambda^{k}_{i} A^{m}_{t} \tag{108}$$

$$\bar{\phi}^{s}\left(\boldsymbol{x}_{t}\right) \equiv p^{s}\left(\boldsymbol{x}_{t}\right) \phi^{m}\left(\boldsymbol{x}_{t}\right) \tag{109}$$

$$q\left(\boldsymbol{x}_{t}\right) = \min(\lambda^{b}_{i}/\omega_{i}, 1) \equiv q_{i} \tag{110}$$

$$\varepsilon^{s*}\left(\boldsymbol{x}_{t}\right) \equiv \frac{\bar{\phi}^{s}\left(\boldsymbol{x}_{t}\right)-\phi^{s}\left(\boldsymbol{x}_{t}\right)}{y_{t}} = \bar{\phi}^{s}_{i}-\phi^{s}_{i} \equiv \varepsilon^{s*}_{i}, \tag{111}$$

the equilibrium conditions reduce to (21)-(25), which is a system of $M(3N+2)$ independent equations to be solved for the $M(3N+2)$ unknowns $\{\phi_i^s, \varepsilon_i^{s*}, Z_i, \lambda_i^s, \lambda_i^b\}_{i\in\mathbb{M},s\in\mathbb{N}}$. Given $\{\phi_i^s, \varepsilon_i^{s*}, Z_i, \lambda_i^s, \lambda_i^b\}_{i\in\mathbb{M},s\in\mathbb{N}}$, for a state $\boldsymbol{x}_t = (A_t^m, y_t, \boldsymbol{\tau}_t)$ with $\boldsymbol{\tau}_t = \boldsymbol{\tau}_i = (\omega_i, \mu_i, r_i)$, $\phi^s(\boldsymbol{x}_t)$ is obtained from (105), $\bar{\phi}^s(\boldsymbol{x}_t)$ from (106) (with $\bar{\phi}_i^s = \varepsilon_i^{s*} + \phi_i^s$), $\phi^m(\boldsymbol{x}_t)$ from (107), A_t^{mk} from (108), $p^s(\boldsymbol{x}_t)$ from (109), and $q(\boldsymbol{x}_t)$ from (110). Notice that an economy with no explicit open-market operations is just special case of this economy with $\omega_t = 0$ for all t (which in turn implies $\lambda_i^b = 0$ for all i, so (23) is dropped from the set of equilibrium conditions).

The following proposition shows that if a monetary equilibrium exists for a given joint policy process for money growth and real rates, $\{\mu_t, r_t\}_{t=0}^\infty$, then there exists a bond policy, $\{\omega_t\}_{t=0}^\infty$ that implements a positive real value of money that is constant over time. This result is useful because it implies the real price of money need not change at the times when monetary policy switches states.

Proposition 12 *Let $\langle(\mu_i, r_i), [\sigma_{ij}]\rangle_{i,j\in\mathbb{M}}$ denote a (Markov chain for the) joint process of money growth and real rates, i.e., a set of states $(\mu_i, r_i)_{i\in\mathbb{M}}$ and a transition matrix $[\sigma_{ij}]_{i,j\in\mathbb{M}}$ such that $\sigma_{ij} = \Pr[(\mu_{t+1}, r_{t+1}) = (\mu_j, r_j) \,|\, (\mu_t, r_t) = (\mu_i, r_i)]$. Consider a process $\langle(\mu_i, r_i), [\sigma_{ij}]\rangle_{i,j\in\mathbb{M}}$ such that there exists a vector $(\phi_i^s, \varepsilon_i^{s*})_{i\in\mathbb{M},s\in\mathbb{N}}$ that solves*

$$\phi_i^s = \frac{\bar{\gamma}\delta}{1+r_i}\sum_{j\in\mathbb{M}}\sigma_{ij}\left[\bar{\varepsilon} + \phi_j^s + \alpha^s\theta\int_{\varepsilon_L}^{\varepsilon_j^{s*}}(\varepsilon_j^{s*} - \varepsilon)dG(\varepsilon)\right] \text{ for } (i,s)\in\mathbb{M}\times\mathbb{N} \tag{112}$$

$$1 = \frac{\bar{\gamma}}{(1+r_i)\mu_i}\sum_{j\in\mathbb{M}}\sigma_{ij}\left[1 + \alpha^s\theta\int_{\varepsilon_j^{s*}}^{\varepsilon_H}\frac{\varepsilon - \varepsilon_j^{s*}}{\varepsilon_j^{s*} + \phi_j^s}dG(\varepsilon)\right] \text{ for } (i,s)\in\mathbb{M}\times\mathbb{N}. \tag{113}$$

Then for any $Z \in (Z_0, \infty)$, there exists a bond policy $(\omega_i)_{i\in\mathbb{M}}$ that implements equilibrium aggregate real balances $(Z_i)_{i\in\mathbb{M}}$ with $Z_i = Z$ for all $i \in \mathbb{M}$. Moreover, the bond policy that implements the contant aggregate real balance Z is

$$\omega_i = \left[1 - \frac{\sum_{s\in\mathbb{N}}\frac{G(\varepsilon_i^{s*})A^s}{1-G(\varepsilon_i^{s*})}(\varepsilon_i^{s*} + \phi_i^s)}{Z}\right]\left[1 + \alpha^s\theta\int_{\varepsilon_i^{s*}}^{\varepsilon_H}\frac{\varepsilon - \varepsilon_i^{s*}}{\varepsilon_i^{s*} + \phi_i^s}dG(\varepsilon)\right] \text{ for } i\in\mathbb{M}, \tag{114}$$

and

$$Z_0 = \max_{i\in\mathbb{N}}\sum_{s\in\mathbb{N}}\frac{G(\varepsilon_i^{s*})A^s}{1-G(\varepsilon_i^{s*})}(\varepsilon_i^{s*} + \phi_i^s).$$

Under bond policy (114), in state $\boldsymbol{x}_t = (A_t^m, y_t, \omega_i, \mu_i, r_i)$, investors assign $\lambda_i^b Z$ real balances to *the bond market and $\lambda_i^s Z$ real balances to the market for stock $s \in \mathbb{N}$, where*

$$\lambda_i^b = 1 - \frac{\sum_{s\in\mathbb{N}} \frac{G(\varepsilon_i^{s*})A^s}{1-G(\varepsilon_i^{s*})}(\varepsilon_i^{s*} + \phi_i^s)}{Z} \in (0,1) \tag{115}$$

$$\lambda_i^s = \frac{\frac{G\left(\varepsilon_i^{s*}\right)A^s}{1-G\left(\varepsilon_i^{s*}\right)}(\varepsilon_i^{s*} + \phi_i^s)}{Z} \in (0,1)\,, \tag{116}$$

and the dollar price of equity in the OTC round is $p^s(\boldsymbol{x}_t) = (\varepsilon_i^{s*} + \phi_i^s)\, A_t^m / Z$.

Proof. Given the vector $(\phi_i^s, \varepsilon_i^{s*})_{i\in\mathbb{M},s\in\mathbb{N}}$ that solves (112) and (113), (23)-(25) imply

$$Z = \frac{\sum_{s\in\mathbb{N}} \frac{G\left(\varepsilon_i^{s*}\right)A^s}{1-G\left(\varepsilon_i^{s*}\right)}(\varepsilon_i^{s*} + \phi_i^s)}{1 - \frac{\omega_i}{1+\alpha^s\theta\int_{\varepsilon_i^{s*}}^{\varepsilon_H} \frac{\varepsilon-\varepsilon_i^{s*}}{\varepsilon_i^{s*}+\phi_i^s}dG(\varepsilon)}} \tag{117}$$

$$\lambda_i^b = \frac{\omega_i}{1 + \alpha^s\theta\int_{\varepsilon_i^{s*}}^{\varepsilon_H} \frac{\varepsilon-\varepsilon_i^{s*}}{\varepsilon_i^{s*}+\phi_i^s}dG(\varepsilon)} \tag{118}$$

$$\lambda_i^s = \left[1 - \frac{\omega_i}{1 + \alpha^s\theta\int_{\varepsilon_i^{s*}}^{\varepsilon_H} \frac{\varepsilon-\varepsilon_i^{s*}}{\varepsilon_i^{s*}+\phi_i^s}dG(\varepsilon)}\right] \frac{\frac{G\left(\varepsilon_i^{s*}\right)A^s}{1-G\left(\varepsilon_i^{s*}\right)}(\varepsilon_i^{s*} + \phi_i^s)}{\sum_{s\in\mathbb{N}} \frac{G\left(\varepsilon_i^{s*}\right)A^s}{1-G\left(\varepsilon_i^{s*}\right)}(\varepsilon_i^{s*} + \phi_i^s)} \tag{119}$$

From (117), it is clear that the bond policy $(\omega_i)_{i\in\mathbb{M}}$ described in the proposition implements aggregate real balance Z. Then (118) and (119) imply (115) and (116), and $\lambda_i^b, \lambda_i^s \in (0,1)$ since $0 < Z_0 < Z$. Finally, $p^s(\boldsymbol{x}_t)$ is obtained from (109). ■

In general, the equilibrium for the general model with N asset classes, open-market operations, and policy uncertainty, involves numerically solving the system of $M(3N+2)$ independent equations and $M(3N+2)$ unknowns given by (21)-(25). In order to gain analytical intuition, the following proposition offers a full characterization of the monetary equilibrium for an economy with N equity classes and open-market operations, but no policy uncertainty. In this context, Corollary 5 deals with implementing a level of real balance that is independent of the growth rate of the money supply.

Proposition 13 *Consider the economy with no policy uncertainty, i.e., $\mu_i = \mu$, $\omega_i = \omega$, and $r_i = r$ for all $i \in \mathbb{M}$. Let $\bar{\mu}^s \equiv \bar{\beta}\left[1 + \frac{\alpha^s\theta\left(1-\bar{\beta}\delta\right)(\bar{\varepsilon}-\varepsilon_L)}{\bar{\beta}\delta\bar{\varepsilon}+\left(1-\bar{\beta}\delta\right)\varepsilon_L}\right]$ and $\bar{\mu}^* = \min_{s\in\mathbb{N}} \bar{\mu}^s$, and assume $\mu \in (\bar{\beta}, \bar{\mu}^*)$ and $\omega \in \left(0, \mu/\bar{\beta}\right)$. Then there exists a unique recursive monetary equilibrium:*

(i) Asset prices are

$$\phi_t^s = \phi^s y_t,$$

where

$$\phi^s = \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}}(\varepsilon^{s*}-\varepsilon)dG(\varepsilon)\right] \tag{120}$$

and $\varepsilon^{s*} \in (\varepsilon_L, \varepsilon_H)$ *is the unique solution to*

$$\frac{\int_{\varepsilon^{s*}}^{\varepsilon_H}(\varepsilon-\varepsilon^{s*})\,dG(\varepsilon)}{\varepsilon^{s*} + \frac{\bar{\beta}\delta}{1-\bar{\beta}\delta}\left[\bar{\varepsilon} + \alpha^s\theta\int_{\varepsilon_L}^{\varepsilon^{s*}}(\varepsilon^{s*}-\varepsilon)dG(\varepsilon)\right]} - \frac{\mu-\bar{\beta}}{\bar{\beta}\alpha^s\theta} = 0. \tag{121}$$

(ii) Aggregate real balances are

$$\phi_t^m A_t^m = Z y_t,$$

where

$$Z = \frac{\sum_{s\in\mathbb{N}}\frac{G(\varepsilon^{s*})A^s}{1-G(\varepsilon^{s*})}(\varepsilon^{s*}+\phi^s)}{1-\frac{\bar{\beta}}{\mu}\omega}. \tag{122}$$

(iii) The price of a bond is

$$q_t = \frac{\bar{\beta}}{\mu}. \tag{123}$$

(iv) The proportion of real balances assigned to the bond market is

$$\lambda^b = \frac{\bar{\beta}}{\mu}\omega. \tag{124}$$

(v) The proportion of real balances assigned to the OTC market for equity s is

$$\lambda^s = \frac{\frac{G(\varepsilon^{s*})A^s}{1-G(\varepsilon^{s*})}(\varepsilon^{s*}+\phi^s)}{\sum_{s\in\mathbb{N}}\frac{G(\varepsilon^{s*})A^s}{1-G(\varepsilon^{s*})}(\varepsilon^{s*}+\phi^s)}\left(1-\frac{\bar{\beta}}{\mu}\omega\right). \tag{125}$$

Proof. With no policy uncertainty, $\phi_i^s = \phi^s$, $\varepsilon_i^{s*} = \varepsilon^{s*}$, $Z_i = Z$, $\lambda_i^s = \lambda^s$, $\lambda_i^b = \lambda^b$ for all

$i \in \mathbb{M}$ and all $s \in \mathbb{N}$, and the equilibrium conditions (21)-(25) reduce to

$$\phi^s = \bar{\beta}\delta \left[\bar{\varepsilon} + \phi^s + \alpha^s\theta \int_{\varepsilon_L}^{\varepsilon^{s*}} (\varepsilon^{s*} - \varepsilon) dG(\varepsilon) \right] \text{ for all } s \in \mathbb{N} \tag{126}$$

$$1 = \frac{\bar{\beta}}{\mu} \left[1 + \alpha^s\theta \int_{\varepsilon^{s*}}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^{s*}}{\varepsilon^{s*} + \phi^s} dG(\varepsilon) \right] \text{ for all } s \in \mathbb{N} \tag{127}$$

$$\max(\omega/\lambda^b, 1) = 1 + \alpha^s\theta \int_{\varepsilon^{s*}}^{\varepsilon_H} \frac{\varepsilon - \varepsilon^{s*}}{\varepsilon^{s*} + \phi^s} dG(\varepsilon) \text{ for all } s \in \mathbb{N} \tag{128}$$

$$Z\lambda^s = \frac{G(\varepsilon^{s*}) A^s}{1 - G(\varepsilon^{s*})} (\varepsilon^{s*} + \phi^s) \text{ for all } s \in \mathbb{N} \tag{129}$$

$$1 - \lambda^b = \sum_{s \in \mathbb{N}} \lambda^s. \tag{130}$$

This is a system of $3N+2$ independent equations in the $3N+2$ unknowns, $\left\{ \{\phi^s, \varepsilon^{*s}, \lambda^s\}_{s \in \mathbb{N}}, \lambda^b, Z \right\}$. Conditions (126) and (127) imply (120) and (121). It is easy to check there exists a unique $\varepsilon^{s*} \in (\varepsilon_L, \varepsilon_H)$ provided $\mu \in (\bar{\beta}, \bar{\mu}^s)$. Given $\{\phi^s, \varepsilon^{*s}\}_{s \in \mathbb{N}}$, conditions (128)-(130) need to be solved for $\left\{ \{\lambda^s\}_{s \in \mathbb{N}}, \lambda^b, Z \right\}$. Conditions (129) imply the values of $\{Z\lambda^s\}_{s \in \mathbb{N}}$. Conditions (127) and (128) imply $\max(\omega/\lambda^b, 1) = \mu/\bar{\beta}$, and since $\bar{\beta} < \mu$, this implies (123) and (124). Finally, condition (130) implies (122) and therefore (129) implies (125). ■

Corollary 5 *Consider the economy of Proposition 13. Let $\{\varepsilon^{s*}(\mu), \phi^s(\mu)\}_{s \in \mathbb{N}}$ denote the vector $\{\varepsilon^{s*}, \phi^s\}_{s \in \mathbb{N}}$ that solves (120) and (121) for a given μ, and let*

$$\mathcal{Z}(\mu) \equiv \sum_{s \in \mathbb{N}} \frac{G[\varepsilon^{s*}(\mu)] A^s}{1 - G[\varepsilon^{s*}(\mu)]} [\varepsilon^{s*}(\mu) + \phi^s(\mu)].$$

(i) The monetary authority can implement any real balance $Z \in [0, \infty)$.

(ii) For any $\mu_0 \in (\bar{\beta}, \bar{\mu}^*)$, any equilibrium aggregate real balance $Z \in (Z_0, \infty)$, where $Z_0 = \mathcal{Z}(\mu_0)$, can be implemented in a way that it is independent of the money growth rate, μ, for any $\mu \in (\mu_0, \bar{\mu}^*)$.

(iii) Any equilibrium aggregate real balance $Z_0 \in (0, \infty)$ can be implemented independently of the money growth rate, μ, provided $\mu \in (\mu_0, \bar{\mu}^*)$, where μ_0 is the unique solution to $\mathcal{Z}(\mu_0) = Z_0$.

Proof. (i) Fix ω. From (121), it is clear that by varying μ in the interval $(\bar{\beta}, \bar{\mu}^*)$, the monetary authority can implement $\varepsilon^{s*} = \varepsilon_L$, as well as $\varepsilon^{s*} = \varepsilon_H$ for all $s \in \mathbb{N}$. The result then follows from (122).

(ii) Fix $\mu_0 \in \left(\bar{\beta}, \bar{\mu}^*\right)$ and let $Z_0 = \mathcal{Z}(\mu_0)$. Then for any $Z \in (Z_0, \infty)$, set

$$\omega = \left(1 - \frac{\mathcal{Z}(\mu)}{Z}\right)\frac{\mu}{\bar{\beta}}. \tag{131}$$

Clearly, $\omega \in \left(0, \mu/\bar{\beta}\right)$ for any $\mu \in (\mu_0, \bar{\mu}^*)$. To conclude, notice that for any $\mu \in (\mu_0, \bar{\mu}^*)$, the bond policy (131) implements the constant aggregate real balance Z.

(iii) Fix $Z_0 \in (0, \infty)$ and let μ_0 denote the unique solution to $\mathcal{Z}(\mu_0) = Z_0$. Then for any $\mu \in (\mu_0, \bar{\mu}^*)$, set

$$\omega = \left(1 - \frac{\mathcal{Z}(\mu)}{Z_0}\right)\frac{\mu}{\bar{\beta}}. \tag{132}$$

Clearly, $\omega \in \left(0, \mu/\bar{\beta}\right)$ for any $\mu \in (\mu_0, \bar{\mu}^*)$, and the bond policy (132) implements the constant aggregate real balance Z_0. ■

D Supplementary material: Robustness

In this section we perform several robustness checks on the empirical and quantitative analyses.

D.1 Delayed return response

Our quantitative theory predicts that returns of more liquid stocks are more responsive than returns of less liquid stocks to monetary policy shocks on the announcement day, and that these differences persist beyond the announcement day. The prediction for announcement days is in line with the empirical estimations we have carried out in Section 5.2 and Section 5.3. However, the evidence in those sections may also be consistent with an alternative hypothesis, namely, that while the more liquid stocks may experience a stronger reaction than less liquid stocks on the day of the announcement, this differential response would dissipate if we gave the less liquid stocks more time to react. We have already pointed out (see footnote 37) that this hypothesis is at odds with the VAR evidence in Section 5.4. In this section we redo the estimations in Section 5.2 and Section 5.3 by looking at two-day cumulative returns after the announcement, and find no support for the alternative hypothesis.

For $j = 1, 2, ..$, define the cumulative marketwide stock return between day t and day $t + j$, by $\bar{\mathcal{R}}^I_{t,t+j} \equiv \prod_{k=1}^{j} \mathcal{R}^I_{t+k}$, the cumulative return of stock s between day t and day $t + j$, by $\bar{\mathcal{R}}^s_{t,t+j} \equiv \prod_{k=1}^{j} \mathcal{R}^s_{t+k}$, and the change in the 3-month Eurodollar future rate between day t and day $t + j$, with $\Delta i_{t,t+j} \equiv i_{t+j} - i_t$.

The first exercise we conduct consists of estimating the marketwide regression

$$\bar{\mathcal{R}}^I_{t-1,t-1+j} = a + b\Delta i_{t-1,t-1+j} + \epsilon_{t-1+j}, \tag{133}$$

for $t \in S_1$, with $j = 2$, where ϵ_{t-1+j} is an exogenous shock to the asset return. Notice (19) is a special case of (133) with $j = 1$. The second exercise consists of estimating

$$\bar{\mathcal{R}}^s_{t-1,t-1+j} = a + b\Delta i_{t-1,t-1+j} + \epsilon_{t-1+j} \tag{134}$$

with $j = 2$, for $t \in S_1$ and $s = 1, ..., 20$, where s represents each of the twenty liquidity portfolios constructed in Section 5.3, and ϵ_{t-1+j} is an exogenous shock to the asset return. Since we want to estimate the effects of the day-t policy surprise on the cumulative return between the end of day $t-1$ and the end of day $t+1$, we instrument for $\Delta i_{t,t+j}$ using the daily *imputed change* in the 30-day federal funds futures rate from the level it has 20 minutes after the FOMC announcement and the level it has 10 minutes before the FOMC announcement, i.e., the variable z_t as described in Appendix B (Section B.2). That is, we estimate b in (133) and in (134) using the following two-stage least squares (2SLS) procedure. Define $\Delta i^{ed}_{t,t+j} \equiv i^{ed}_{t+j} - i^{ed}_t$, where i^{ed}_t denotes the rate implied (for day t) by the 3-month Eurodollar futures contract with closest expiration date at or after day t. First, run the regression $\Delta i^{ed}_{t-1,t-1+j} = \kappa_0 + \kappa z_t + \eta_{t-1+j}$ on sample S_1 (where η_{t-1+j} is an error term) to obtain the OLS estimates of κ_0 and κ, namely $\hat{\kappa}_0$ and $\hat{\kappa}$. Second, construct the fitted values $\hat{z}_{t-1,t-1+j} \equiv \hat{\kappa}_0 + \hat{\kappa} z_t$ and run the regression (19) (or (134)) setting $\Delta i_{t-1,t-1+j} = \hat{z}_{t-1,t-1+j}$. The resulting marketwide and portfolio-by-portfolio estimates are reported in Table 5. All estimates are negative, and again, the magnitude of the response tends to be stronger for more liquid portfolios. For example, the two-day return of portfolio 20 responds 2.23 times more than the two-day return of portfolio 1, while (from Table 2) the announcement-day return of portfolio 20 responds 2.55 times more than the announcement-day return of portfolio 1. Thus, even much of the tilting in the announcement-day return responses to the policy shock is still noticeable when looking at two-day cumulated returns.

The third exercise we conduct consists of estimating the following regression of delayed individual stock returns (for the universe of stocks listed in the NYSE) on changes in the policy rate, an interaction term between the change in the policy rate and individual stock daily turnover rate, and several controls, i.e.,

$$\begin{aligned} \bar{\mathcal{R}}^s_{t-1,t-1+j} = \beta_0 + \beta_1 \Delta i_{t-1,t-1+j} + \beta_2 \mathcal{T}^s_t + \beta_3 \left(\Delta i_{t-1,t-1+j} - \Delta i\right) \times \bar{\mathcal{T}}^s_t \\ + D_s + D_t + \beta_4 \left(\Delta i_{t-1,t-1+j}\right)^2 + \beta_5 \left(\mathcal{T}^s_t\right)^2 + \varepsilon^s_{t-1,t-1+j}, \end{aligned} \tag{135}$$

with $j = 2$, for all $t \in S_1$ and all individual stocks, s, where D_s is a stock fixed effect, D_t is a quarterly time dummy, and $\varepsilon^s_{t-1,t-1+j}$ is the error term corresponding to stock s on policy announcement day t, $\bar{\mathcal{T}}^s_t \equiv \mathcal{T}^s_t - \mathcal{T}$, and Δi and $\mathcal{T}$ denote the sample averages of $\Delta i_{t-1,t-1+j}$ and $\mathcal{T}^s_t$, respectively. We estimate seven different specifications based on (135). These seven specifications correspond to specifications (I), (II), (III), (IV), (VI), (VII), and (VIII) in Section 5.3. In every specification, the measure of daily turnover of a stock s, namely $\mathcal{T}^s_t$, is measured as in the estimation of (20). In specification (I), $\Delta i_{t-1,t-1+j}$ is instrumented for with z_t, as explained above. In all other specifications, we proxy for $\Delta i_{t-1,t-1+j} \equiv v^1_{t-1,t-1+j}$ and $(\Delta i_{t-1,t-1+j} - \Delta i) \times \bar{\mathcal{T}}^s_t \equiv v^2_{t-1,t-1+j}$ as follows. We first run the following two regressions (i.e., for $i = 1, 2$)

$$v^i_{t-1,t-1+j} = \kappa^i_0 + \kappa^i_1 z_t + \kappa^i_2 \left(z_t \times \mathcal{T}^s_t\right) + \eta^i_{t-1+j} \tag{136}$$

on sample S_1 (where η^i_{t-1+j} is an error term) to obtain the OLS estimates of $\left(\kappa^i_0, \kappa^i_1, \kappa^i_2\right)_{i=1,2}$, namely $\left(\hat{\kappa}^i_0, \hat{\kappa}^i_1, \hat{\kappa}^i_2\right)_{i=1,2}$. We then construct the fitted values $\hat{v}^i_{t-1,t-1+j} \equiv \kappa^i_0 + \kappa^i_1 z_t + \kappa^i_2 \left(z_t \times \mathcal{T}^s_t\right)$ for $i = 1, 2$, and run the regression (135) setting $\Delta i_{t-1,t-1+j} = \hat{v}^1_{t-1,t-1+j}$ and $(\Delta i_{t-1,t-1+j} - \Delta i) \times \bar{\mathcal{T}}^s_t = \hat{v}^2_{t-1,t-1+j}$. Whenever $(\Delta i_{t-1,t-1+j})^2$ appears in a particular specification of (135), we also include z^2_t as an additional regressor in the first-stage regressions (136).

The results are in Table 6. The estimate of interest, β_3, is large, negative, and statistically significant in all specifications. This means that the magnitude of the negative effect of unexpected changes in the policy rate on two-day cumulated equity returns is still larger for stocks with higher turnover liquidity.

D.2 Disaggregative announcement-day effects with additional controls

In addition to the specifications (I)-(IX) of (20) discussed in Section 5.3, in this section we report the results of three more specifications, labeled (X), (XI), and (XII), that incorporate additional control variables. The general regression we fit is now

$$\begin{aligned}
\mathcal{R}^s_t = \beta_0 &+ \beta_1 \Delta i_t + \beta_2 \mathcal{T}^s_t + \beta_3 \overline{\mathcal{T}^s_t} \times \overline{\Delta i_t} \\
&+ D_s + D_t + D_I + \beta_4 (\Delta i_t)^2 + \beta_5 (\mathcal{T}^s_t)^2 \\
&+ \beta_6 \beta^s_{1t} + \beta_7 \overline{\beta^s_{1t}} \times \overline{\Delta i_t} \\
&+ \beta_8 \beta^s_{2t} + \beta_9 \overline{\beta^s_{2t}} \times \overline{\Delta i_t} \\
&+ \beta_{10} \beta^s_{3t} + \beta_{11} \overline{\beta^s_{3t}} \times \overline{\Delta i_t} \\
&+ \beta_{12} L^s_t + \beta_{13} \overline{L^s_t} \times \overline{\Delta i_t} + \varepsilon_{st},
\end{aligned} \tag{137}$$

where D_I is a dummy for the Fama-French ten-industry classification, L^s_t is the measure of the bank leverage of company s at date t constructed by Ippolito et al. (2018), and β^s_{jt} is the "beta" corresponding to factor "$f_{j,t}$" with $j \in \{1, 2, 3\}$ for announcement day t that we estimated from the multi-factor regression (82) (recall $f_{1,t} = MKT_t$, $f_{2,t} = HML_t$, and $f_{3,t} = SMB_t$; see Section B.3 for details).[64] All other variables are as defined in Section 5.3. Also as in Section 5.3, a "" on top of a variable denotes deviation from the sample average, i.e., $\overline{\beta^s_{jt}} \equiv (\beta^s_{jt} - \beta_j)$ and $\overline{L^s_t} \equiv (L^s_t - L)$, where β_j and L denote the sample averages of β^s_{jt} and L^s_t. The industry dummy D_I addresses the potential concern that the turnover rate of a stock may be correlated

[64]In Section B.3 we used $\beta^s_j(k)$ to denote the estimate for the beta corresponding to factor j for stock s, estimated on the sample consisting of all trading days between the policy announcement days t_{k-1} and t_k. Since t_k is the date of the k^{th} announcement, we have $\beta^s_{jt_k} = \beta^s_j(k)$.

with the industry to which the stock belongs. Incorporating the annuoncement-date "betas" $\{\beta_{jt}^s\}_{j=1}^3$ addresses the potential concern that the differential return response to policy rate shocks for stocks with different turnover liquidity may be driven by heterogeneous exposure to other aggregate factors. Specifically the betas $\{\beta_{jt}^s\}_{j=1}^3$ control for the effects on stock returns of the three standard Fama-French factors—after controlling for the risk exposure to aggregate turnover of the stock market (as captured by β_0^s in (82)).[65] The additional control variable, bank leverage L_t^s, addresses the potential concern that the differential return responses that we attribute to the turnover rate of a stock may instead reflect some other fundamentals of a firm, such as its reliance on debt.

Specifications (I)-(IX) are as in Section 5.3. Specification (X) adds D_I to specification (IX). Specification (XI) adds $\{\beta_{jt}^s\}_{j=1}^3$ and the corresponding cross terms, $\{\overline{\beta_{jt}^s} \times \overline{\Delta i_t}\}_{j=1}^3$, to specification (X). Specification (XII) adds L_t^s and the cross term $\overline{L_t^s} \times \overline{\Delta i_t}$ to specification (XI). In every case, the coefficient of interest is β_3.

The results for specifications (I)-(XI) for our baseline sample period (January 3, 1994 through December 31, 2007) are reported in Table 7. The results for specification (XII) reported in Table 8 are estimated for the sample period June 26, 2002 to December 31, 2007 since the company-specific leverage data from Ippolito et al. (2018) starts on June 26, 2002. For comparison purposes, in Table 8 we also report the results of specifications (IX), (X), and (XI) for the same sample period. The main conclusion is that in all specifications, the estimate of β_3 is negative, large in magnitude, and significant.

D.3 NASDAQ stocks

In this section we use daily time series for all individual stocks in the National Association of Securities Dealer Automated Quotation system (NASDAQ) from CRSP to estimate the aggregate and disaggregative return responses of Section 5.3 for the same sample period. We perform the same estimations as in Section 5.3. The estimates for marketwide return and turnover are reported in Table 9, which is analogous to Table 1. The estimates obtained from the portfolio-by-portfolio regressions are reported in Table 10, which is analogous to Table 2. The estimates from the nine specifications based on (20) are reported in Table 11, which is analogous to Table 3. The tilting in returns across liquidity portfolios in response to the

[65] Because the Fama-French factors are reduced-form factors that may partly capture partly the effect of the turnover liquidity transmission mechanism, we control for the return exposure to the aggregate turnover rate in order to obtain the betas that are net of the influence of the turnover liquidity factor.

monetary policy shock is even stronger than for the NYSE stocks.

D.4 Value-weighted returns

Let $\mathbb{P}$ denote a portfolio of stocks, i.e., a collection of stocks, each denoted by s, and let $N(\mathbb{P})$ denote the number of stocks in $\mathbb{P}$. In the portfolio-by-portfolio regressions of Section 5.3, we defined the average return of portfolio $\mathbb{P}$ on day t as

$$\sum_{s\in\mathbb{P}} \frac{1}{N(\mathbb{P})} \mathcal{R}_t^s.$$

In this section we redo the same estimations using the value-weighted return, defined as

$$\sum_{s\in\mathbb{P}} \omega_t^s \mathcal{R}_t^s,$$

with

$$\omega_t^s \equiv \frac{P_{t-1}^s K_t^s}{\sum_{i\in\mathbb{P}} P_{t-1}^i K_t^i},$$

where K_t^s denotes the number of outstanding shares for stock s on day t. The results for the NYSE are summarized in Table 12. All the estimates are negative, as predicted by the theory. Also, the magnitude of the (statistically significant) estimates tends to increase with the turnover liquidity of the portfolio.

D.5 Results for the 2002-2007 subsample

The empirical finding that surprise increases in the nominal policy rate cause sizable reductions in real stock returns on announcement days of the FOMC is well established for sample periods ranging roughly from the early 1990's until 2002. For example, Bernanke and Kuttner (2005) use a sample that runs from June 1989 to December 2002, and Rigobon and Sack (2004) use a sample that runs from January 1994 to November 2001. We have found that both their empirical results for stocks, and our additional findings regarding the turnover-liquidity mechanism, hold for the longer sample that runs from January 1994 to December 2007. Table 13 reports the estimates for specifications (I)-(IX) of (20), for the sample period 2002-2007. The table shows that our empirical findings, and the results for stocks in Bernanke and Kuttner (2005) and Rigobon and Sack (2004) also hold for this more recent subsample.

D.6 Nominal-real interest rate passthrough

In the baseline calibration of Section 6 we set $w = .8$, which implies a 100 bp increase in the nominal rate is associated with a 80 bp increase in the real rate and a 20 bp increase in expected inflation. As a robustness check we have also set $w = 1$ and recalibrated the model to fit the same data targets as the baseline calibration, and found that the quantitative performance of the theory is very similar to the case with $w = .8$. Here we report results for the case with $w = 0$, which implies a 100 bp increase in the nominal rate is associated with a 100 bp increase in expected inflation and the real rate remains constant. Specifically, we consider an economy with $w = 0$, recalibrate the model to fit the same data targets as the baseline calibration, and carry out Exercise 1 as described in Section 6.3. The theory is able to generate most of the announcement-day tilting in cross-sectional returns. The results are shown in Figure 11.

Portfolio	Return	
	Estimate	Std dev
1	-4.20***	1.21
2	-4.15***	1.25
3	-4.61***	1.29
4	-4.35***	1.32
5	-4.87***	1.41
6	-4.45***	1.54
7	-5.25***	1.43
8	-4.91***	1.55
9	-4.65***	1.65
10	-5.90***	1.64
11	-6.07***	1.47
12	-6.06***	1.70
13	-6.80***	1.59
14	-6.12***	1.57
15	-7.79***	1.66
16	-7.08***	1.78
17	-8.91***	2.07
18	-9.32***	1.97
19	-8.90***	2.19
20	-9.35***	2.56
NYSE	-6.18***	1.49

Table 5: Two-day responses of stock returns to monetary policy across liquidity portfolios (HFIV estimates). *** denotes significance at 1% level, ** significance at 5% level, * significance at 10% level.

Variable	(I)	(II)	(III)	(IV)	(VI)	(VII)	(VIII)
$\Delta i_{t-1,t+1}$	-6.13	-6.25	-6.29	-6.10	-5.59	-5.59	-5.90
	(.141)	(.144)	(.142)	(.161)	(.158)	(.157)	(.187)
$\mathcal{T}_t^s$	3364	3593	2777	2124	6090	5020	5374
	(207)	(210)	(274)	(231)	(441)	(548)	(464)
$\overline{\mathcal{T}_t^s} \times (\Delta i_{t-1,t+1} - \Delta i)$		-434.20	-429.91	-248.50	-423.18	-422.76	-232.91
		(39.68)	(39.21)	(39.14)	(39.17)	(38.82)	(38.85)
D_s			yes			yes	
D_t				yes			yes
$(\Delta i_{t-1,t+1})^2$					.044	.046	.012
					(.009)	(.009)	(.011)
$(\mathcal{T}_t^s)^2$					-103202	-83014.5	-131721
					(20442)	(22947)	(20333)
R^2	.0157	.0152	.0140	.0645	.0202	.0191	.0669

Table 6: Effects of monetary policy on two-day stock returns of NYSE individual stocks (both in basis points). Each column reports the coefficients from a separate pooled OLS regression based on (20). Number of observations: 205,657. Standard errors in parenthesis. All estimates are significant at 1% level, except for $(\Delta i_{t-1,t+1})^2$ in regression (VIII).

Variable	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)	(X)	(XI)
Δi_t	-5.37	-5.56	-5.60	-5.83	-5.82	-5.21	-5.23	-5.30	-5.30	-5.30	-5.36
	(.11)	(.11)	(.13)	(.12)	(.13)	(.099)	(.098)	(.110)	(.112)	(.112)	(.127)
$\overline{\mathcal{T}_t^s} \times \overline{\Delta i_t}$		-553	-557	-492	-491	-554	-560	-485	-487	-485	-539
		(29)	(30)	(30)	(32)	(29)	(31)	(30)	(32)	(32)	(34)
D_s			yes		yes		yes		yes	yes	yes
D_t				yes	yes			yes	yes	yes	yes
D_I										yes	yes
$(\Delta i_t)^2$						.04	.04	.05	.05	.05	.06
						(.008)	(.008)	(.009)	(.010)	(.010)	(.010)
$(\mathcal{T}_t^s)^2$						-53852	-57208	-63775	-56275	-55883	-67669
						(14751)	(15093)	(14887)	(15597)	(15650)	(16444)
$\overline{\beta_{1t}^s} \times \overline{\Delta i_t}$											.08
											(.21)
$\overline{\beta_{2t}^s} \times \overline{\Delta i_t}$											.37
											(.17)
$\overline{\beta_{3t}^s} \times \overline{\Delta i_t}$											.38
											(.11)
R^2	.0241	.0286	.0286	.0778	.0778	.0290	.0290	.0783	.0784	.0781	.0933

Table 7: Effects of monetary policy on stock returns of individual NYSE stocks (both in basis points). Each column reports the coefficients from a separate pooled OLS regression based on (137). Number of observations: 205,760. Standard errors in parentheses. All estimates are significant at the 1 percent level except $\overline{\beta_{2t}^s} \times \overline{\Delta i_t}$, which is significant at the 5 percent level and $\overline{\beta_{1t}^s} \times \overline{\Delta i_t}$, which is insignificant.

Variable	(IX)	(X)	(XI)	(XII)
Δi_t	-8.98	-8.98	-9.40	-9.38
	(.315)	(.316)	(.350)	(.347)
$\overline{\mathcal{T}_t^s} \times \overline{\Delta i_t}$	-272	-273	-251	-253
	(40)	(40)	(41)	(41)
D_s	yes	yes	yes	yes
D_t	yes	yes	yes	yes
D_I		yes	yes	yes
$\overline{\beta_{1t}^s} \times \overline{\Delta i_t}$			-4.69	-4.71
			(.38)	(.38)
$\overline{\beta_{2t}^s} \times \overline{\Delta i_t}$			-1.48	1.49
			(.20)	(.20)
$\overline{\beta_{3t}^s} \times \overline{\Delta i_t}$			.87	.87
			(.11)	(.12)
$\overline{L_t^s} \times \overline{\Delta i_t}$				-2.30
				(2.06)
R^2	.1528	.1502	.1913	.1913

Table 8: Effects of monetary policy on stock returns of individual NYSE stocks (both in basis points). Each column reports the coefficients from a separate pooled OLS regression based on (137), estimated for the sample period June 26, 2002 to December 31, 2007. Number of observations: 44,734. Standard errors in parentheses. All estimates are significant at the 1 percent level, except for $\overline{L_t^s} \times \overline{\Delta i_t}$, which is insignificant.

	E-based		H-based		HFIV	
	Estimate	Std. dev.	Estimate	Std. dev.	Estimate	Std. dev.
Return	-6.37***	2.11	-11.12***	3.74	-12.97***	3.58
Turnover	-.00004***	.00001	-.00006**	.00003	-.00006***	.00001

Table 9: Empirical response of NASDAQ marketwide stock returns to monetary policy. *** denotes significance at the 1 percent level, ** significance at the 5 percent level, * significance at the 10 percent level.

		E-based		H-based		HFIV	
Portfolio	Turnover	Estimate	Std. dev.	Estimate	Std. dev.	Estimate	Std. dev.
1	.08	-1.64**	.64	-1.76	1.19	-3.38***	1.18
2	.14	-1.51**	.71	-3.05	1.60	-2.87**	1.42
3	.21	-1.96***	.65	-4.87***	1.86	-4.49***	1.36
4	.28	-2.25**	.98	-4.71**	1.91	-5.27***	1.54
5	.36	-4.07***	1.01	-7.52***	2.55	-7.16***	1.79
6	.44	-3.48***	1.24	-6.85***	2.57	-7.78***	2.19
7	.54	-3.75***	1.37	-6.22***	2.40	-7.45***	2.71
8	.64	-5.04***	1.42	-9.26***	3.26	-8.94***	2.55
9	.76	-5.81***	1.80	-9.84***	3.46	-10.13***	3.31
10	.89	-4.46**	1.73	-8.24**	3.34	-11.23***	3.23
11	1.04	-6.28***	1.98	-11.58***	4.18	-12.21***	3.61
12	1.21	-6.12***	1.79	-11.30***	4.13	-12.12***	3.50
13	1.41	-6.69**	2.62	-10.56***	4.03	-13.85***	4.35
14	1.61	-7.71***	2.45	-12.89***	4.47	-14.72***	4.19
15	1.88	-8.92***	2.98	-15.20***	5.18	-18.29***	4.96
16	2.22	-9.25***	3.06	-15.80***	5.38	-19.16***	5.11
17	2.65	-9.14**	3.69	-15.22***	5.63	-20.11***	6.75
18	3.26	-11.18***	4.05	-19.63***	6.73	-22.85***	6.36
19	4.26	-12.23***	4.37	-21.40***	7.30	-24.80***	6.46
20	6.65	-15.87***	6.04	-26.68***	9.43	-32.69***	8.33
NASDAQ	1.53	-6.37***	2.11	-11.12***	3.74	-12.97***	3.58

Table 10: Empirical responses of NASDAQ stock returns to monetary policy across liquidity portfolios (1994-2007 sample). *** denotes significance at 1% level, ** significance at 5% level, * significance at 10% level.

Variable	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)
Δi_t	-6.32	-6.56	-6.63	-6.37	-6.38	-3.88	-3.91	-3.50	-3.50
	(.129)	(.128)	(.128)	(.139)	(.139)	(.121)	(.121)	(.139)	(.139)
$\mathcal{T}_t^s$	1655	1655	1093	1451	892	3214	2853	2894	2414
	(95)	(94)	(124)	(93)	(123)	(196)	(267)	(194)	(266)
$\overline{\mathcal{T}_t^s} \times \overline{\Delta i_t}$		-643	-649	-632	-636	-640	-646	-625	-628
		(23)	(22)	(22)	(22)	(22)	(22)	(22)	(21)
D_s			yes		yes		yes		yes
D_t				yes	yes			yes	yes
$(\Delta i_t)^2$						.263	.268	.285	.286
						(.009)	(.009)	(.011)	(.011)
$(\mathcal{T}_t^s)^2$						-44321	-43028	-42754	-39000
						(5776)	(6708)	(5725)	(6648)
R^2	.0096	.0145	.0144	.0317	.0316	.0179	.0179	.0342	.0340

Table 11: Effects of monetary policy on stock returns of individual NASDAQ stocks (both in basis points). Each column reports the coefficients from a separate pooled OLS regression based on (20). Number of observations: 482,825. Standard errors in parenthesis. All estimates are significant at 1% level.

		E-based		H-based		HFIV	
Portfolio	Turnover	Estimate	Std. dev.	Estimate	Std. dev.	Estimate	Std. dev.
1	.17	-2.74**	1.14	-6.04**	2.96	-4.06*	2.26
2	.32	-3.67***	.92	-10.21**	4.61	-6.25***	2.35
3	.43	-5.06***	1.75	-9.13**	4.50	-8.32**	3.55
4	.52	-2.93**	1.73	-10.75*	5.58	-1.48	3.08
5	.59	-4.25***	2.07	-11.45**	5.20	-3.55	4.18
6	.66	-5.64***	1.46	-11.82***	3.75	-7.21***	2.61
7	.73	-4.51***	1.61	-9.83***	3.59	-5.10	3.39
8	.80	-6.23***	1.59	-10.75**	4.39	-9.44***	3.53
9	.87	-4.81***	1.50	-10.95**	5.49	-6.31**	2.90
10	.94	-5.56***	1.29	-12.76**	5.01	-8.38***	2.81
11	1.01	-8.30***	2.10	-13.00***	4.85	-13.06***	4.20
12	1.11	-6.74***	1.97	-11.06**	4.67	-12.24***	4.74
13	1.21	-8.57***	2.10	-13.93***	4.37	-14.38***	3.67
14	1.32	-6.38***	1.66	-14.72***	5.16	-10.42***	3.12
15	1.45	-9.78***	2.42	-16.24***	4.75	-16.21***	3.61
16	1.60	-10.21***	3.20	-15.47***	5.04	-19.31***	6.05
17	1.79	-10.06***	1.85	-19.65***	6.63	-16.28***	3.66
18	2.07	-10.04***	2.50	-18.32***	6.16	-18.07***	4.79
19	2.50	-12.00***	3.09	-22.33***	7.46	-21.38***	5.59
20	3.57	-9.22***	2.84	-18.88***	6.86	-18.69***	5.03

Table 12: Empirical responses of stock returns to monetary policy across NYSE value-weighted liquidity portfolios (1994-2007 sample). *** denotes significance at 1% level, ** significance at 5% level, * significance at 10% level.

Variable	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)	(IX)
Δi_t	-8.49	-8.21	-8.30	-8.20	-8.15	-9.47	-9.55	-9.11	-9.06
	(.19)	(.19)	(.18)	(.22)	(.21)	(.19)	(.19)	(.25)	(.22)
$\mathcal{T}_t^s$	2390	2356	2879	480	377	3550	4867	1882	1037
	(194)	(194)	(253)	(192)	(273)	(440)	(564)	(440)	(595)
$\overline{\mathcal{T}_t^s} \times \overline{\Delta i_t}$		-436	-430	-297	-305	-395	-391	-280	-288
		(39)	(32)	(38)	(32)	(38)	(31)	(38)	(32)
D_s			yes		yes		yes		yes
D_t				yes	yes			yes	yes
$(\Delta i_t)^2$						.57	.56	.40	.39
						(.017)	(.016)	(.025)	(.022)
$(\mathcal{T}_t^s)^2$						-47089	-68872	-43233	-33071
						(17542)	(18930)	(17503)	(19449)
R^2	.0343	.0376	.0374	.1387	.1385	.0506	.0502	.1431	.1429

Table 13: Effects of monetary policy on stock returns of individual NYSE stocks (both in basis points). Each column reports the coefficients from a separate pooled OLS regression based on (20) for the sample period 2002-2007. Number of observations: 78,942. Standard errors in parentheses. All estimates are significant at the 1 percent level.

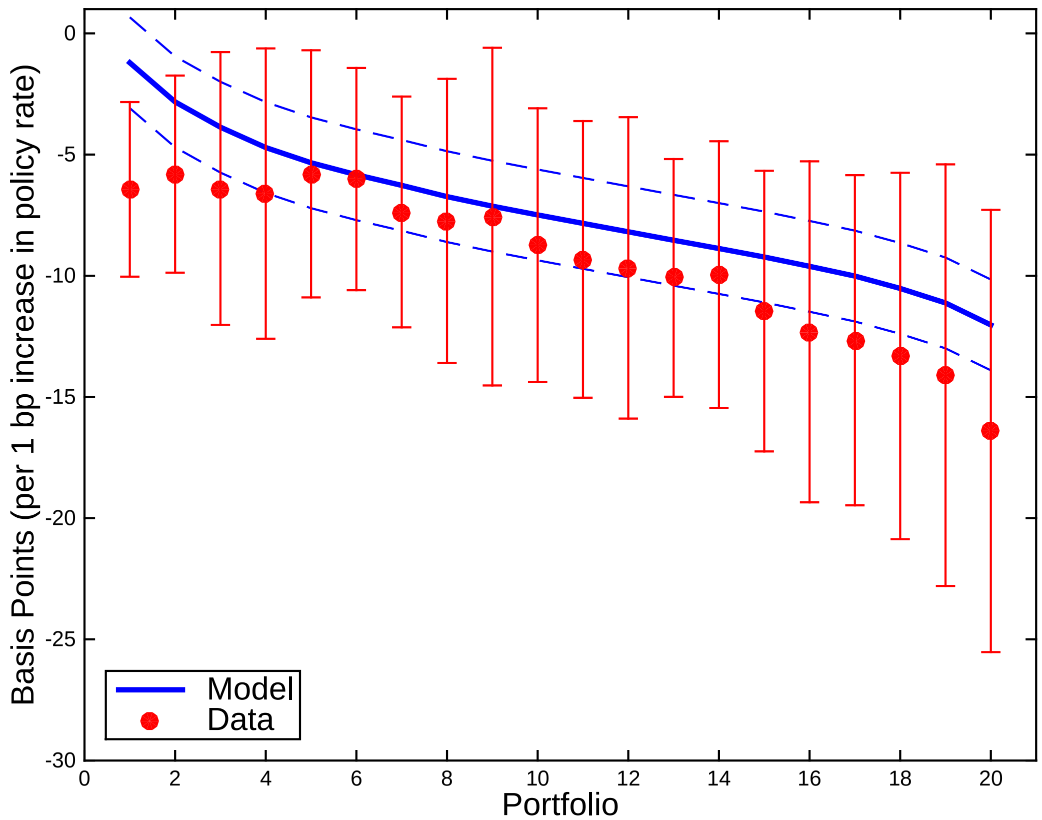

Figure 11: Cross-sectional announcement-day responses of stock returns to policy rate for the model with $w = 0$.

E Supplementary material: Literature

The empirical component of our paper (Section 5) is related to a large empirical literature that studies the effect of monetary policy shocks on asset prices. Like many of these studies, we identify monetary policy shocks by focusing on the reaction of asset prices in a narrow time window around FOMC monetary policy announcements. Cook and Hahn (1989), for example, use this kind of event-study identification strategy (with an event window of one day) to estimate the effects of changes in the federal funds rate on bond rates. Kuttner (2001) conducts a similar analysis but shows the importance of focusing on unexpected policy changes, which he proxies for with federal funds futures data. Cochrane and Piazzesi (2002) estimate the effect of monetary policy announcements on the yield curve using a one-day window around the FOMC announcement and the daily change in the one-month Eurodollar rate to proxy for unexpected changes in the federal funds rate target. Bernanke and Kuttner (2005) use daily event windows around FOMC announcements to estimate the effect of unexpected changes in the federal funds rate (measured using federal funds futures data) on the return of broad stock indices. Gürkaynak, Sack and Swanson (2005) focus on intraday event windows around FOMC announcements (30 minutes or 60 minutes wide) to estimate the effects on the S&P500 return and several Treasury yields of unexpected changes in the federal funds target and "forward guidance" (i.e., information on the future path of policy contained in the announcement). More recently, Hanson and Stein (2015) estimate the effect of monetary policy shocks on the nominal and real Treasury yield curves using a two-day window around the announcement. Nakamura and Steinsson (2015) also estimate the effects of monetary policy shocks on the nominal and real Treasury yield curves, but they use a 30-minute window around the announcement. Gertler and Karadi (2015) also use a 30-minute window around the announcement to estimate the response of bond yields and credit spreads to monetary policy shocks. Rigobon and Sack (2004) propose a heteroskedasticity-based estimator to correct for possible simultaneity biases remaining in these event-study regressions.

Relatively fewer papers have attempted to identify the precise mechanism through which surprise increases in the federal funds rate lead to a reduction in stock prices. Bernanke and Kuttner (2005), for example, take one step in this direction by analyzing the response of more disaggregated indices, in particular 10 industry-based portfolios. They find that the precision of their estimates is not sufficient to reject the hypothesis of an equal reaction for all 10 industries.

Firms differ along many dimensions, however, and a number of studies have focused on how these may be related to different responses of their stock prices to policy shocks. Ehrmann and Fratzscher (2004), for example, find that firms with low cash flows, small firms, firms with low credit ratings, firms with high price-earnings multiples, or firms with high Tobin's q exhibit a higher sensitivity to monetary policy shocks. Ippolito et al. (2013, 2018) find that the stock prices of bank-dependent firms that borrow from financially weaker banks display a stronger sensitivity to monetary policy shocks, while bank-dependent firms that hedge against interest rate risk display a lower sensitivity to monetary policy shocks. Gorodnichenko and Weber (2016) document that after monetary policy announcements, the conditional volatility of stock market returns rises more for firms with stickier prices than for firms with more flexible prices. Relative to this literature, our contribution is to document and offer a theory of the *turnover-liquidity transmission mechanism* that channels monetary policy to asset prices.

From a theoretical standpoint, the model we develop in this paper bridges the search-theoretic monetary literature that has largely focused on macro issues and the search-theoretic financial OTC literature that focuses on microstructure considerations. Specifically, we embed an OTC financial trading arrangement similar to Duffie et al. (2005) into a Lagos and Wright (2005) economy. Despite several common ingredients with those papers, our formulation is different from previous work along two important dimensions.

In the standard formulations of the Lagos-Wright framework, money (and sometimes other assets) are used as payment instruments to purchase consumption goods in bilateral markets mediated by search. We instead posit that money is used as a medium of exchange in OTC markets for financial assets. In the standard monetary model, money and other liquid assets help to allocate goods from producers to consumers, while in our current formulation, money helps to allocate financial assets among traders with heterogeneous valuations. This shift in the nature of the gains from trade offers a different perspective that delivers novel insights into the interaction between monetary policy and financial markets. For example, from a normative standpoint, the new perspective emphasizes a new angle on the welfare cost of inflation that is associated with the distortion of the optimal allocation of financial assets across investors with high and low valuations when real balances are scarce. From a positive perspective, it explains the positive correlation between nominal bond yields and real equity yields, something that the conventional formulation in which monetary or real assets are used to buy consumption goods cannot do.

As a model of financial trade, an appealing feature of Duffie et al. (2005) is its realistic OTC market structure consisting of an interdealer market and bilateral negotiated trades between investors and between investors and dealers. In Duffie et al. (2005), agents who wish to buy assets pay sellers with linear-utility transfers. In addition, utility transfers from buyers to sellers are unconstrained, so effectively there is no bound on what buyers can afford to purchase in financial transactions. Our formulation keeps the appealing market structure of Duffie et al. (2005) but improves on its stylized model of financial transactions by considering traders who face standard budget constraints and use fiat money to purchase assets. These modifications make the standard OTC formulation amenable to general equilibrium analysis and deliver a natural transmission mechanism through which monetary policy influences asset prices and the standard measures of financial liquidity that are the main focus of the microstructure strand of the OTC literature.

Our theoretical work is related to several previous studies, e.g., Geromichalos et al. (2007), Jacquet and Tan (2012), Lagos and Rocheteau (2008), Lagos (2010a, 2010b, 2011), Lester et al. (2012), and Nosal and Rocheteau (2013), which introduce a real asset that can (at least to some degree) be used along with money as a medium of exchange for consumption goods in variants of Lagos and Wright (2005). These papers identify the liquidity value of the asset with its usefulness in exchange and find that when the asset is valuable as a medium of exchange, this manifests itself as a "liquidity premium" that makes the real asset price higher than the expected present discounted value of its financial dividend. High anticipated inflation reduces real money balances; this tightens bilateral trading constraints, which in turn increases the liquidity value and the real price of the asset. In contrast, we find that real asset prices are decreasing in the rate of anticipated inflation. There are some models that also build on Lagos and Wright (2005) where agents can use a real asset as collateral to borrow money that they subsequently use to purchase consumption goods. In those models, anticipated inflation reduces the demand for real balances, which in turn can reduce the real price of the collateral asset needed to borrow money (see, e.g., He et al., 2012, and Li and Li, 2012). The difference is that in our setup, inflation reduces the real asset price by constraining the reallocation of the financial asset from investors with low valuations to investors with relatively high valuations.[66]

[66]In the model that we have developed here, money is the only asset used as means of payment. It would be straightforward, however, to enrich the asset structure so that investors may choose to carry other real assets that can be used as means of payment in the OTC market, e.g., along the lines of Lagos and Rocheteau (2008) or Lagos (2010a, 2010b, 2011). As long as money is valued in equilibrium, we anticipate that the main results

We share with two recent papers, Geromichalos and Herrenbrueck (2016) and Trejos and Wright (2016), the general interest in bringing models of OTC trade in financial markets within the realm of modern monetary general equilibrium theory. Trejos and Wright (2016) offer an in-depth analysis of a model that nests Duffie et al. (2005) and the prototypical "second generation" monetary search model with divisible goods, indivisible money, and a unit upper bound on individual money holdings (e.g., Shi, 1995 or Trejos and Wright, 1995). Trejos and Wright (2016) emphasize the different nature of the gains from trade in both classes of models. In monetary models, agents value consumption goods differently and use assets to buy goods, while in Duffie et al. (2005), agents trade because they value assets differently, and goods that are valued the same by all investors are used to pay for asset purchases. In our formulation, there are gains from trading assets, as in Duffie et al. (2005), but agents pay with money, as in standard monetary models. Another difference with Trejos and Wright (2016) is that rather than assuming indivisible assets and a unit upper bound on individual asset holdings, as in Shi (1995), Trejos and Wright (1995), and Duffie et al. (2005), we work with divisible assets and unrestricted portfolios, as in Lagos and Wright (2005) and Lagos and Rocheteau (2009).

Geromichalos and Herrenbrueck (2016) extend Lagos and Wright (2005) by incorporating a real asset that by assumption cannot be used to purchase goods in the decentralized market (as usual, at the end of every period agents choose next-period money and asset portfolios in a centralized market). The twist is that at the very beginning of every period, agents learn whether they will want to buy or sell consumption goods in the subsequent decentralized market, and at that point they have access to a bilateral search market where they can retrade money and assets. This market allows agents to rebalance their positions depending on their need for money, e.g., those who will be buyers seek to buy money and sell assets. So although assets cannot be directly used to purchase consumption goods as in Geromichalos et al. (2007) or Lagos and Rocheteau (2008), agents can use assets to buy goods indirectly, i.e., by exchanging them for cash in the additional bilateral trading round at the beginning of the period. Geromichalos and Herrenbrueck use the model to revisit the link between asset prices and inflation. Mattesini and Nosal (2016) study a related model that combines elements of Geromichalos and Herrenbrueck (2016) and elements of Lagos and Zhang (2015) but considers a new market structure for the interdealer market.

The fact that the equilibrium asset price is larger than the expected present discounted value

emphasized here would continue to hold.

that any agent assigns to the dividend stream is reminiscent of the literature on speculative trading that can be traced back to Harrison and Kreps (1978). As in Harrison and Kreps and more recent work, e.g., Scheinkman and Xiong (2003a, 2003b) and Scheinkman (2013), speculation in our model arises because traders have heterogeneous asset valuations that change over time: investors are willing to pay for the asset more than the present discounted value that they assign to the dividend stream, in anticipation of the capital gain they expect to obtain when reselling the asset to higher-valuation investors in the future. In terms of differences, in the work of Harrison and Kreps or Scheinkman and Xiong, traders have heterogeneous stubborn beliefs about the stochastic dividend process, and their motive for trading is that they all believe (at least some of them mistakenly) that by trading the asset they can profit at the expense of others. In our formulation, traders simply have stochastic heterogeneous valuations for the dividend, as in Duffie et al. (2005). Our model offers a new angle on the speculative premium embedded in the asset price, by showing how it depends on the underlying financial market structure and the prevailing monetary policy that jointly determine the likelihood and profitability of future resale opportunities. Through this mechanism, our theory can generate a positive correlation between trade volume and the size of speculative premia, a key stylized fact that the theory of Scheinkman and Xiong (2003b) also explains. In Lagos and Zhang (2015) we use a model similar to the one developed in this paper to explain the correlation between the real yield on stocks and the nominal yield on Treasury bonds at low frequencies—a well known puzzling empirical observation often referred to as the "Fed Model." In that paper we also show the model can exhibit rational expectations dynamic sunspot equilibria with recurring belief driven events that resemble liquidity crises, i.e., times of sharp persistent declines in asset prices, trade volume, and dealer participation in market-making activity, accompanied by large increases in spreads and abnormally long trading delays. Asriyan et al. (2017) also study dynamic sunspot equilibria in an environment where the value of the asset is determined by a resale value option as in Harrison and Kreps (1978), but their key mechanism emphasizes information frictions (adverse selection) rather than OTC-style search frictions.

Piazzesi and Schneider (2016) also emphasize the general idea that the cost of liquidity can affect asset prices. In their model, the cost of liquidity to end users depends on the cost of leverage to intermediaries, while our model and our empirical work instead center on the role of the nominal policy rate, which represents the cost of holding the nominal assets used routinely to settle financial transactions (e.g., bank reserves, real money balances).